2021 Summer Course

Optical Oceanography and Ocean Color Remote Sensing

Light and Radiometry

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Today

- Just what is light?

- Radiometry: How do you quantitatively describe and measure light?
  - How much light is there?
  - Where is it going?
  - What wavelength does it have?

This lecture will present the theory, the next lecture by Ken will discuss measurement.

References: The first chapter of *The Ocean Optics Web Book*, https://www.oceanopticsbook.info/view/light-and-radiometry/introduction
Chapter 1 of *The Ocean Optics Book*

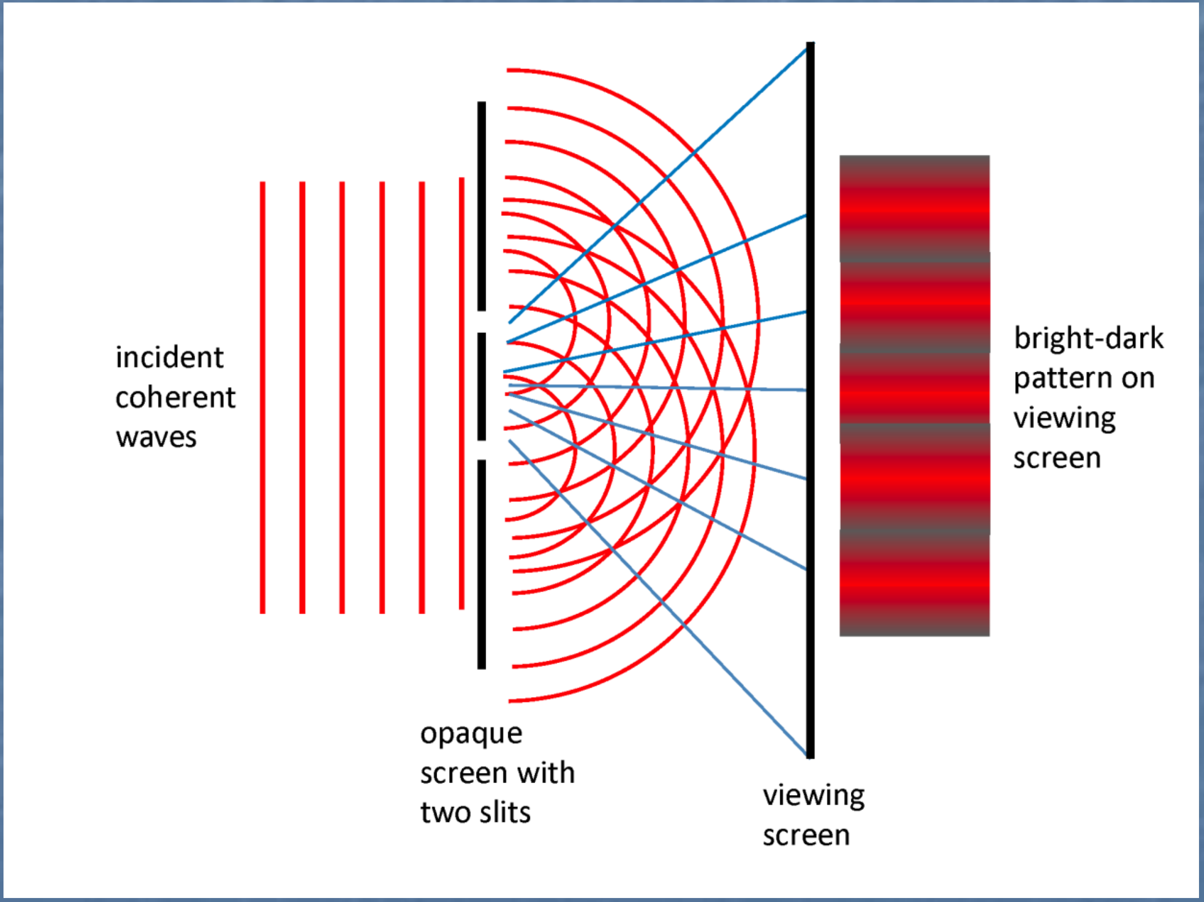
A Brief History of Light

Is light a particle, a wave, both, or neither?

- ancient Greece, Democritus: Everything is made of particles (atoms)
- 11th century, Alhazen: Light is rays of particles
- 1630s, Descartes: light is waves
- 1670s, Newton: light is particles ("corpuscles")
- same time: Huygens and Fresnel: light is waves
- 1803 Young: light is waves (double slit interference)
- late 1800's, Maxwell: light is propagating electric and magnetic fields; a wave
- 1905: Einstein: light is absorbed as discrete quanta (photoelectric effect)
- early-mid 1900's: quantum mechanics: light and matter have both particle and wave properties ("wave-particle duality")
- 1926: "photon" invented as the name for light quanta
- mid-late 1900s: quantum electrodynamics (QED): light is photon "particles," but very strange particles: photons cannot be localized; they take all possible paths from source to detector; they fill all of space between the source and detector, a single photon can interfere with itself (single-photon double-slit experiment)
- today, elementary particle theory: everything is particles, but all particles have wave properties

Young's Two-slit Experiment (1803)

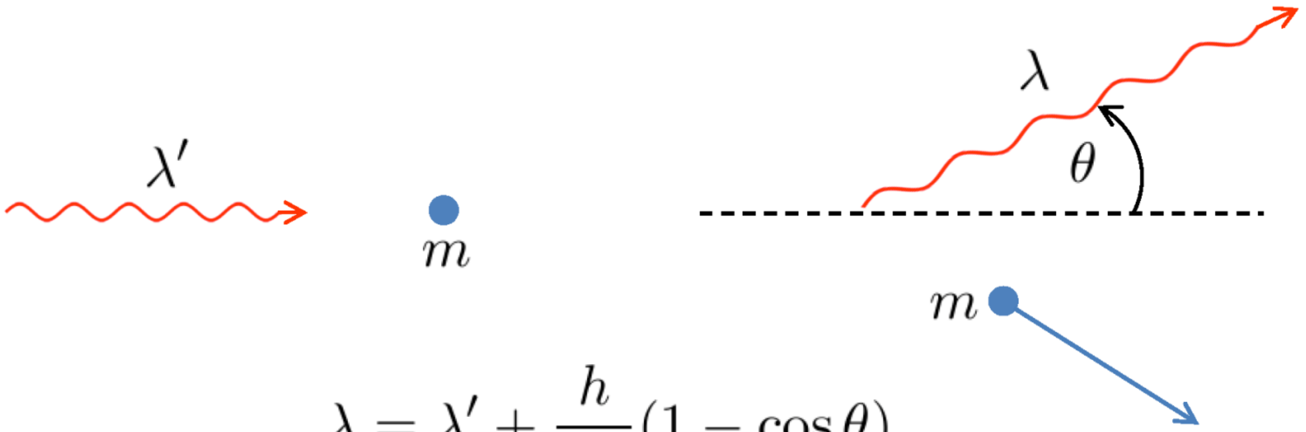
Interference is one "proof" that light is a wave



This result cannot be explained if light is "classical" particles

Compton Scattering of Light by Electrons (1923)

This result “proves” that light is a particle

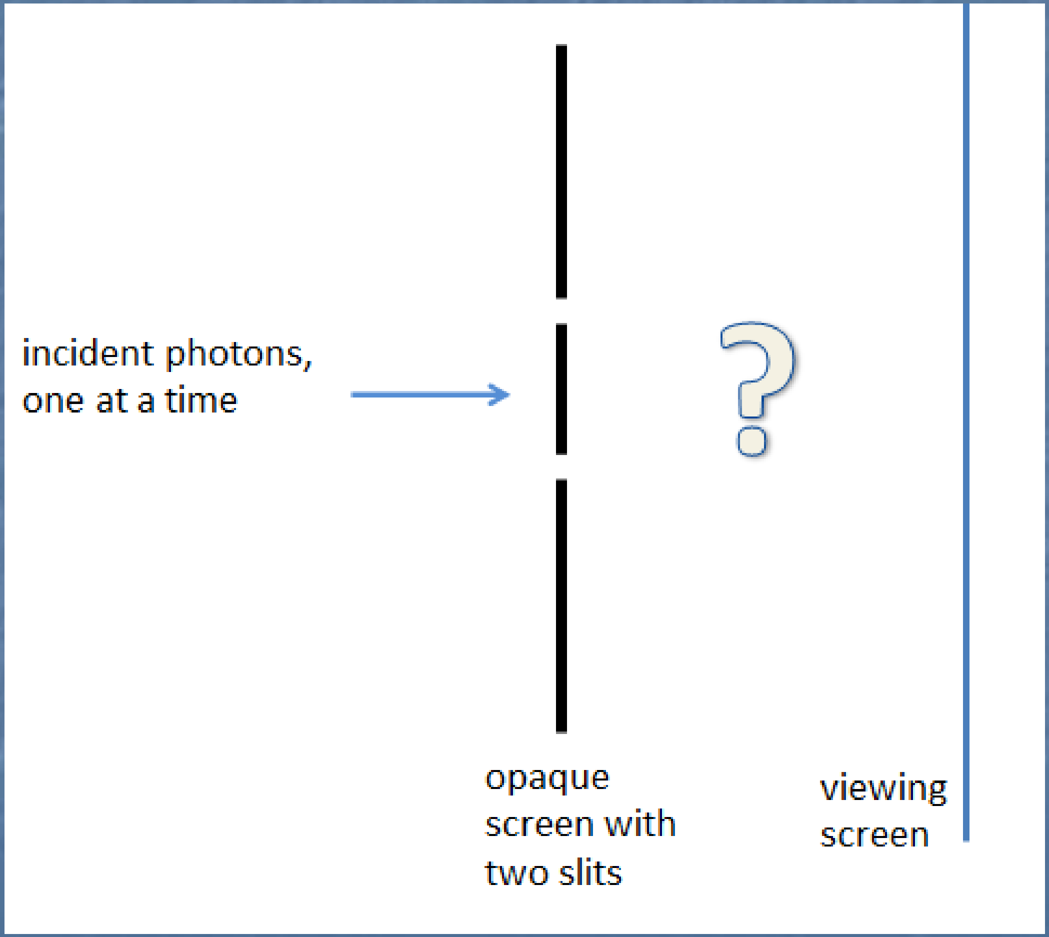


$$\lambda = \lambda' + \frac{h}{mc}(1 - \cos\theta)$$

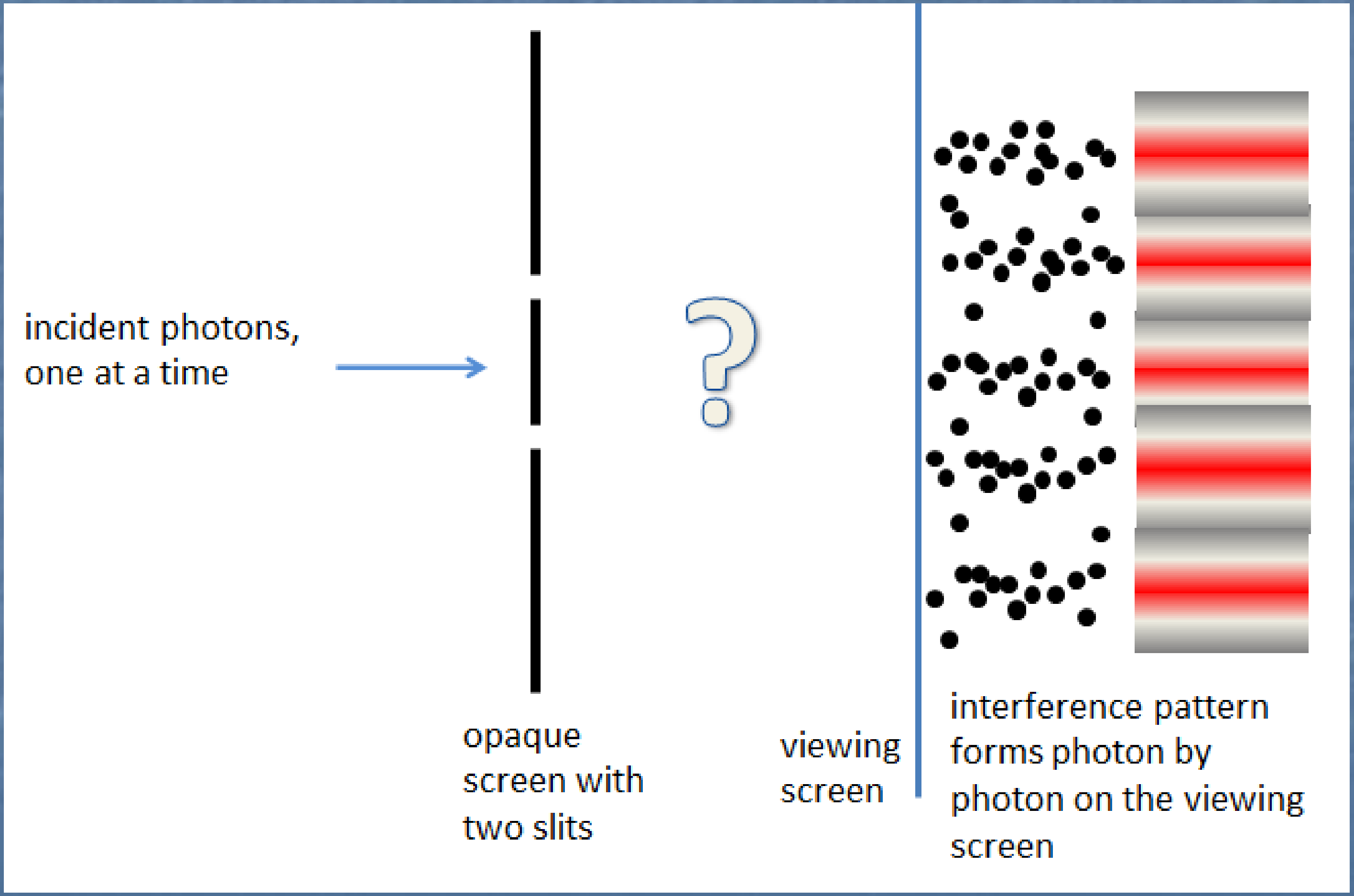
The formula is derived by treating the photon and the electron as particles and then applying the relativistic forms of conservation of energy and conservation of linear momentum. (1927 Nobel Prize in Physics.)

This result cannot be explained if light is “classical” waves

Young's Two-slit Experiment with Single Photons

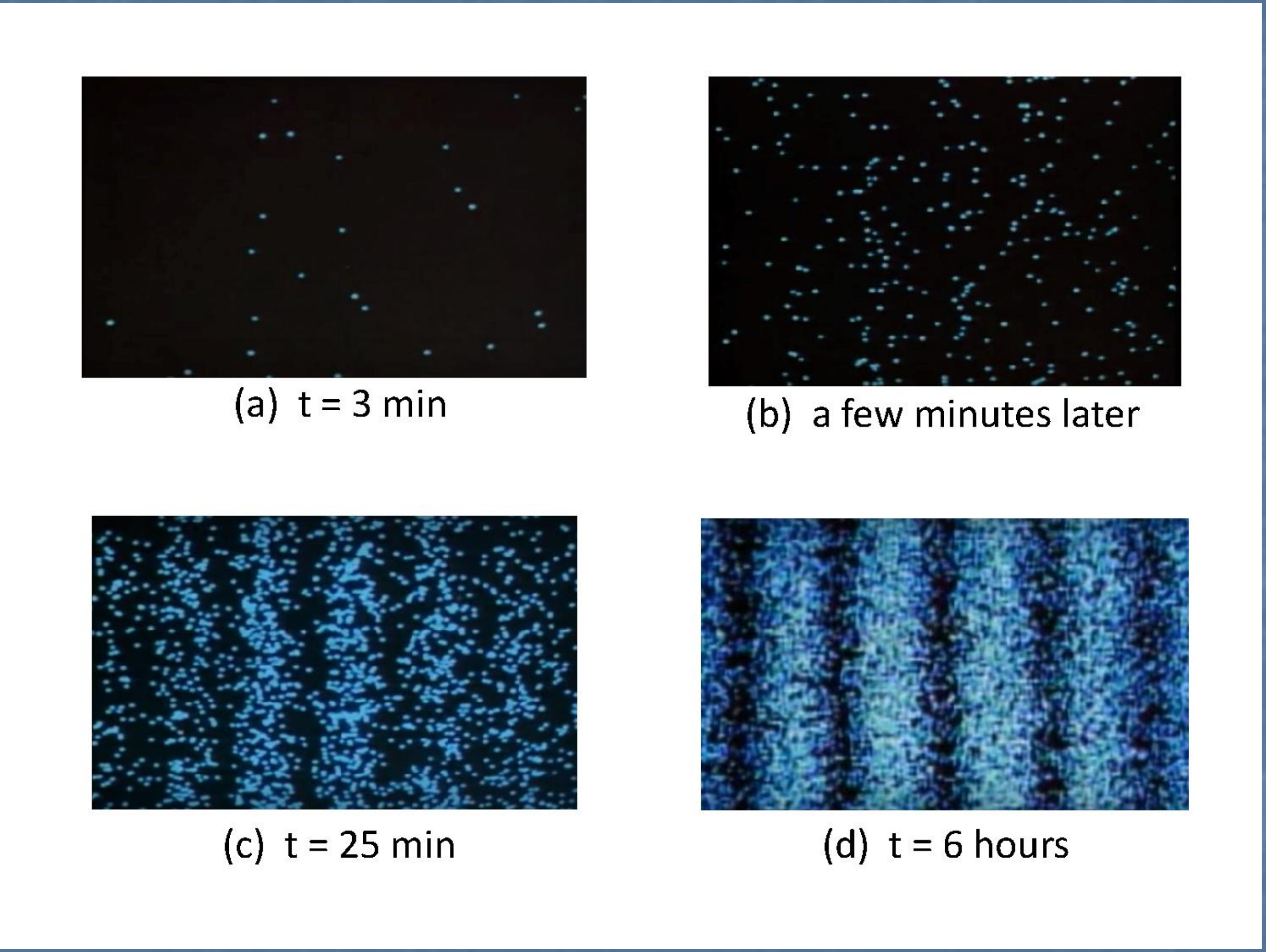


Interference by Single Photons



Interference by Single Photons

Single photon interference (from Hammatsu video)



https://www.youtube.com/watch?reload=9&v=I9Ab8BLW3kA

also in the Library as single-photon-interference.mp4

Interference by Single Photons

R. P. Feynman called the experimental result of interference by single photons “The Fundamental Mystery.” It has two profound consequences:

- *We are forced to abandon the idea that photons are localized particles* in the classical sense of having a well-defined (even small) size. A localized particle could not pass through both slits at the same time and then interfere with itself.

- *We are forced to abandon the idea that photons take a particular path from one point to another*. In QED calculations (using so-called Feynman path integrals), *a photon simultaneously takes all possible paths from one point to another*. Only after all of the calculations are done for all possible paths and the results for the different paths are combined does the final result look like the classical idea of a light ray traveling from one point to the next by a single path.

You can say that a photon was created at point A (e.g. an atom in the filament of a light bulb) and you can say is was absorbed at point B (e.g. by an atom in a particular pixel of a CCD array), but you can say *nothing* about the path it took from A to B.

Photon Properties

A photon is defined by its

- energy (or frequency or wavelength or linear momentum)
- angular momentum
- state of polarization

"position" and "path" and "time" are not defined and have no meaning for photons

Photons can NOT be "localized" like electrons or basketballs or other particles with a non-zero rest mass. You can say "it is 95% certain that there is an electron in the left half of this box." You can not say that for a photon. In quantum mechanics, there is no "position operator" for photons, which would give the probability of finding a photon at a certain location at a particular time.

"particle" and "wave" are idealized classical physics models for nature, but light is more complicated and behaves very, very strangely by human terms.

Thinking About Light

Feynman: “Nobody knows (what photons are), and it’s best if you try not to think about it.”

However, if you do want to think more deeply about photons and light:

- Start with the Web Book page on “The Nature of Light” at https://www.oceanopticsbook.info/view/light-and-radiometry/the-nature-of-light or Section 1.1 of the Ocean Optics Book

- Read *QED: The Strange Story of Light and Matter* by R. P. Feynman

- Watch the video single-photon-interference.mp4 in the Library (from https://youtu.be/I9Ab8BLW3kA)

Remember that much (perhaps most) of what is said about photons on websites and even in some physics textbooks is overly simplified, outdated, or just simply wrong. Never think of photons as little balls bouncing around in space.

All You Need to Know about Photons

q photon energy [J]

ν frequency [s^{-1}]

λ wavelength [m]

$h = 6.626 \cdot 10^{-34}$ J s Planck's constant

$c = 2.998 \cdot 10^{8}$ m s^{-1} speed of light (in a vacuum)

Photon energy is related to frequency and wavelength by

$$\boxed{q = h\nu = \frac{hc}{\lambda} \quad [J]}$$

The linear momentum is related to wavelength, energy, and frequency by

$$p = \frac{h}{\lambda} = \frac{q}{c} = \frac{h\nu}{c} \quad [\mathrm{kg\,m\,s^{-1}}]$$

The angular momentum is

$$\ell = \frac{h}{2\pi} \quad [\mathrm{kg\,m^2\,s^{-1}}]$$

Energy and linear momentum depend on frequency, but all photons have the same angular momentum (“spin 1 particles”)

Example Calculations

How many photons of visible light are there on a typical day at sea level?

From *Light and Water*, Table 1.4, typical day, 400-700 nm: 400 $\mathrm{W\,m^{-2}}$, so for $\lambda = 550$ nm $= 550 \cdot 10^{-9}$ m (green light)

$$\frac{400\ \mathrm{J\,s^{-1}\,m^{-2}}}{\frac{(6.63\cdot 10^{-34}\ \mathrm{J\,s})\,(3\cdot 10^{8}\ \mathrm{m\,s^{-1}})}{550\cdot 10^{-9}\ \mathrm{m}}} \approx 10^{21}\ \text{photons}\ \mathrm{m^{-2}\,s^{-1}}$$

How many photons are there per cubic meter in the water?

The speed of light in water is *c*/(index of refraction) ≈ *c*/1.34, so

$$\frac{10^{21}\ \text{photons}\ \mathrm{m^{-2}\,s^{-1}}}{\frac{3\cdot 10^{8}\ \mathrm{m\,s^{-1}}}{1.34}} \approx 4\cdot 10^{12}\ \frac{\text{photons}}{\mathrm{m^3}} \approx \frac{\text{number of phytoplankton}}{\mathrm{m^3}}$$

Each phytoplankton is being hit by photons many times per second.

Radiometry

Radiometry is the science of measuring electromagnetic (radiant) energy.

- How to specify directions
- How do you describe how much light there is, where it is going, etc.?

Radiance—the fundamental quantity for describing light: the starting point for radiative transfer theory

Irradiances—easier to measure than radiance and often more useful: the starting point for measurments

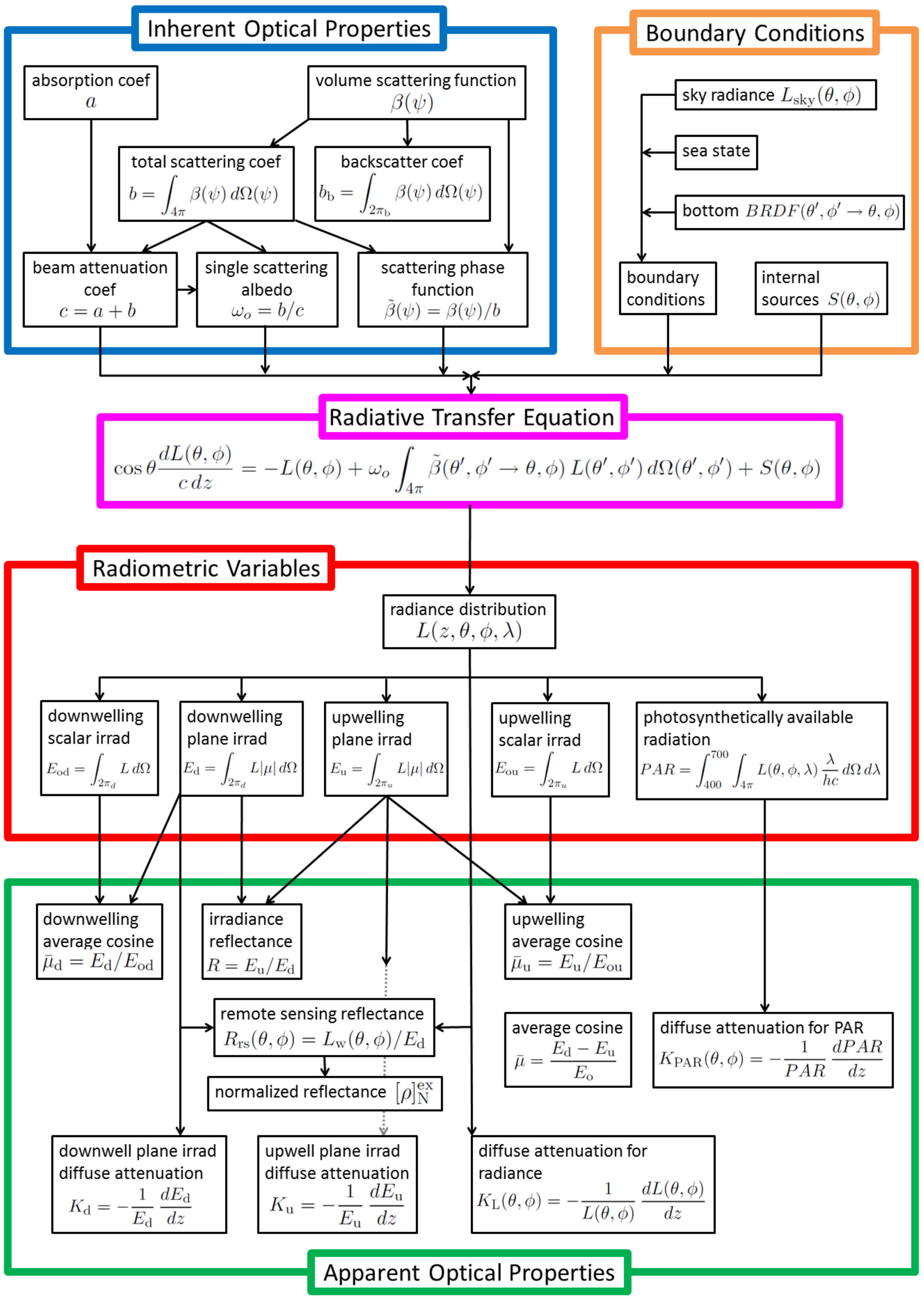
Radiometry

Two types of detectors:

• thermal—instrument response is proportional to the *energy* (e.g., absorbed and converted to heat)

• quantum—instrument response is proportional to the *number of photons* absorbed

Calibration of radiometric instruments is very difficult
(~2% accuracy)



unit vectors for direction

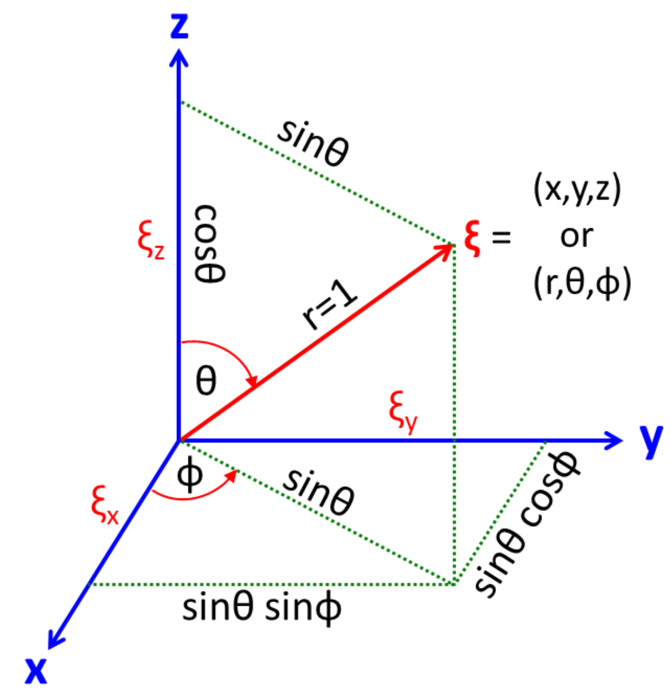
Specifing directions

Warning on directions:

In radiative transfer theory (i.e., in the radiative transfer equation, in *Light and Water*, and in HydroLight), θ and φ always refer to the direction the light is going.

Experimentalists often let θ and φ refer to the direction the instrument was pointed to measure the radiance.

I call the instrument direction the viewing direction, θ_v and φ_v, where $\theta_v = \pi - \theta$ and $\varphi_v = \pi + \varphi$.



$\hat{\xi}$ is a unit vector specifying direction (θ, ϕ)

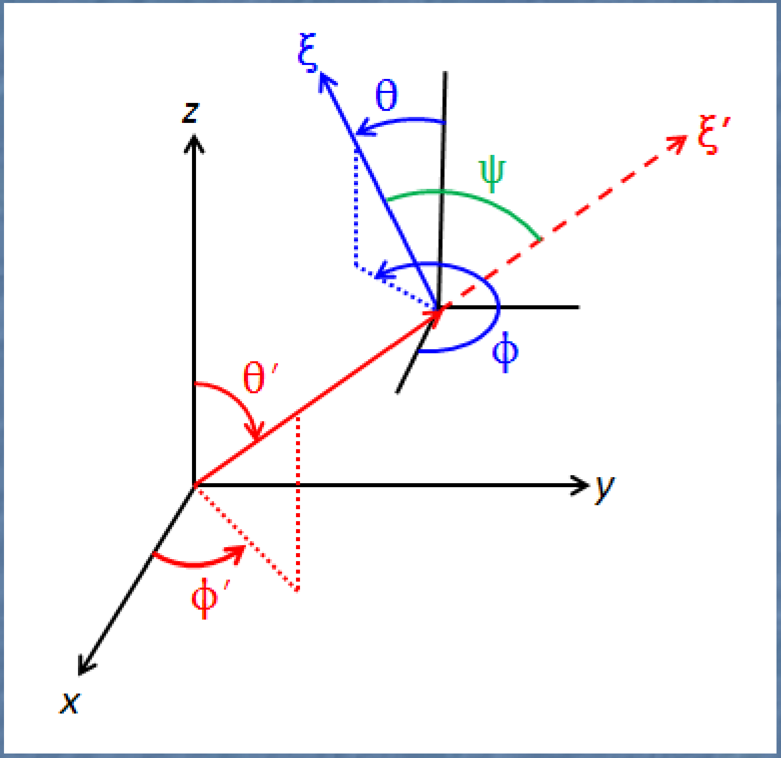
$$|\hat{\xi}| = 1 = \hat{\xi} \cdot \hat{\xi} = \xi_x^2 + \xi_y^2 + \xi_z^2$$

$$\begin{aligned} \hat{\xi} &= \xi_x \hat{x} + \xi_y \hat{y} + \xi_z \hat{z} \\ &= (\sin\theta\cos\phi)\hat{x} + (\sin\theta\sin\phi)\hat{y} + (\cos\theta)\hat{z} \end{aligned}$$

$$\theta = \cos^{-1}(\xi_z) \qquad \mu \equiv \cos\theta$$

$$\phi = \tan^{-1}\left(\frac{\xi_y}{\xi_x}\right)$$

Computing the (scattering) angle ψ between two directions



$$\hat{\xi}' \cdot \hat{\xi} \equiv |\hat{\xi}'||\hat{\xi}|\cos\psi = \cos\psi.$$

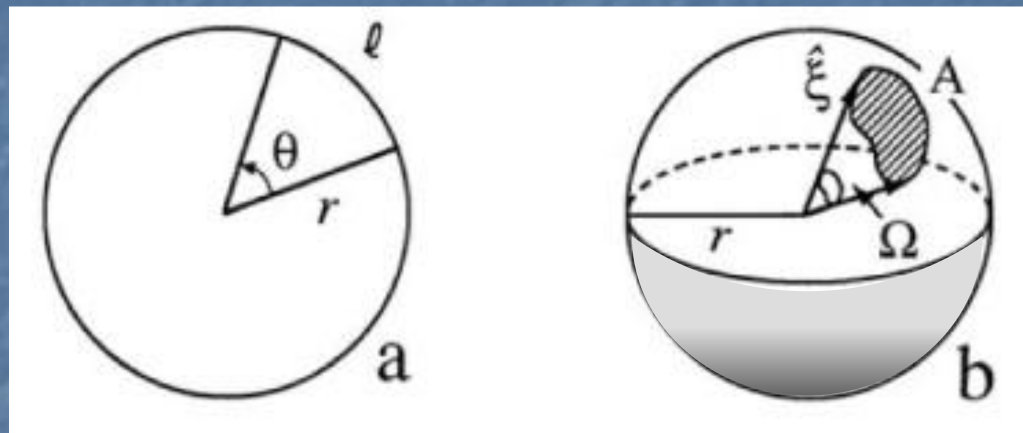
$$\begin{aligned}\hat{\xi} &= \xi_x \hat{x} + \xi_y \hat{y} + \xi_z \hat{z} \\ &= (\sin\theta\cos\phi)\hat{x} + (\sin\theta\sin\phi)\hat{y} + (\cos\theta)\hat{z}\end{aligned}$$

$$\begin{aligned}\cos\psi &= \xi'_x \xi_x + \xi'_y \xi_y + \xi'_z \xi_z \\ &= \cos\theta'\cos\theta + \sin\theta'\sin\theta\cos(\phi' - \phi) \\ &= \mu'\mu + \sqrt{1-\mu'^2}\sqrt{1-\mu^2}\cos(\phi' - \phi)\end{aligned}$$

My personal convention: primed variables are unscattered or incident directions; unprimed variables are scattered or final directions. θ is always polar angle; ψ is always scattering angle.

Defining Solid Angles

Solid angle is the 3D equivalent of 2D plane angle



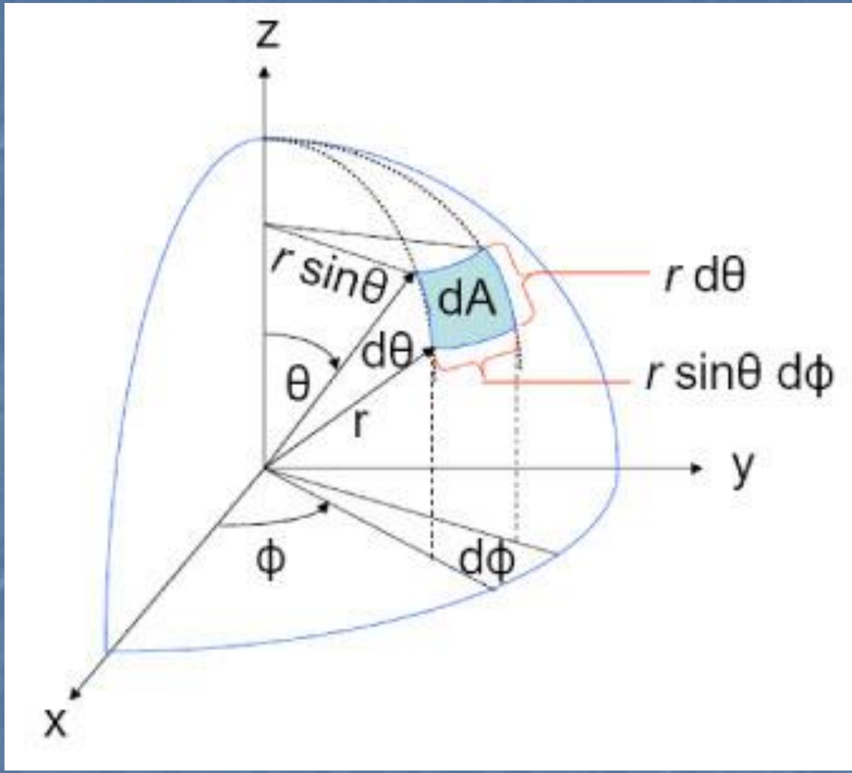
$$\begin{aligned} \text{angle} &\equiv \frac{\text{arc length}}{\text{radius}} \\ \theta &= \frac{\ell}{r} \quad (\text{radian}) \\ \text{circle} &= 2\pi \text{ rad} \end{aligned}$$

$$\begin{aligned} \text{solid angle} &\equiv \frac{\text{area}}{\text{radius squared}} \\ \Omega &= \frac{A}{r^2} \quad (\text{steradian}) \\ \text{sphere} &= 4\pi \text{ sr} \end{aligned}$$

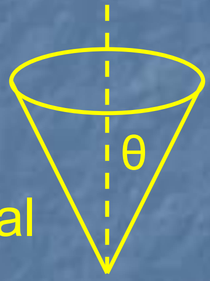
Area of the lower 48 states: 8.08×10^6 km^2
Radius of the Earth: 6384 km

Solid angle of the 48 states as seen from the center of the Earth:
$\Omega = 8.08 \times 10^6 \text{ km}^2 / (6384 \text{ km})^2 = 0.198$ sr

Computing Solid Angles



Example: What is the solid angle of a cone with half-angle θ?

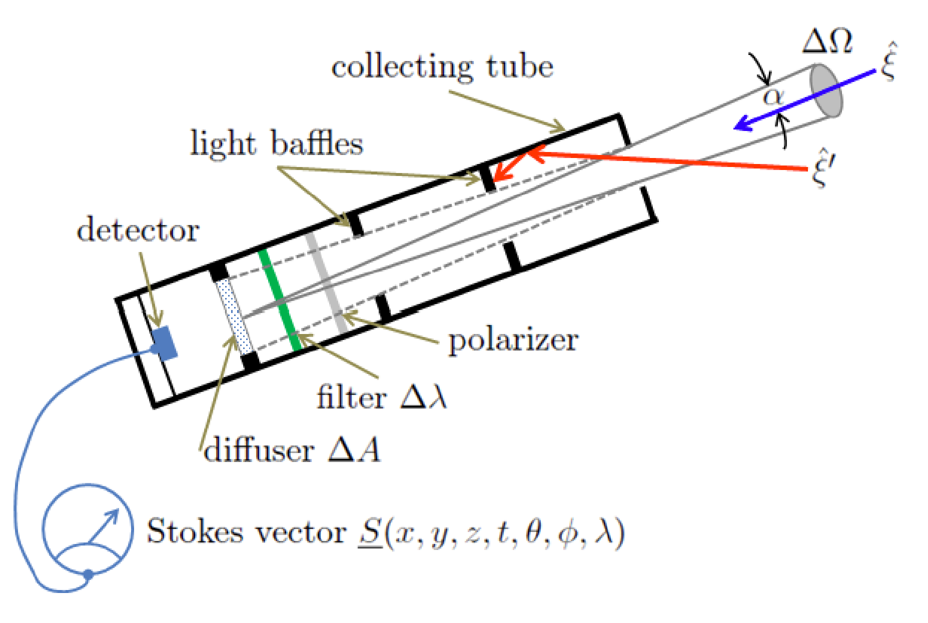


Place the cone pointing to the "north pole" of a spherical coordinate system.

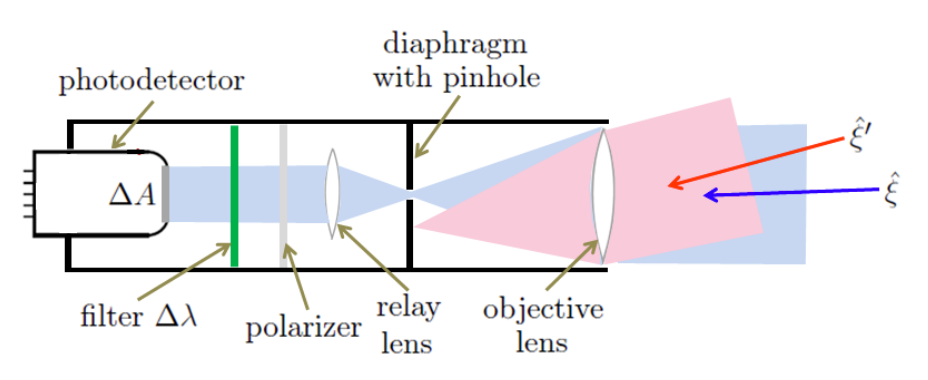
$$\Omega = \int_{\phi'=0}^{2\pi} \int_{\theta'=0}^{\theta} \sin\theta' d\theta' d\phi' = 2\pi(1-\cos\theta)$$
$$= \int_{\phi'=0}^{2\pi} \int_{\mu'=\mu}^{1} d\mu' d\phi' = 2\pi(1-\mu)$$

$$d\Omega = \frac{dA}{r^2} = \frac{(r sin\theta\, d\phi)(r d\theta)}{r^2} = \sin\theta\, d\theta\, d\phi$$
$$= d\mu d\phi$$

“Well Collimated Radiometers”



Traditional design
Gershun tube radiometer
(A. Gershun, 1930s)



Modern design

With polarizers you can measure the Stokes vector $\underline{S}$; without a polarizer, you measure the total radiance *L*, which is the first component of $\underline{S}$

Well Collimated Radiometers

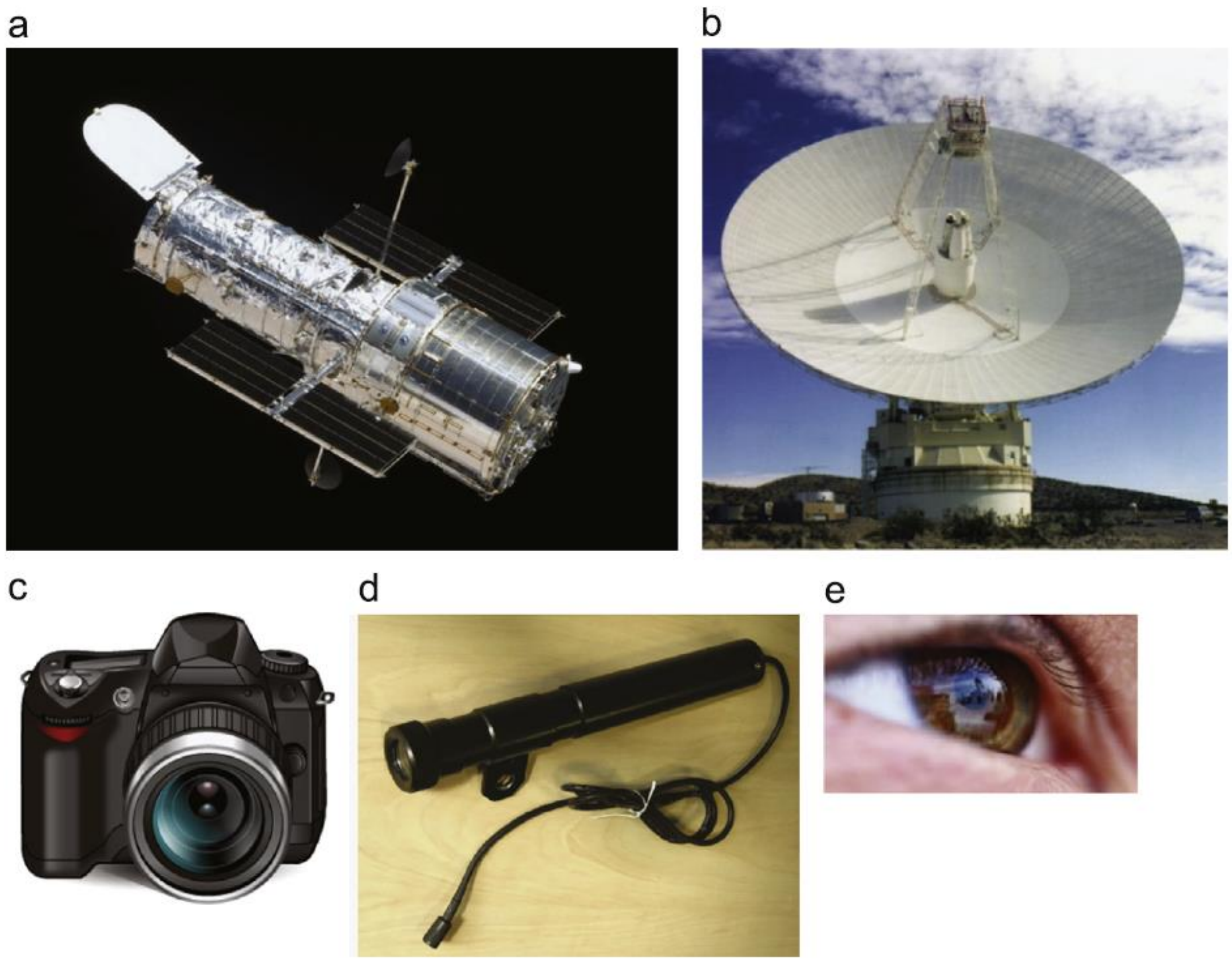


Fig. 15. (a) NASA's Hubble Space Telescope. (b) NASA's 64-m Goldstone radio telescope. (c) Digital photographic camera. (d) Gershun tube [120]. (e) Human eye.

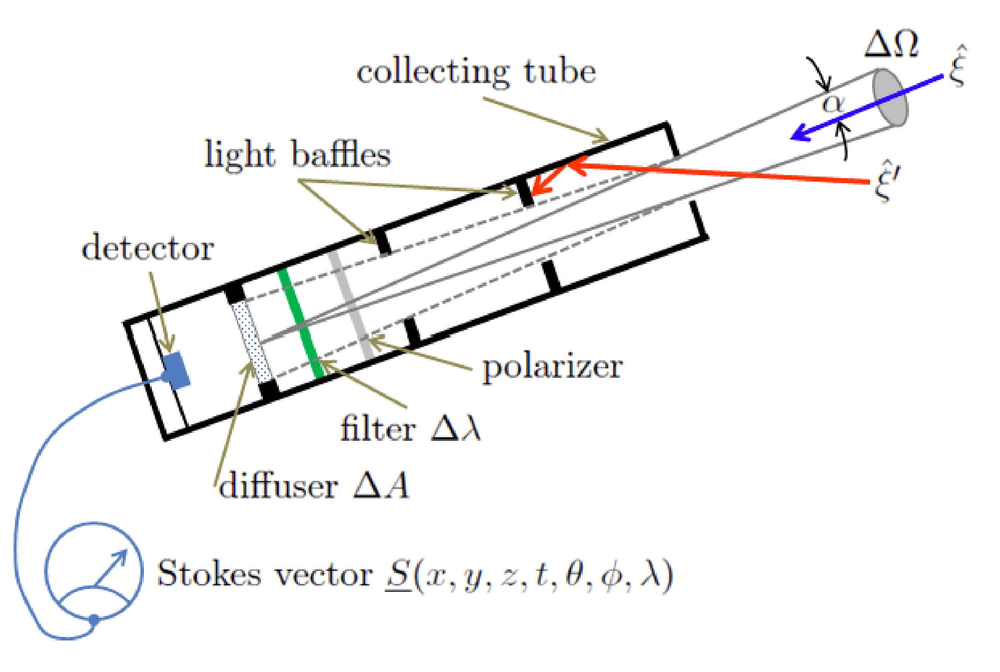
Mishchenko (2014)

Spectral Radiance

If you know the spectral radiance *L*, you know everything there is to know about the total light field (except for polarization)

Full specification of the radiance at a given location and time includes its state of polarization, wavelength, and direction of propagation

"spectral" can mean either "per unit of wavelength or frequency" or "as a function of wavelength"



$$L(\vec{x}, t, \hat{\xi}, \lambda) \equiv \frac{\Delta Q}{\Delta t \, \Delta A \, \Delta\Omega \, \Delta\lambda}$$

$$(\mathrm{J\ s^{-1}\ m^{-2}\ sr^{-1}\ nm^{-1}})$$

$$(\mathrm{W\ m^{-2}\ sr^{-1}\ nm^{-1}})$$

HydroLight computes $L(z,\theta,\varphi,\lambda)$

Polarization

To specify the polarization state of the radiance requires four numbers: the elements of the Stokes vector $\underline{S} = [I, Q, U, V]^T$

I is the total radiance, without regard for the state of polarizaiton. *I* is usually called the radiance *L* in oceanography.

Q, U, V describe the linear and circular polarization of the light.

We will see how this is done in Ken's Lecture 17

For now, we consider only the unpolarized radiance *L* (the first component *I* of the Stokes vector).

Definitions

Operational Definition: This is how a quantity is actually measured with an instrument.

$$L(\vec{x}, t, \hat{\xi}, \lambda) \equiv \frac{\Delta Q}{\Delta t\, \Delta A\, \Delta\Omega\, \Delta\lambda}$$

Conceptual Definition: This is what a theoretician uses. Conceptual definitions often involve limits or derivatives, which cannot be measured in practice. Example: the definition of radiance as used in radiative transfer theory:

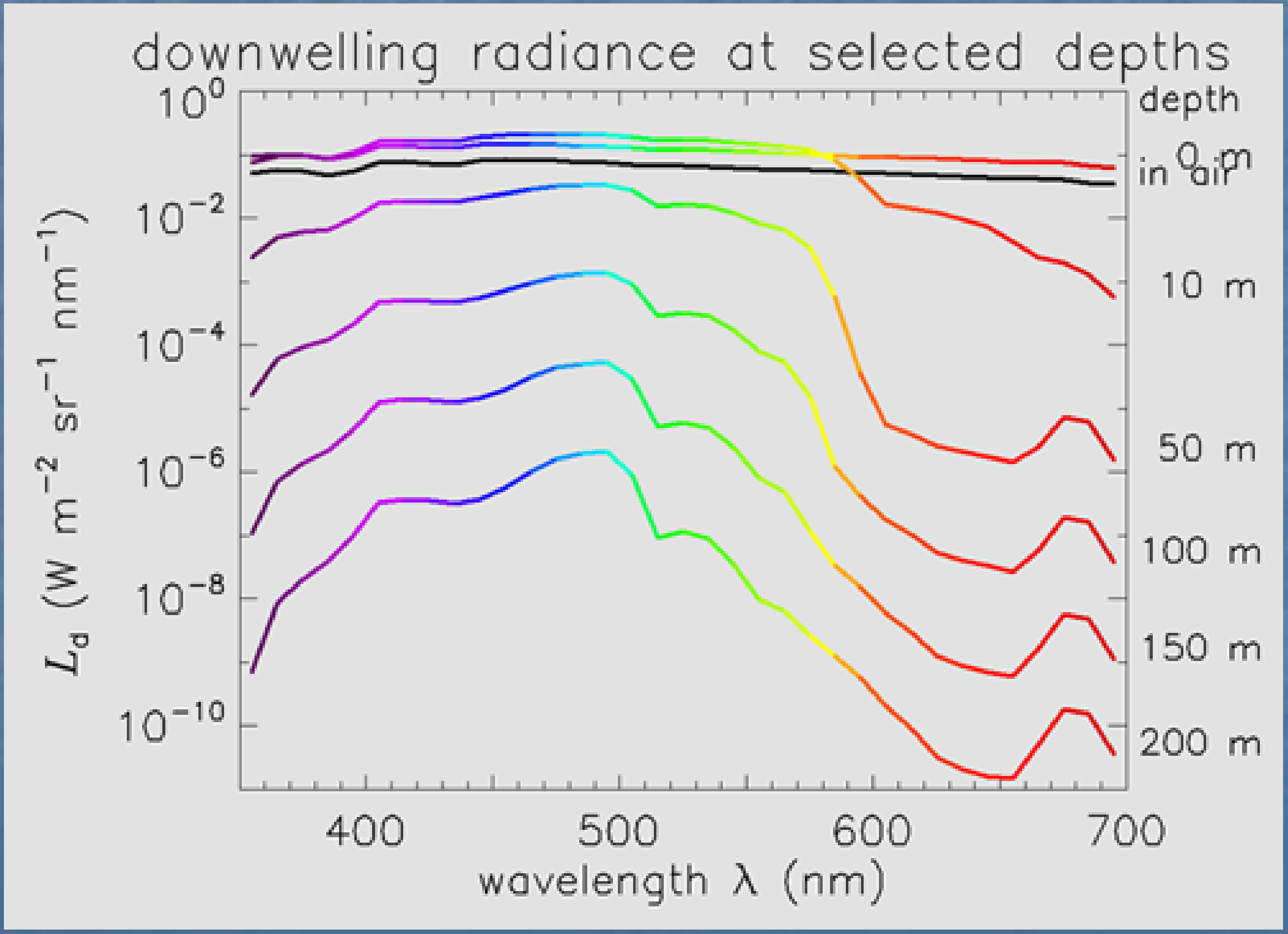
$$L(\vec{x}, t, \hat{\xi}, \lambda) \equiv \frac{\partial^4 Q}{\partial t\, \partial A\, \partial\Omega\, \partial\lambda}$$

then

$$Q = \int_{\Delta t} \int_{\Delta A} \int_{\Delta\Omega} \int_{\Delta\lambda} L(\vec{x}, t, \hat{\xi}, \lambda)\, d\lambda\, d\Omega\, dA\, dt$$

Radiance is always hard to plot because of so many variables

Example plot: Radiance $L(z,\theta,\varphi,\lambda)$ as a function of z and λ for the zenith-viewing direction (the downwelling radiance L_d: light traveling straight down, detector pointed straight up)



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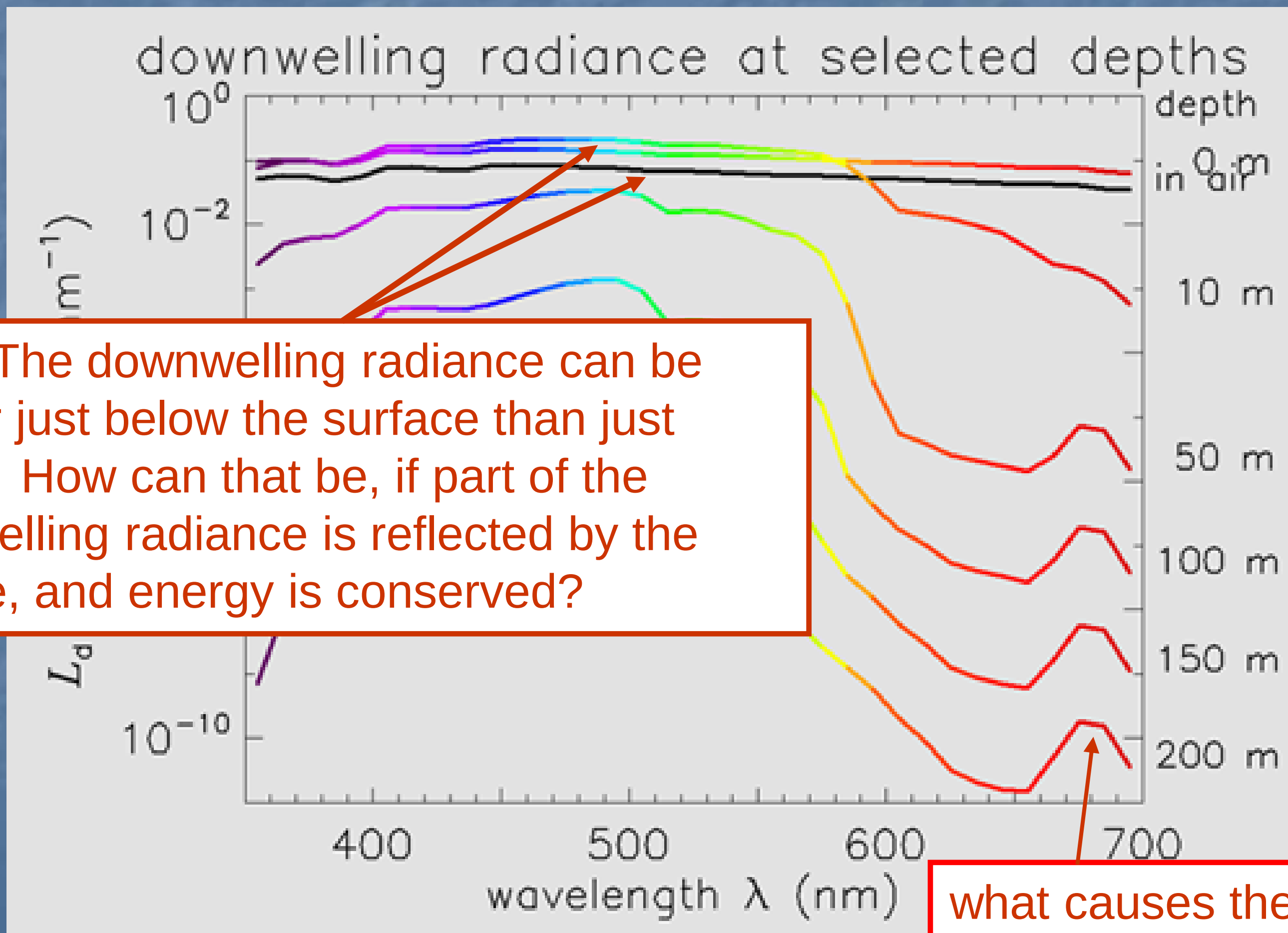
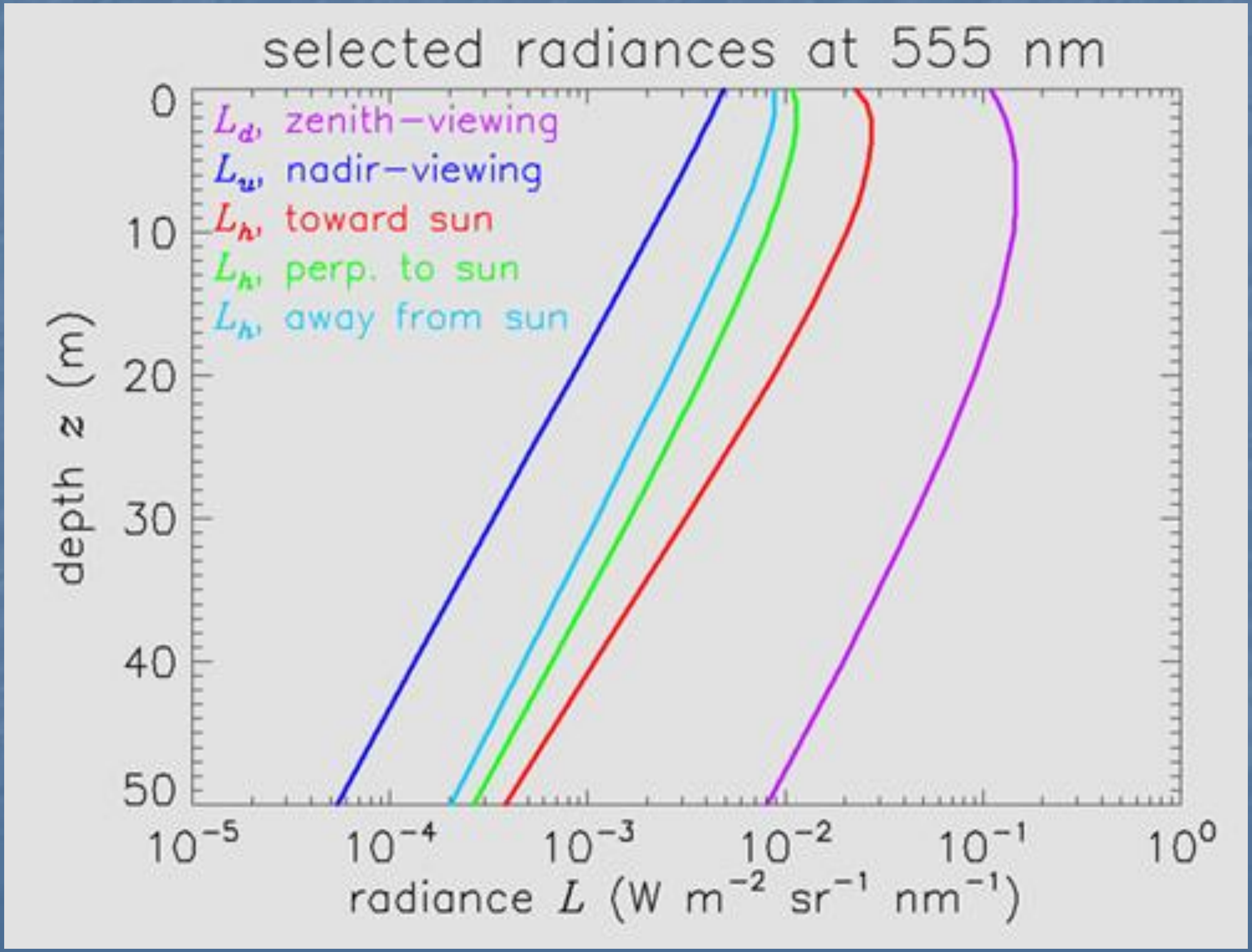


Note: The downwelling radiance can be greater just below the surface than just above. How can that be, if part of the downwelling radiance is reflected by the surface, and energy is conserved?

what causes these bumps?

Example plot: Radiance $L(z,\theta,\varphi,\lambda)$ as a function of z and selected directions for one wavelength, λ = 555 nm.



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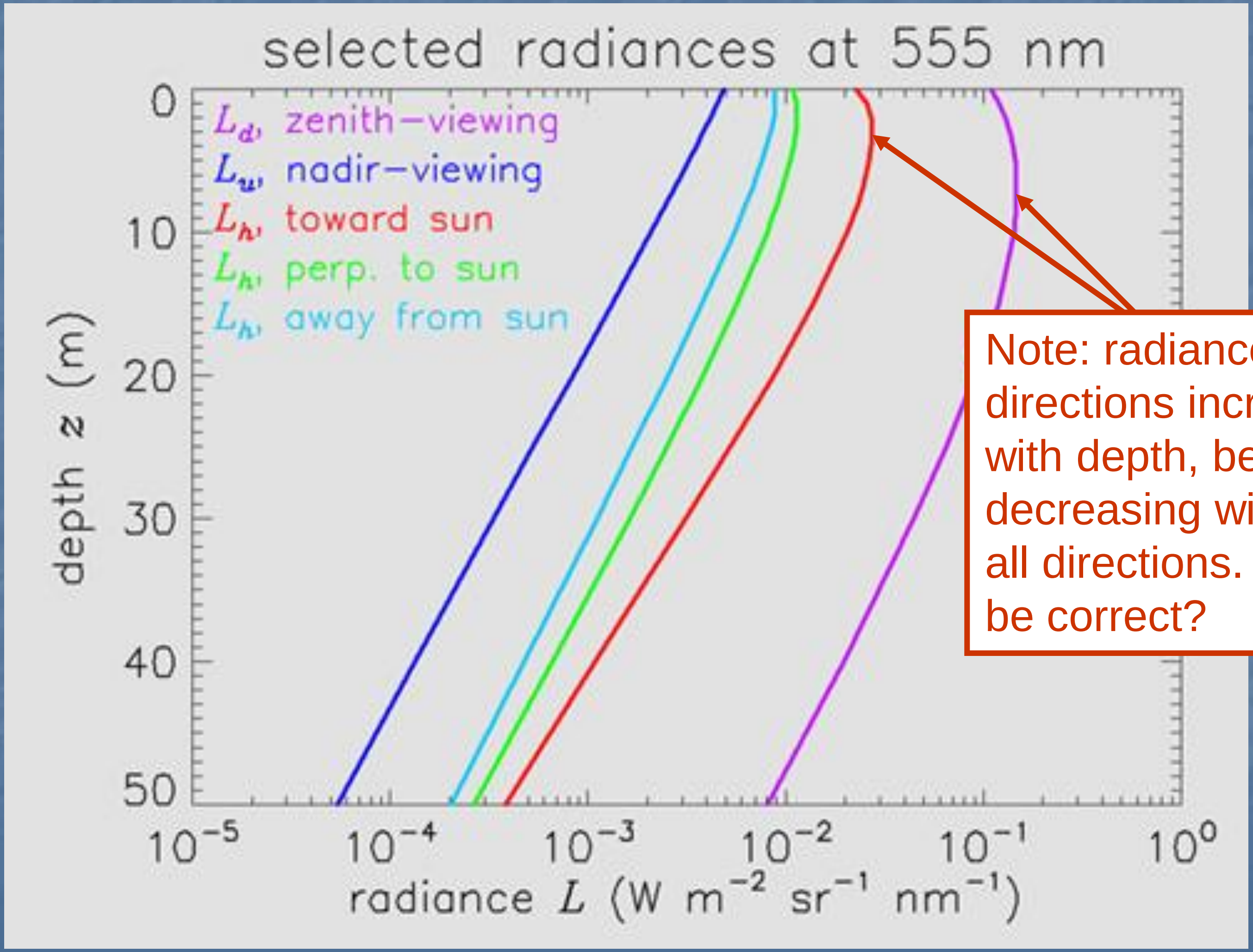
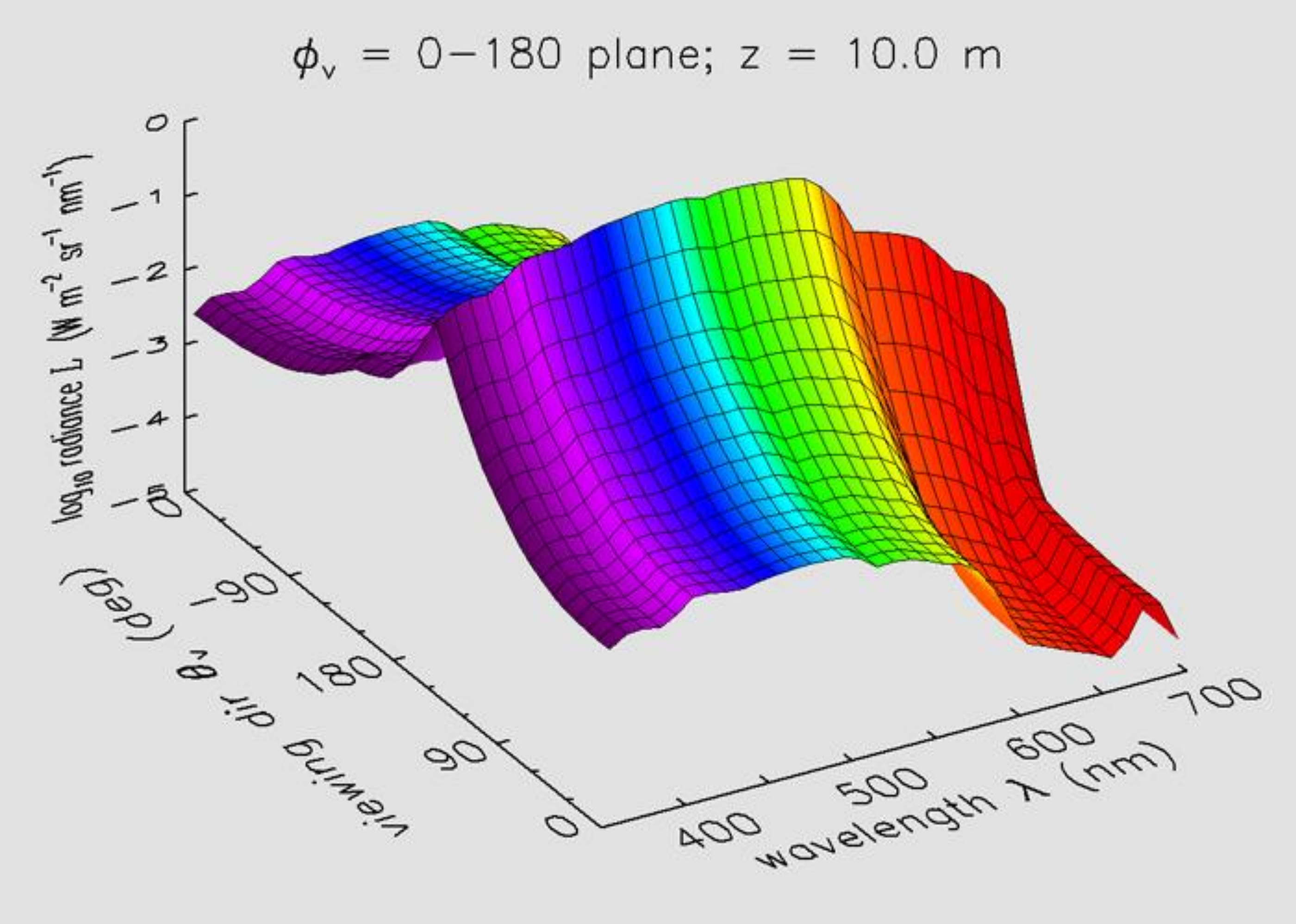


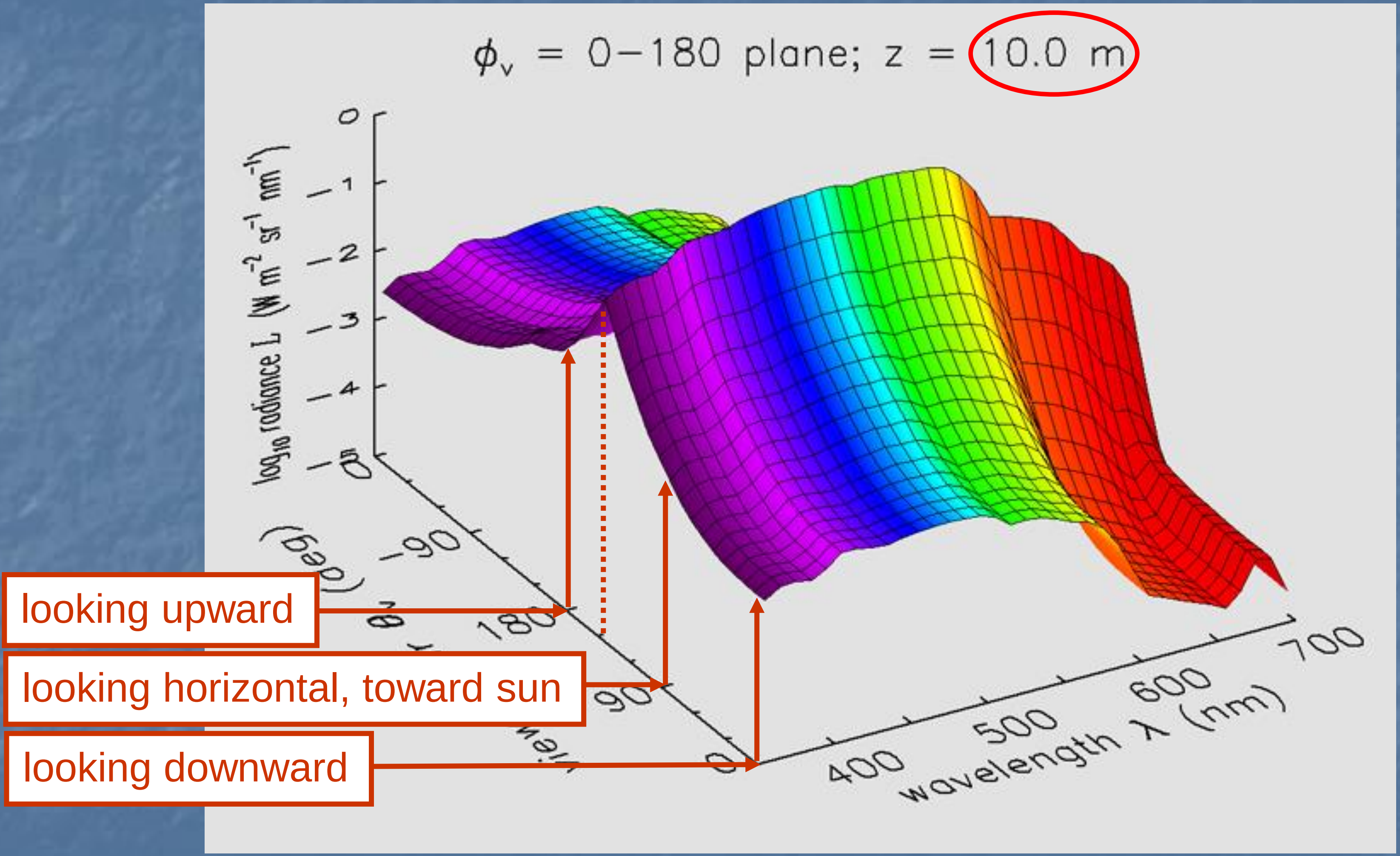
Note: radiance in some directions increases with depth, before decreasing with depth in all directions. Can this be correct?

Example plot: Radiance $L(z,\theta,\varphi,\lambda)$ as a function of polar angle θ and wavelength λ, for depth z = 10 m and φ in the plane of the sun



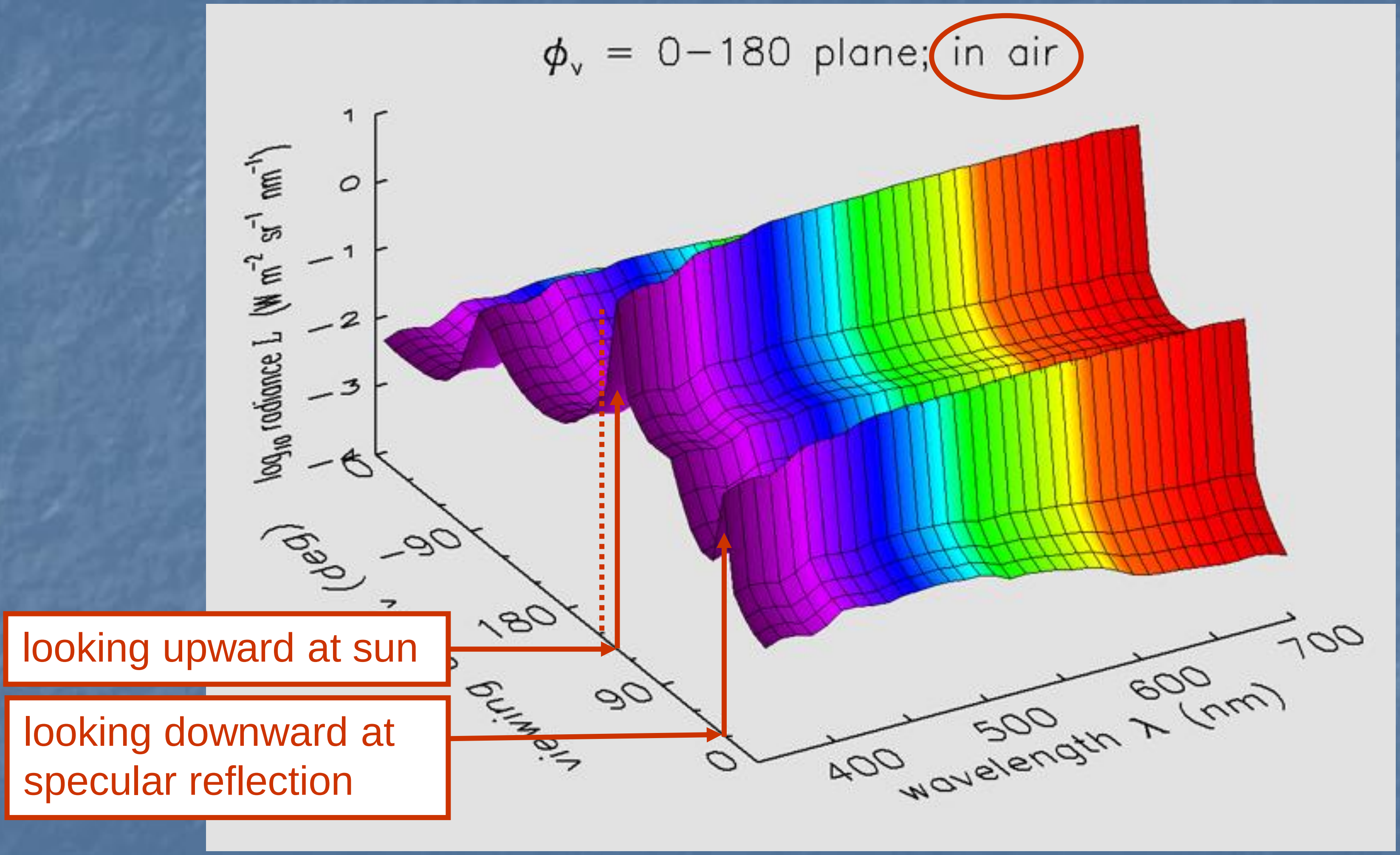
Note: $+z$ is downward, so θ = 0 is light heading straight down, viewed by looking straight up in the θ_v = 180 deg direction.

Example plot: Radiance $L(z,\theta,\varphi,\lambda)$ as a function of polar angle θ and wavelength λ, for depth z = 10 m and φ in the plane of the sun



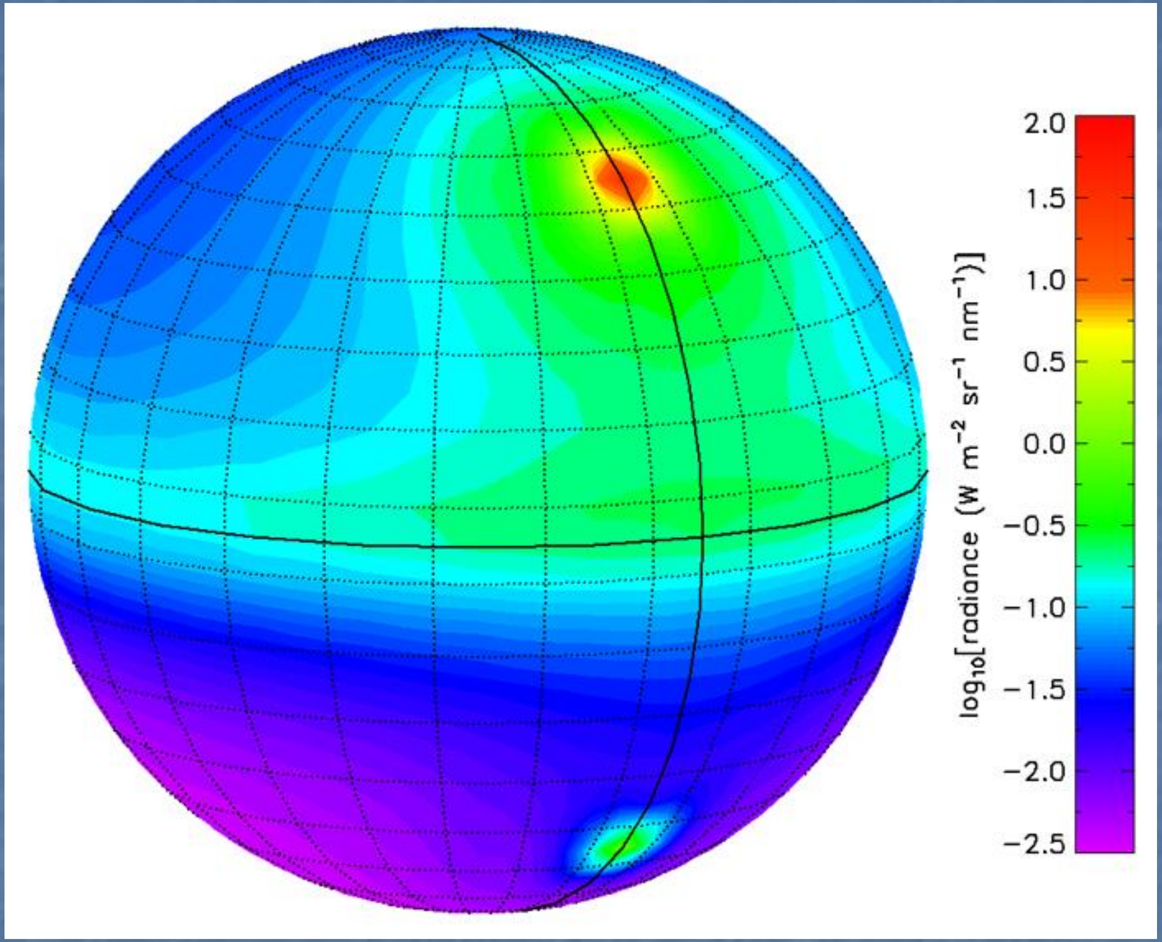
Note: $+z$ is downward, so θ = 0 is light heading straight down, viewed by looking straight up in the θ_v = 180 deg direction.

Example plot: Radiance $L(z,\theta,\varphi,\lambda)$ as a function of polar angle θ and wavelength λ, just above the sea surface and φ in the plane of the sun



Note: $+z$ is downward, so θ = 0 is light heading straight down, viewed by looking straight up in the θ_v = 180 deg direction.

Example plot: Radiance $L(z,\theta,\varphi,\lambda)$ just above the sea surface as a function of θ and φ for $\lambda = 555$ nm.



See https://www.oceanopticsbook.info/view/light-and-radiometry/visualizing-radiances for more discussion of these plots.

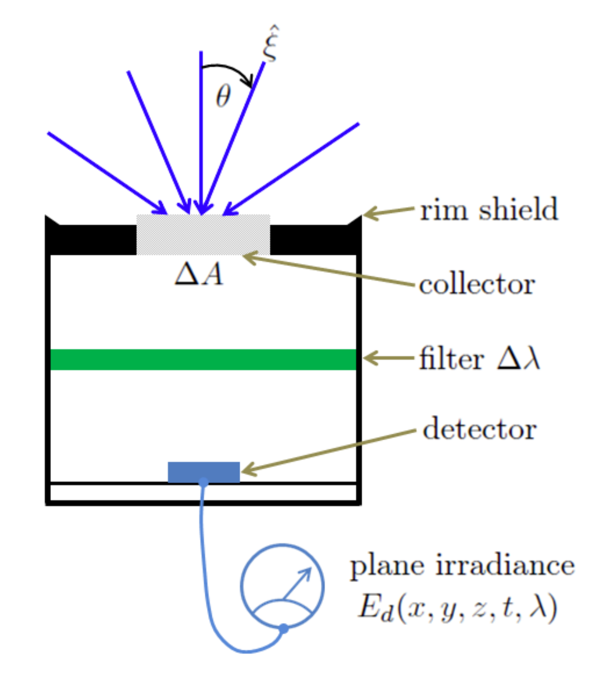
Spectral Plane Irradiance

The most commonly measured radiometric variable

The collector *surface* is equally sensitive to light from any direction.

However, the effective (projected) area of the detector as "seen" by light in direction θ is ΔA cos(θ).

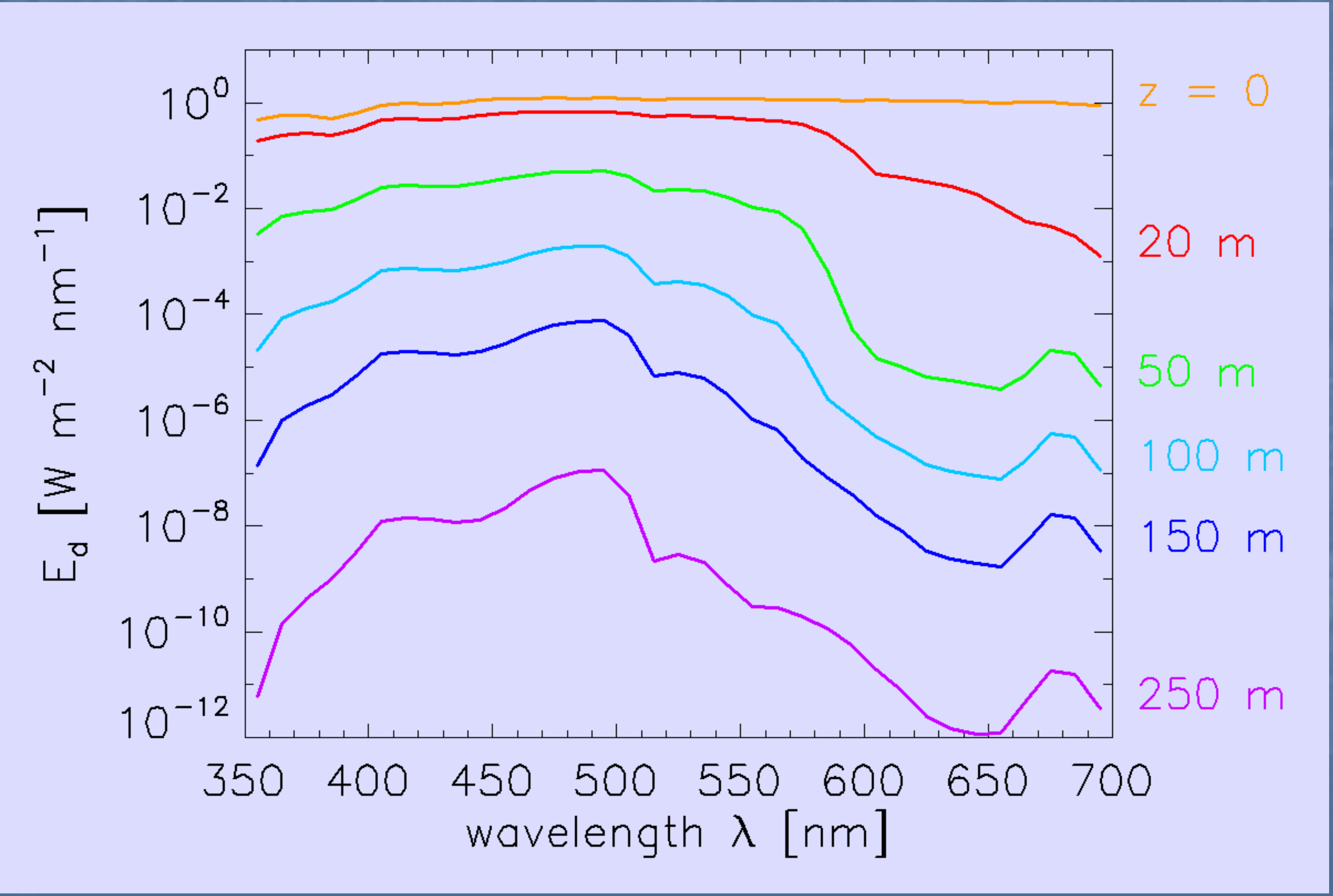
So must weight the radiance by cos(θ) when computed from *L*



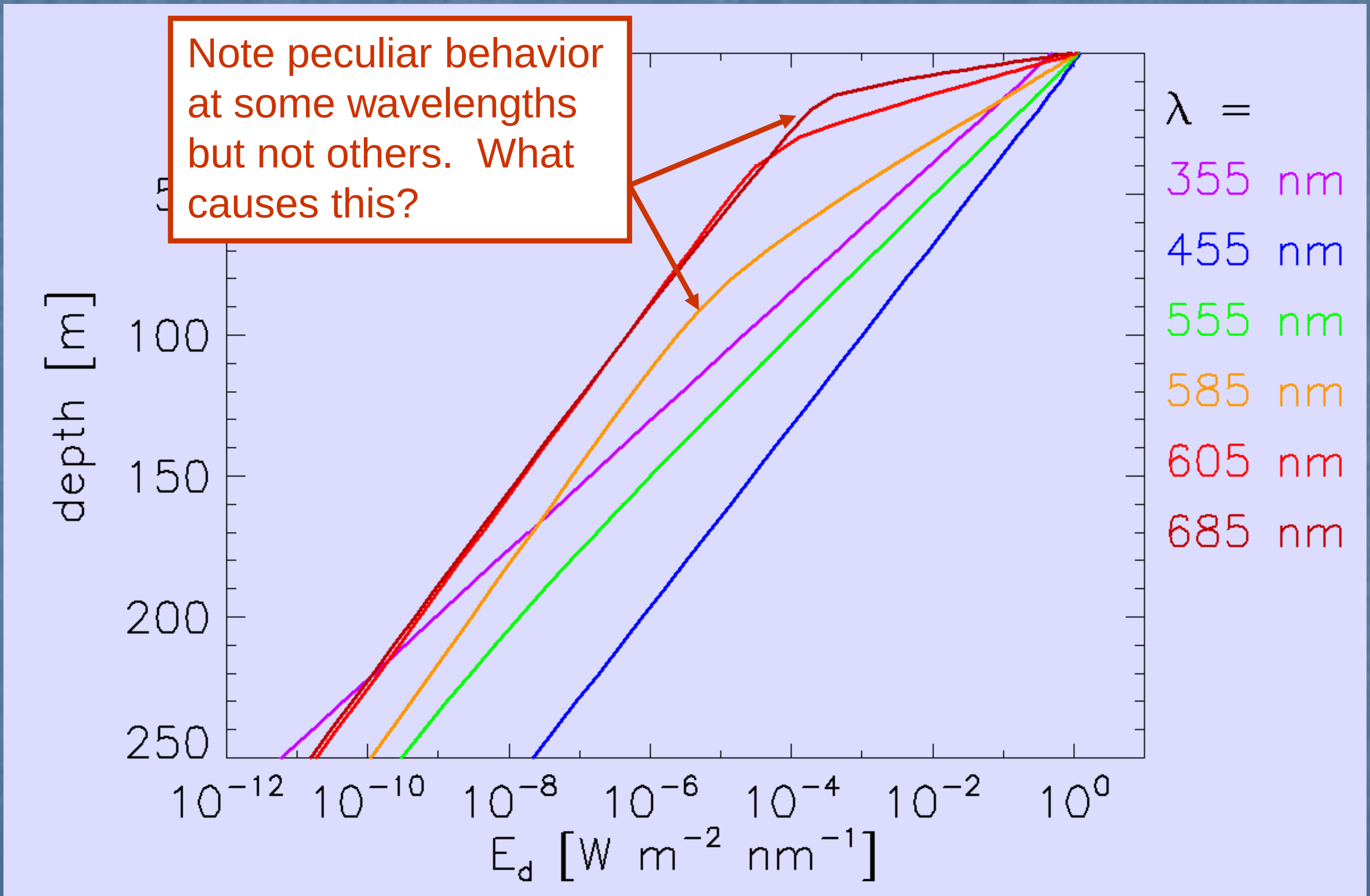
$$E_d(\vec{x}, t, \lambda) \equiv \frac{\Delta Q}{\Delta t \, \Delta A \, \Delta \lambda} \quad (\mathrm{W\ m^{-2}\ nm^{-1}})$$

$$E_d(\vec{x}, t, \lambda) = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi/2} L(\vec{x}, t, \theta, \phi, \lambda) |\cos\theta| \sin\theta d\theta \, d\phi$$

Example plot: E_d as a function of wavelength for selected depths

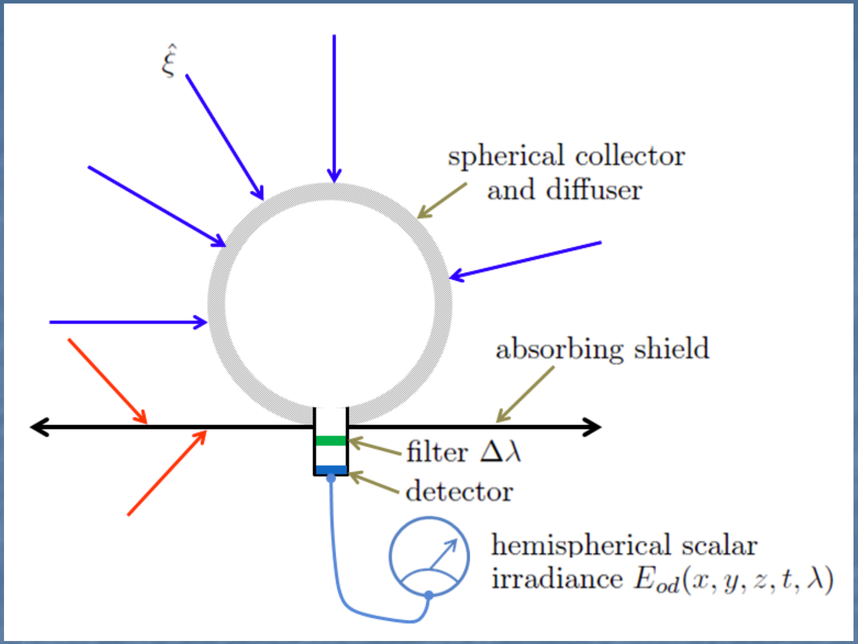


Example plot: E_d as a function of depth for selected wavelengths



Spectral Scalar Irradiance

The radiometric variable that is most relevant to photosynthesis and water heating because those processes are independent of the direction the light is traveling



The detector has the same effective area for radiance in any downward direction, so no cos(θ) factor on *L*

$$E_{od}(\vec{x}, t, \lambda) \equiv \frac{\Delta Q}{\Delta t \, \Delta A \, \Delta\lambda} \quad (\mathrm{W\ m^{-2}\ nm^{-1}})$$

$$E_{od}(\vec{x}, t, \lambda) = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi/2} L(\vec{x}, t, \theta, \phi, \lambda) \sin\theta \, d\theta \, d\phi$$

$$E_o(\vec{x}, t, \lambda) = E_{od}(\vec{x}, t, \lambda) + E_{ou}(\vec{x}, t, \lambda)$$

Spectral Vector Irradiance

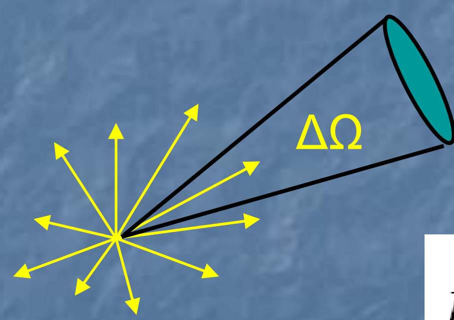
can be related to absorption by Gershun's law (derived later; see the Web Book):

$$
\begin{aligned}
(\vec{E})_z &= \hat{z} \cdot \vec{E} \\
&= \int_{\Xi} L(\vec{x}, t, \hat{\xi}, \lambda) \cos\theta \, d\Omega(\hat{\xi}) \\
&= \int_{\theta=0}^{90} L(\ldots\theta\ldots) \cos\theta \, d\Omega \; + \; \int_{\theta=90}^{180} L(\ldots\theta\ldots) \cos\theta \, d\Omega \\
&= E_d - E_u
\end{aligned}
$$

$a = - (1/E_o)\, d(E_d – E_u)/dz$

$E_z = E_{net} = E_d – E_u$ is the *net downward irradiance*

Spectral Intensity



useful for describing *point* light sources

$$
I(\vec{x}, t, \hat{\xi}, \lambda) = \frac{\Delta Q}{\Delta t \, \Delta\Omega \, \Delta\lambda} \quad (\mathrm{W \ sr^{-1} \ nm^{-1}})
$$

"Intensity" is NOT a synonym for radiance (Palmer, 1993)

Photosynthetically Available Radiation (PAR)

Historically used in simple models for phytoplankton growth

More sophisticated ecosystem models today use the spectra scalar irradiance $E_o(\lambda)$ because different phytoplankton pigments absorb light differently at different wavelengths.

RADIOMETRY uses ENERGY units

PHOTOSYNTHESIS depends on the NUMBER on photons absorbed. The convenient measure of how many photons are available for photosynthesis is

$$PAR \equiv \int_{400\,\mathrm{nm}}^{700\,\mathrm{nm}} E_o(\lambda)\,\frac{\lambda}{hc}\,d\lambda$$

$$(\mathrm{photons\ s^{-1}\ m^{-2}})$$

PAR is often expressed as micro Einsteins s^{-1} m^{-2}

1 Einstein = 1 mole of photons
= 6.023 x 10^{23} photons

Warnings on Terminology

In atmospheric optics, spectral radiance is called "specific intensity" and irradiance is called "flux". Some people call irradiance "flux" and some call irradiance "flux density". Other fields (medical optics, astrophysics, the paint industry, etc.) have their own terminology and notation. It is very confusing.

Spectral vs band-integrated radiance and irradiance:

Spectral downwelling plane irradiance $E_d(\lambda)$ is per unit wavelength interval, with units of W m^{-2} nm^{-1}

Band-integrated downwelling plane irradiance is the spectral irradiance integrated over some finite wavelength band, with units of W m^{-2}, e.g.,

$$E_d = \int_{410}^{420} E_d(\lambda)\, d\lambda$$

It is often not easy to figure out exactly what is being measured or discussed in a paper. **Units and magnitudes matter!** I reject papers that are not clear or have inconsistent or wrong units.

Sea Kayaking in Panama, Feb 2012