

UNIVERSITY  
OF MIAMI



# Polarization in Sky and water

At least an hours worth of it

Ken Voss, Ocean Optics Summer class, 2021

Polarization important as another parameter of the light field you can measure for additional information....or it may be a problem in your measurement. Simplest light field....plane wave

Can describe plane wave propagating in z direction as:

$$E_l(z,t) = a_l \cos(\omega t - kz + \delta_l)$$

$$E_r(z,t) = a_r \cos(\omega t - kz + \delta_r)$$

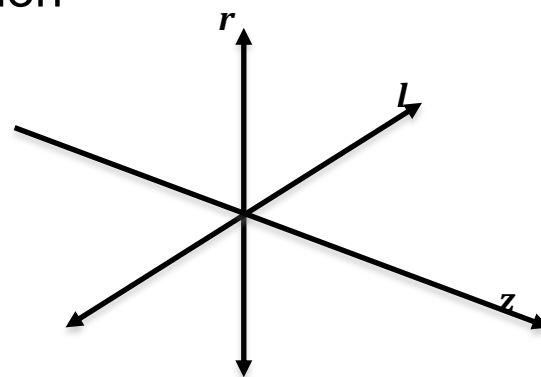
Let:  $\delta = \delta_r - \delta_l$

Some simple cases can be seen:

$\delta=0$ , (or  $a_l$  or  $a_r=0$ ) light is linearly polarized

$\delta=\pi/2$ ,  $a_l = a_r$  light is circularly polarized

Everything else is called elliptically polarized



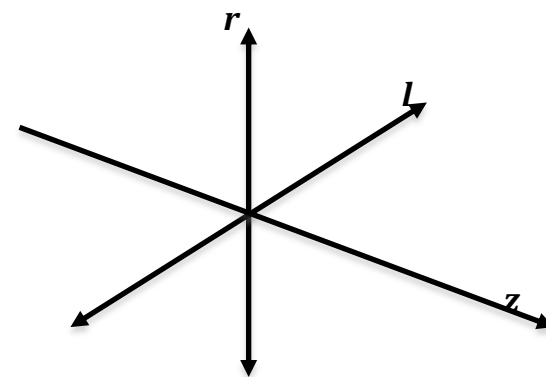
Now in general,  $\omega$ , for visible light is on the order of  $6 \times 10^{14}$  Hz, in general, we can't really measure these oscillations directly. Also the natural light field is not a single plane wave, but a superposition of many waves. So we need a more convenient description...the Stokes vector.

$$I = \langle a_l^2 \rangle + \langle a_r^2 \rangle = \langle E_l E_l^* \rangle + \langle E_r E_r^* \rangle = I_l + I_r$$

$$Q = \langle a_l^2 \rangle - \langle a_r^2 \rangle = \langle E_l E_l^* \rangle - \langle E_r E_r^* \rangle = I_l - I_r$$

$$U = \langle 2a_l a_r \cos \delta \rangle = \langle E_l E_r^* \rangle + \langle E_r E_l^* \rangle = I_{45} - I_{-45}$$

$$V = \langle 2a_l a_r \sin \delta \rangle = i(\langle E_l E_r^* \rangle - \langle E_r E_l^* \rangle) = I_{RCP} - I_{LCP}$$



## Some Simple Stokes Vectors:

$$\begin{bmatrix} I \\ Q \\ U \\ V \end{bmatrix} = \begin{bmatrix} I \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Unpolarized light

$$\begin{bmatrix} I \\ Q \\ U \\ V \end{bmatrix} = \begin{bmatrix} I \\ I \\ 0 \\ 0 \end{bmatrix}$$

Polarized along  $l$

$$\begin{bmatrix} I \\ Q \\ U \\ V \end{bmatrix} = \begin{bmatrix} I \\ -I \\ 0 \\ 0 \end{bmatrix}$$

Polarized along  $r$

$$\begin{bmatrix} I \\ Q \\ U \\ V \end{bmatrix} = \begin{bmatrix} I \\ 0 \\ 0 \\ I \end{bmatrix}$$

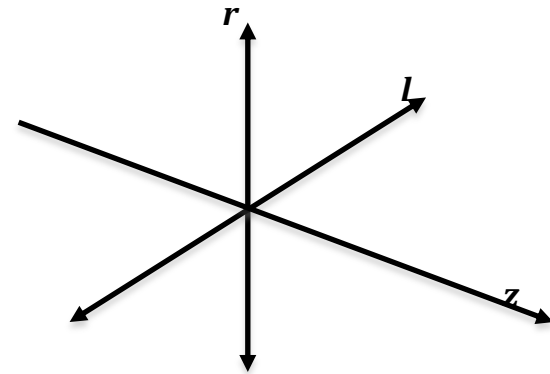
Right Circularly  
polarized light

$$I = I_l + I_r$$

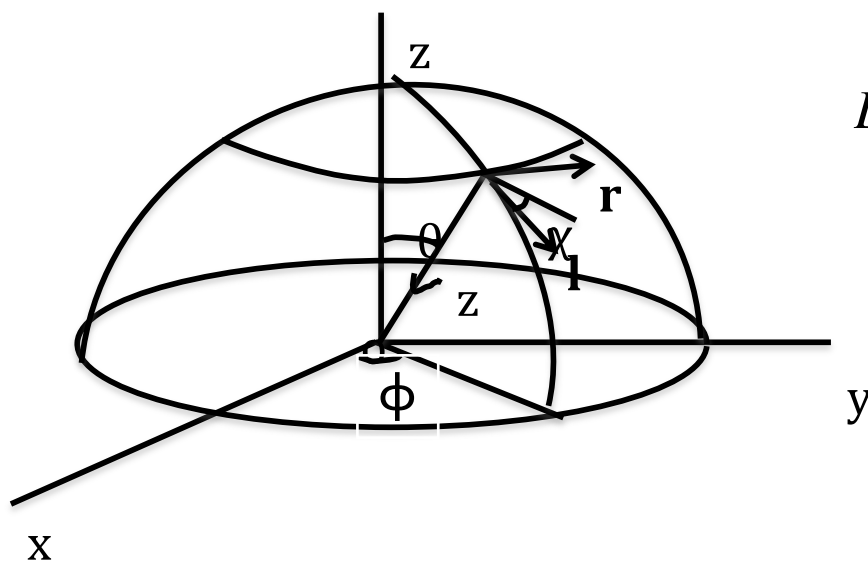
$$Q = I_l - I_r$$

$$U = I_{45} - I_{-45}$$

$$V = I_{RCP} - I_{LCP}$$



Can't figure out where to put this slide...so here it is....



Degree of polarization

$$DoP = \frac{(Q^2 + U^2 + V^2)^{1/2}}{I}, \quad 0 \leq DoP \leq 1$$

Plane of polarization

$$\tan 2\chi = \frac{U}{Q}, \quad -90^\circ \leq \chi \leq 90^\circ$$

For completeness, ellipticity

$$\sin 2\beta = \frac{2a_l a_r \sin \delta}{a_l^2 + a_r^2} = \frac{V}{(Q^2 + U^2 + V^2)^{1/2}}$$

Can transform these Stokes vectors in a linear process, using Mueller Matrices:

$$\begin{bmatrix} I' \\ Q' \\ U' \\ V' \end{bmatrix} = \begin{bmatrix} M_{11} & M_{12} & M_{13} & M_{14} \\ M_{21} & M_{22} & M_{23} & M_{24} \\ M_{31} & M_{32} & M_{33} & M_{34} \\ M_{41} & M_{42} & M_{43} & M_{44} \end{bmatrix} \begin{bmatrix} I \\ Q \\ U \\ V \end{bmatrix}$$

Modify incoming Stokes Vector to outgoing Stokes Vector.

Examples, linear polarizer,  $\theta$ , is angle between reference plane and polarizer axis of acceptance.

$$\frac{1}{2} \begin{bmatrix} 1 & \cos 2\theta & \sin 2\theta & 0 \\ \cos 2\theta & \cos^2 2\theta & \sin 2\theta \cos 2\theta & 0 \\ \sin 2\theta & \sin 2\theta \cos 2\theta & \sin^2 2\theta & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\frac{I+Q}{2} \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} I+Q \\ I+Q \\ 0 \\ 0 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} I \\ Q \\ U \\ V \end{bmatrix}$$

# Two crossed polarizers

$$\begin{bmatrix} I' \\ Q' \\ U' \\ V' \end{bmatrix} = 1/2 \begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} 1/2 \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} I \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$0 = 1/4 \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = 1/4 \begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} I \\ I \\ 0 \\ 0 \end{bmatrix}$$



# Two crossed polarizers, with another one in between

$$\frac{1}{2} \begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \frac{1}{2} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \frac{1}{2} \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\frac{1}{8} \begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} I \\ I \\ 0 \\ 0 \end{bmatrix}$$

Two crossed polarizers, with  
another one in between continued

$$= 1/8 \begin{bmatrix} I \\ -I \\ 0 \\ 0 \end{bmatrix} = 1/8 \begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} I \\ 0 \\ I \\ 0 \end{bmatrix}$$

# How about a circular polarizer?

- Need a quarter wave plate after a polarizer

$$\begin{bmatrix} I' \\ Q' \\ U' \\ V' \end{bmatrix} = 1/2 \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix} 1/2 \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} I \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} I' \\ Q' \\ U' \\ V' \end{bmatrix} = 1/4 \begin{bmatrix} I \\ 0 \\ 0 \\ I \end{bmatrix} = 1/4 \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} I \\ 0 \\ I \\ 0 \end{bmatrix}$$

To determine the Stokes vectors, need at least 4 carefully determined measurements. To determine Mueller matrix, need 16 measurements:

$$\begin{bmatrix} I' \\ Q' \\ U' \\ V' \end{bmatrix} = \begin{bmatrix} M_{11} & M_{12} & M_{13} & M_{14} \\ M_{21} & M_{22} & M_{23} & M_{24} \\ M_{31} & M_{32} & M_{33} & M_{34} \\ M_{41} & M_{42} & M_{43} & M_{44} \end{bmatrix} \begin{bmatrix} I \\ Q \\ U \\ V \end{bmatrix}$$

Typically, normalize by M11 (which is the normal volume scattering function). This emphasizes the polarization properties of the matrix.

Instruments to measure the Stokes Vector are variations on radiance instruments.

Two types, either single direction at a time (such as 4 co-bore sighted Gershun tubes or 3, since the circular polarization is very small or zero for the most part) or some sort of imaging device, such as fisheye camera with polarizers.

## Example Sky instrument

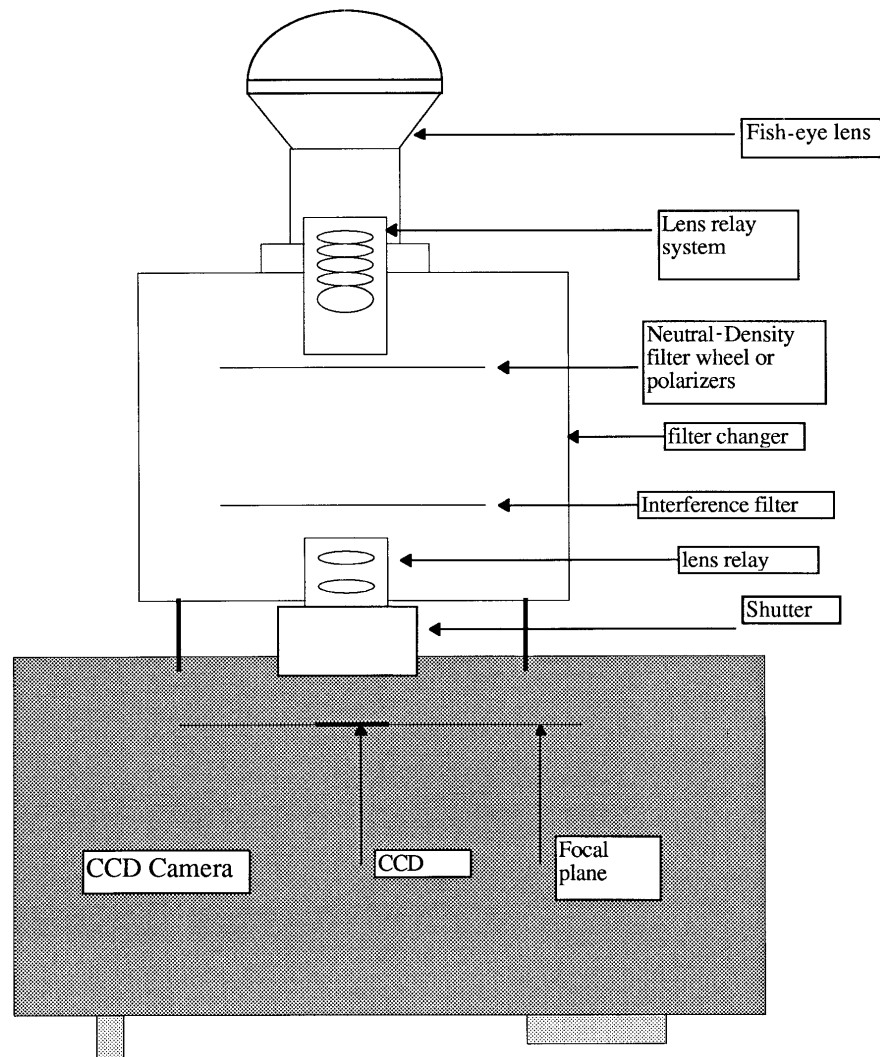
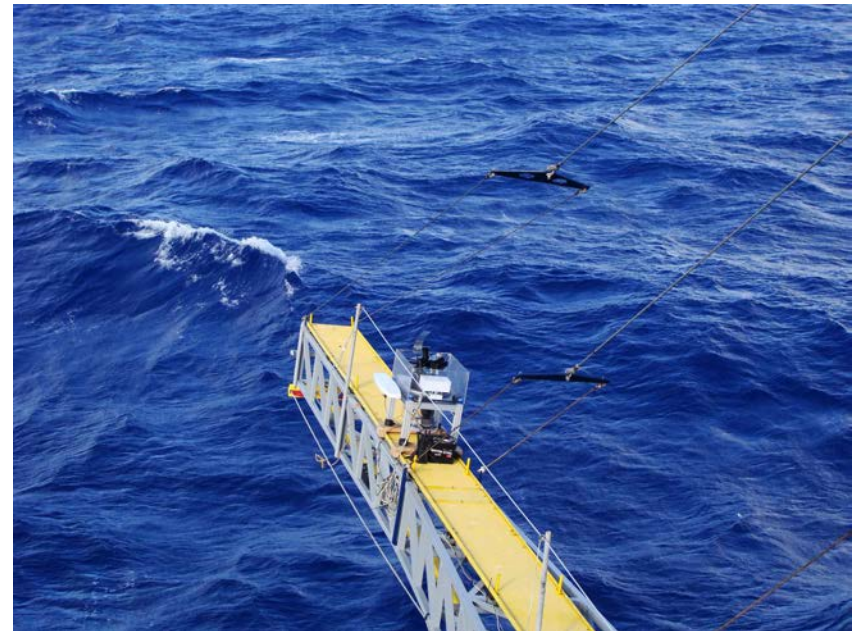
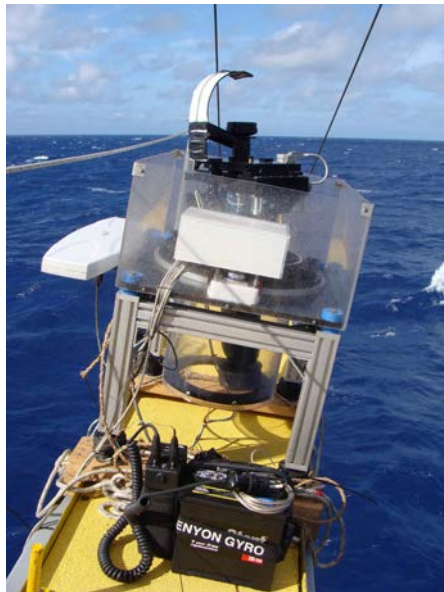


Fig. 1. Block diagram of the RADS-IIP instrument.

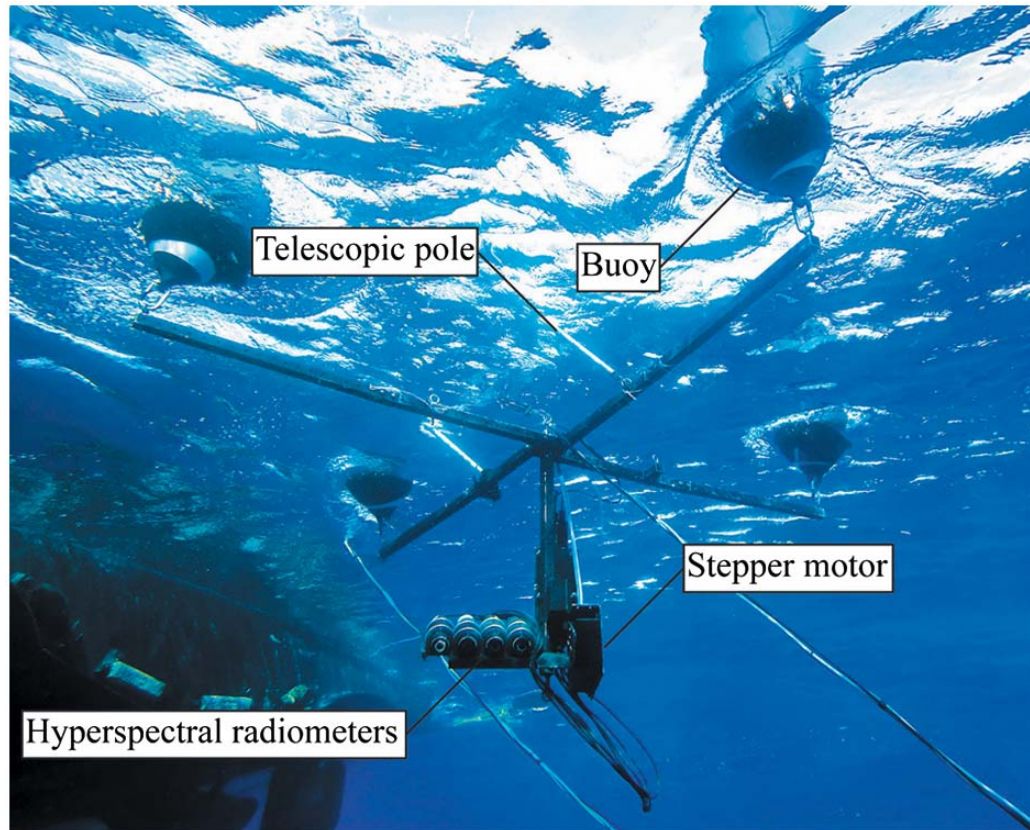
## Instruments

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- Sky scene relatively stable, hence time sequence can be used
  - Example K. J. Voss and Y. Liu, “Polarized radiance distribution measurements of skylight: I. system description and characterization”, 1997, Applied Optics, **36** :6083-6094.
- Must have stabilization to use on ships/moving platforms: Adapt system used by filming crews



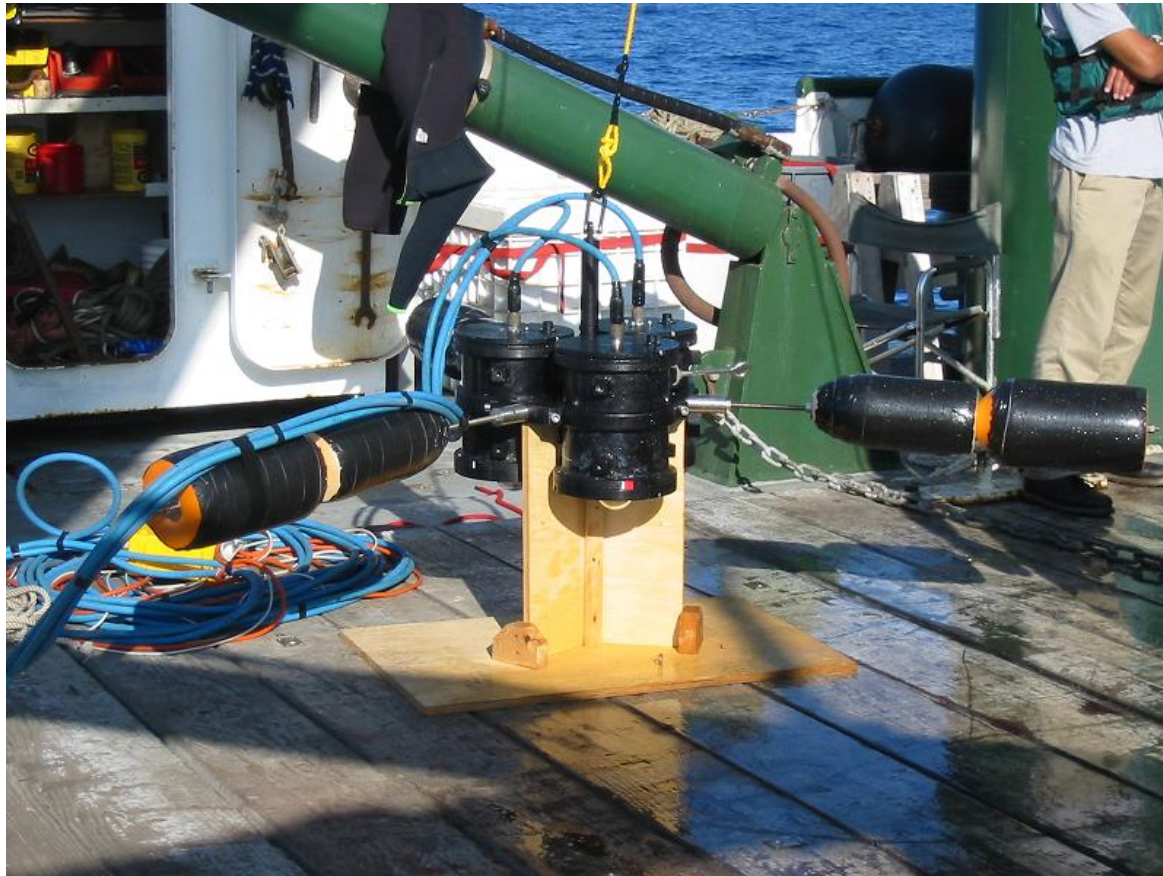
In-water...need simultaneous measurements...either with Gershun tube radiometers fitted with polarizers



Estimating particle composition and size distribution from polarized water-leaving radiance, Alberto Tonizzo,<sup>1,\*</sup> Alex Gilerson,<sup>1</sup> Tristan Harmel,<sup>1</sup> Amir Ibrahim, Jacek Chowdhary, Barry Gross, Fred Moshary, and Sam Ahmed<sup>1</sup>, Applied Optics, Vol 50, 5047-5058



Or simultaneous fisheye systems, with polarizers:



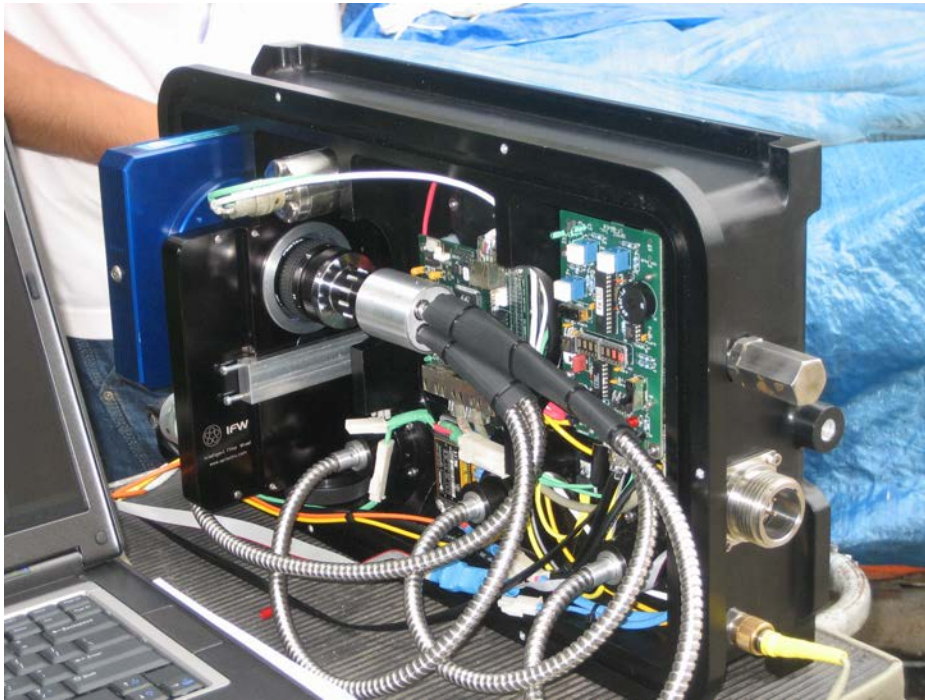
{Polrads

## Instruments

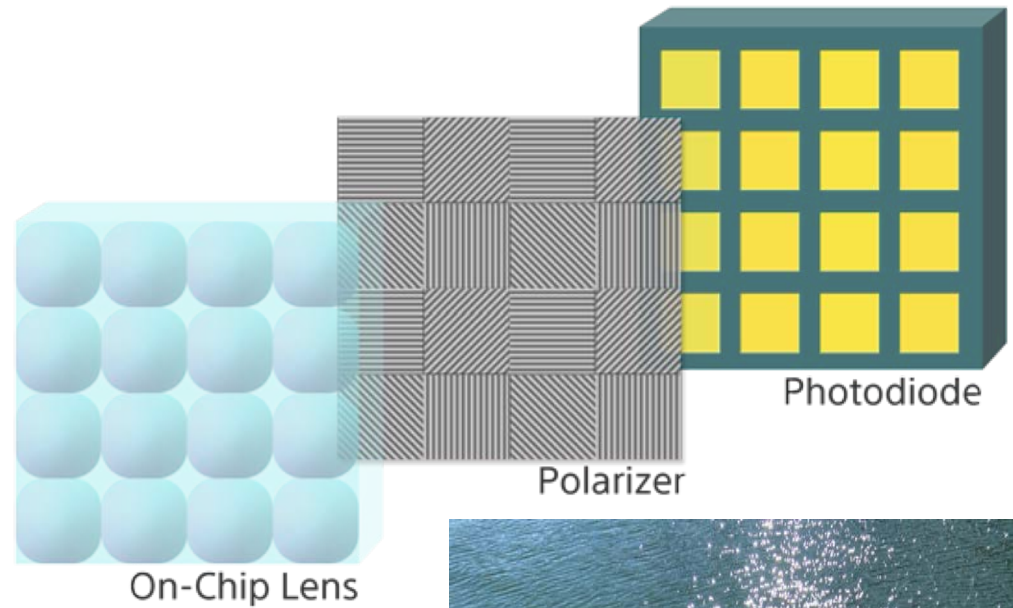
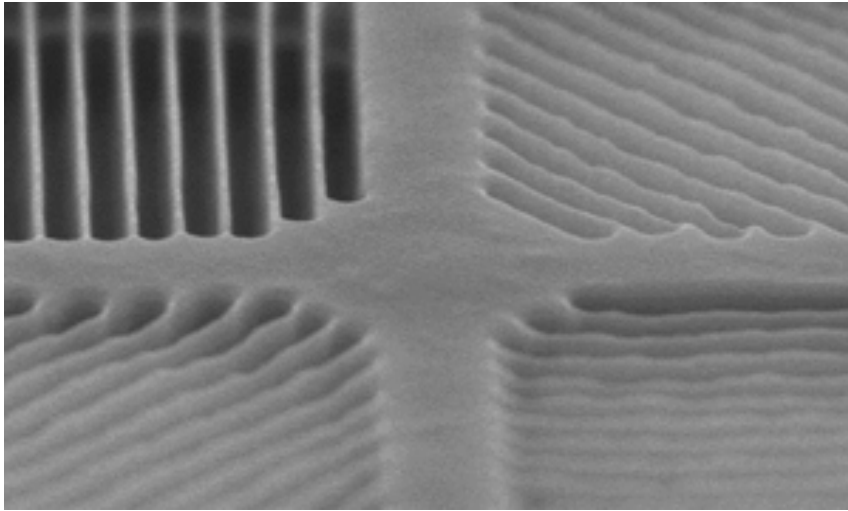
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### DPOL instrument:

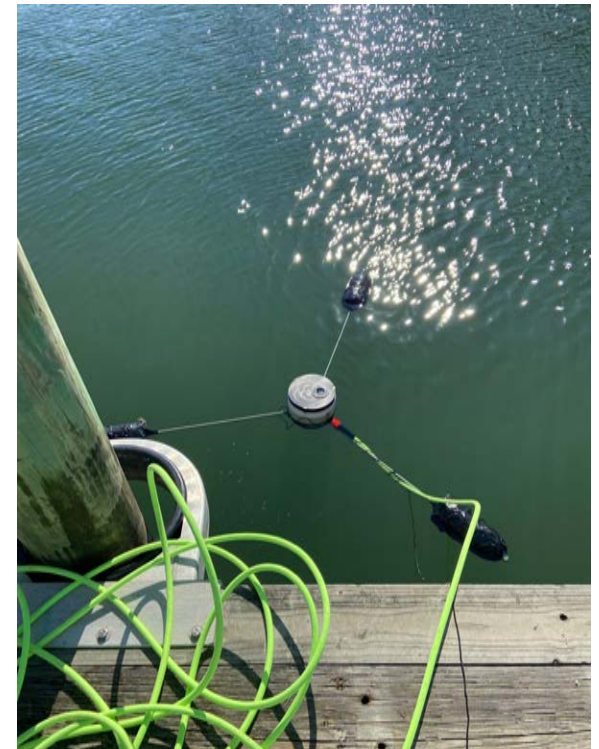
- System has 4 lenses, polarizer's in each
- Fiber Optic bundle collects light from each image into one super image
- Super image focused onto camera through spectral filter changer



# New system, PixPol



Polarsens image sensor (images from Sony web page)



To measure the Mueller matrix need 16 measurements (set of 4 different input Stokes vectors permuted with 4 polarization analyzers). Either do this serially or another method was:

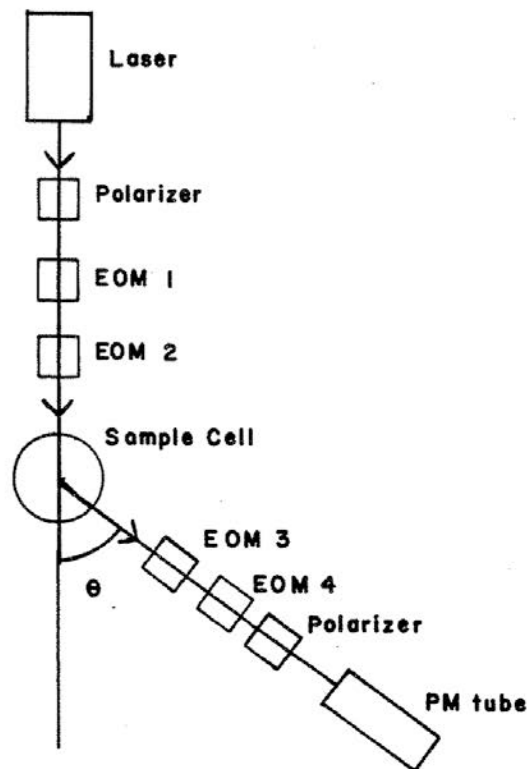
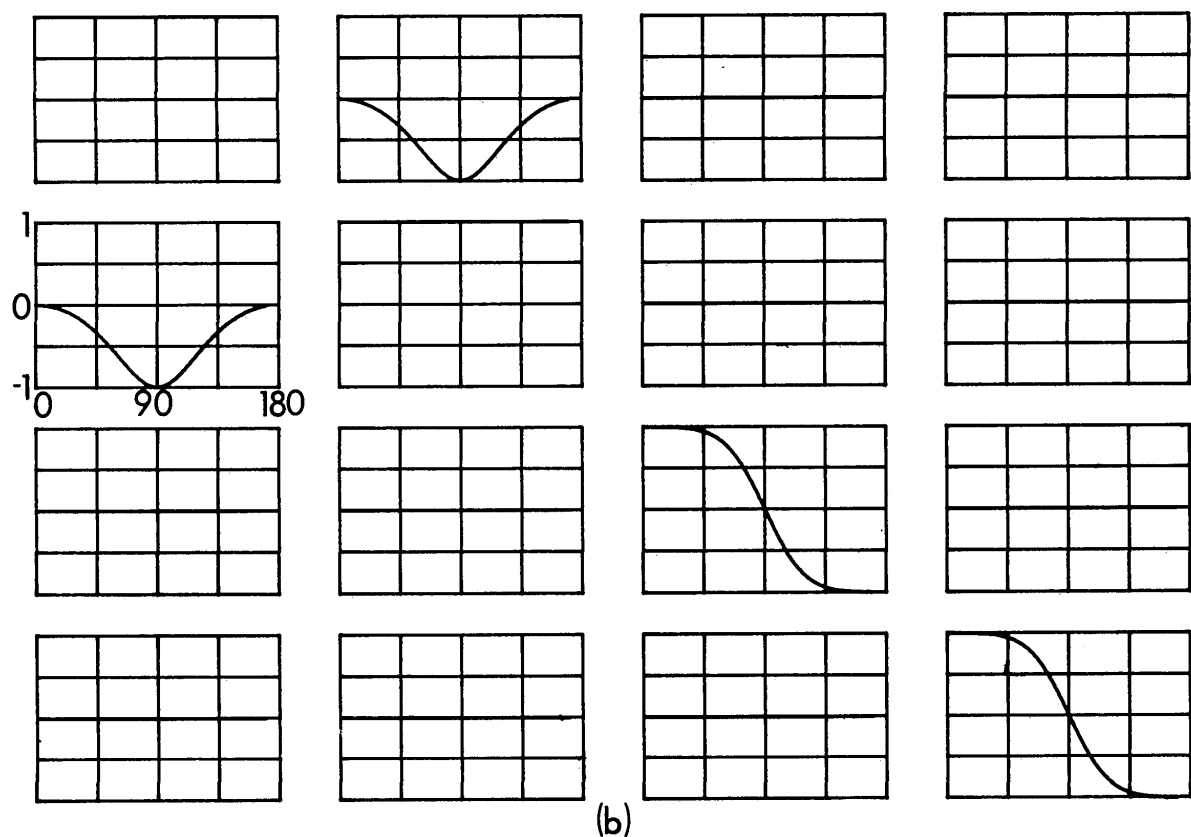


Figure 11. General Instrument Layout

# Mueller matrix for Rayleigh scattering.



Low index particles also scatter this way (Rayleigh-Gans approx.)

Mueller matrix for Rayleigh scattering. Lets look at some specific angles:

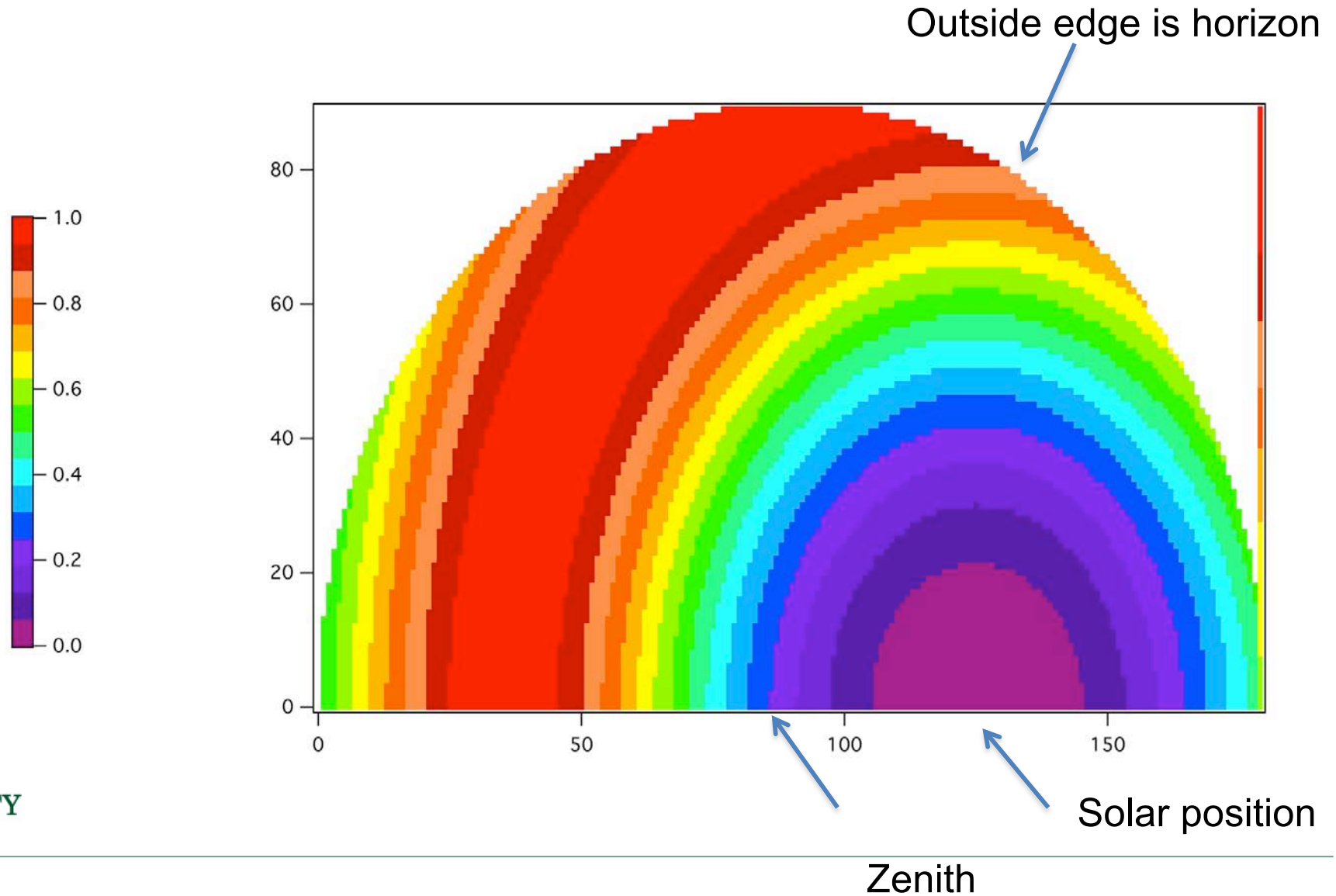
$$M(0^\circ) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$M(90^\circ) = \begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

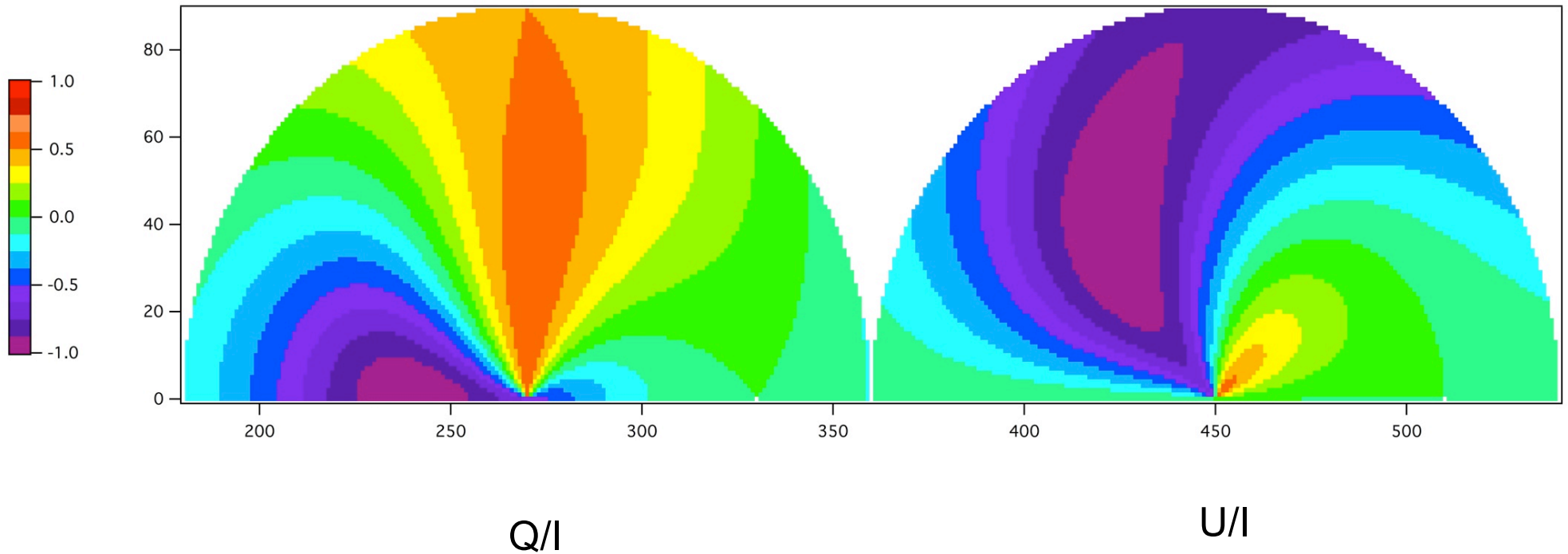
$$M(180^\circ) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

What do these do?

# Case for Rayleigh single scattering in Sky, Degree of polarization.



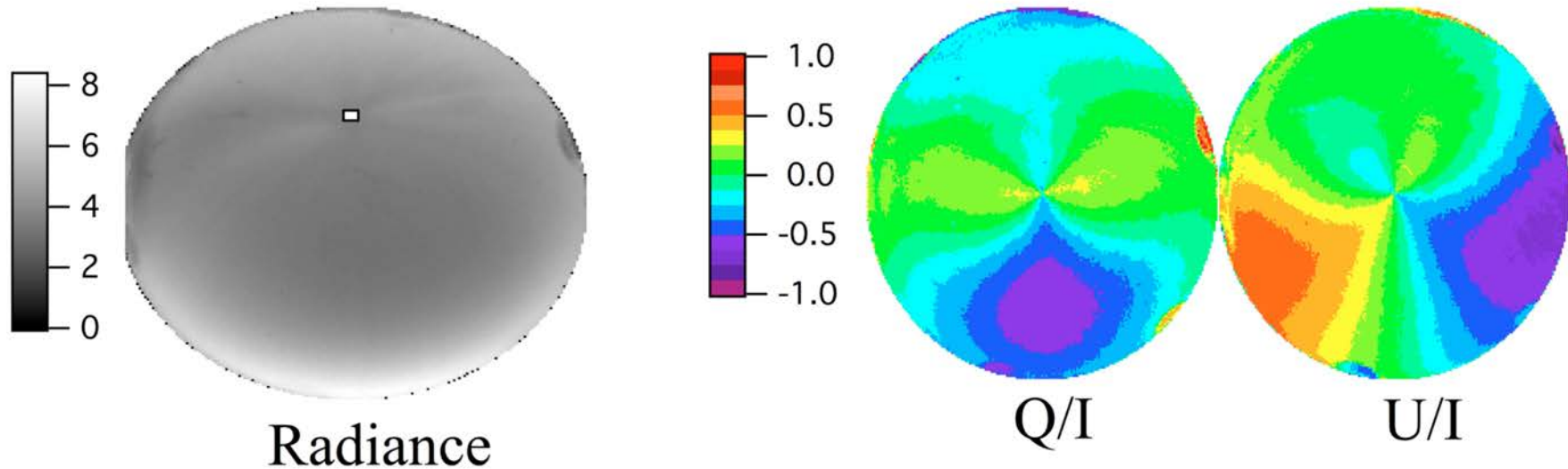
# Case for Rayleigh single scattering in Sky.



$V/I = 0$

60 degree zenith angle





*The radiance units are  $mW\ cm^{-2}\ nm^{-1}\ sr^{-1}$ ,  $Q/I$  and  $U/I$  are dimensionless*

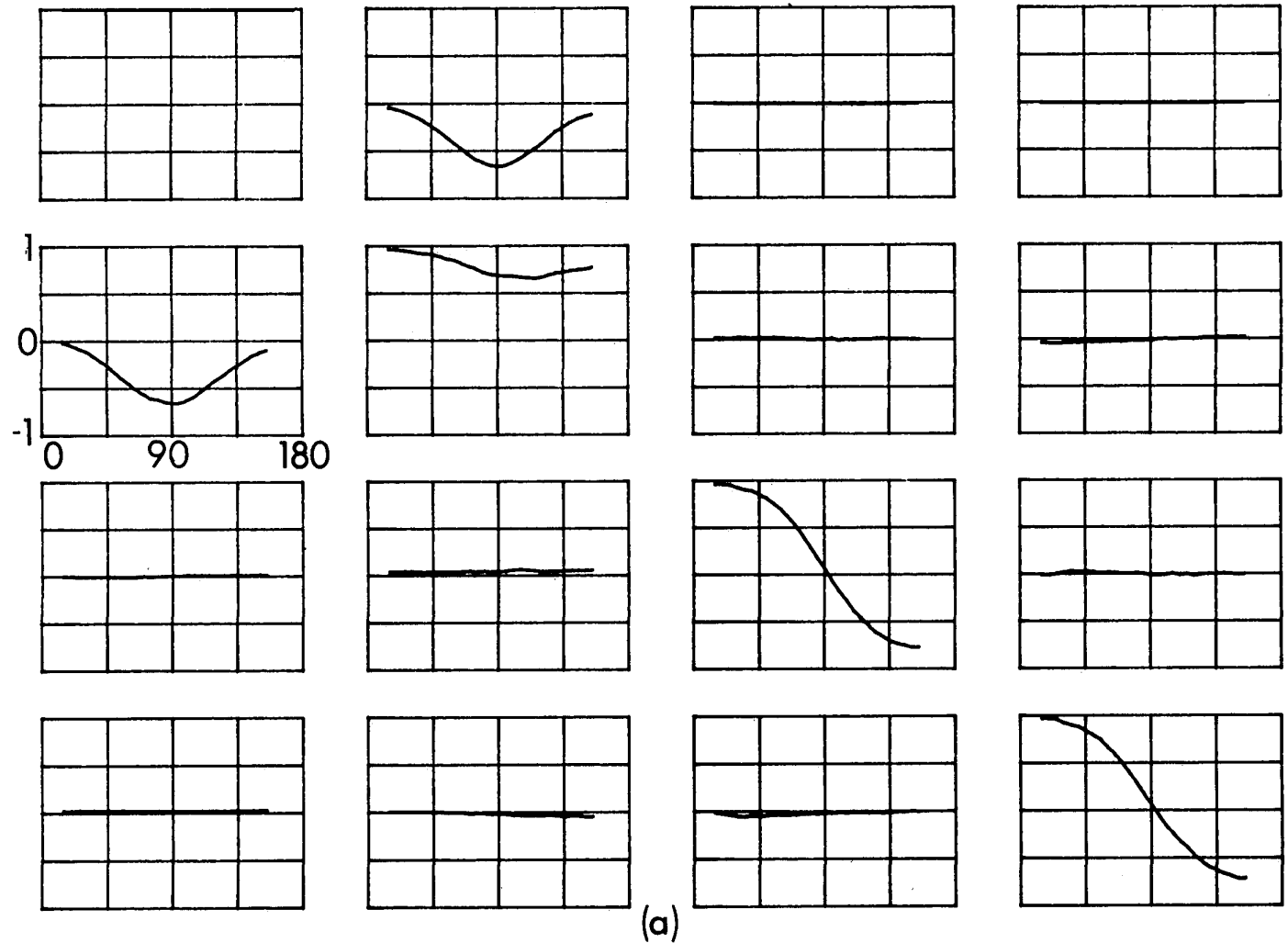
*Hawaii (December 2, 2005,  $20.83^{\circ}$  N,  $157.18^{\circ}$  W, 10:25 local time).*

*Wavelength is 436 nm  
solar zenith angle is  $51^{\circ}$ .*

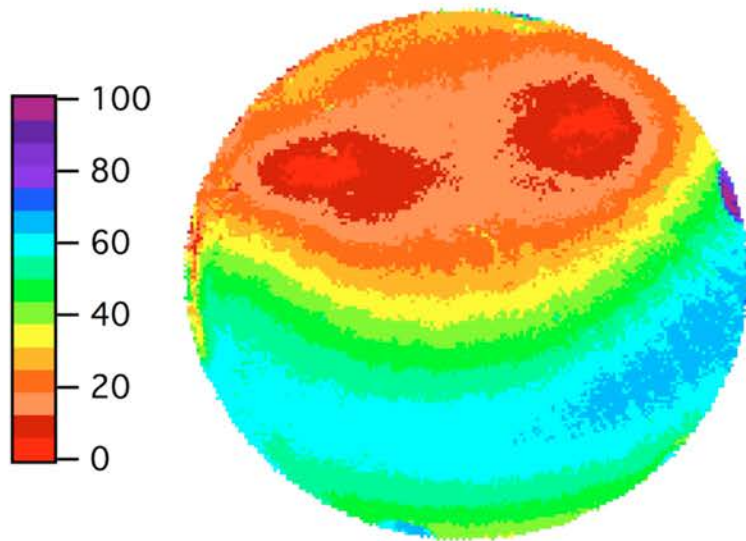
*Measured with POLRads instrument (recent article in OE)*

# Mueller matrix for Ocean water (Voss and Fry, 1984).

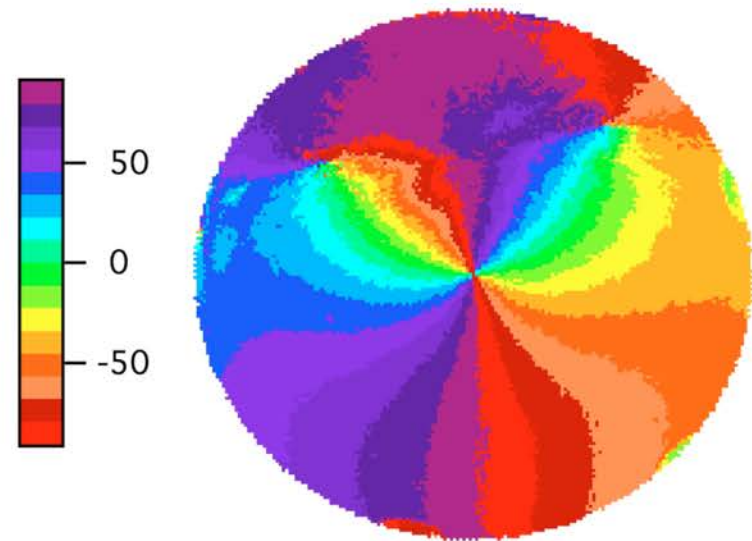
What are the differences?



## Example continued

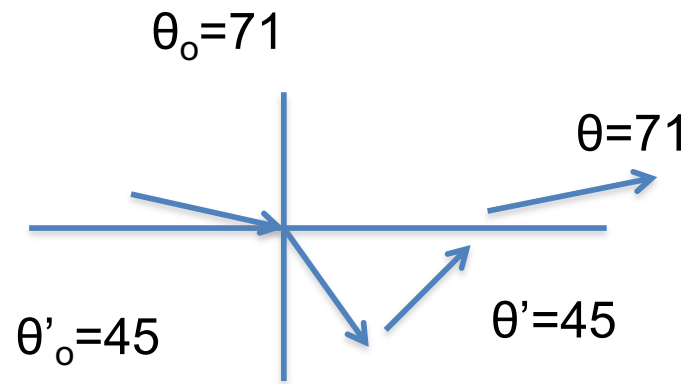


Degree of Polarization



Plane of Polarization

Note high degree of polarization, 90 degrees to refracted solar position, probably lost in glitter pattern, however still significant polarization in other areas.



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# Why do you care?

Other recent applications of polarization:

Use of polarized light for enhanced imaging: scattered light is more polarized than light from target (many people, early work by Gilbert, AO, 1967)..keeps getting rediscovered.

Polarized detection by animals..and polarized light camouflage (Molly Cummings, UT, Roger Hanlon, MBL).

Depolarization due to minerals?

# 2 possible remote sensing applications, change in M12/M11

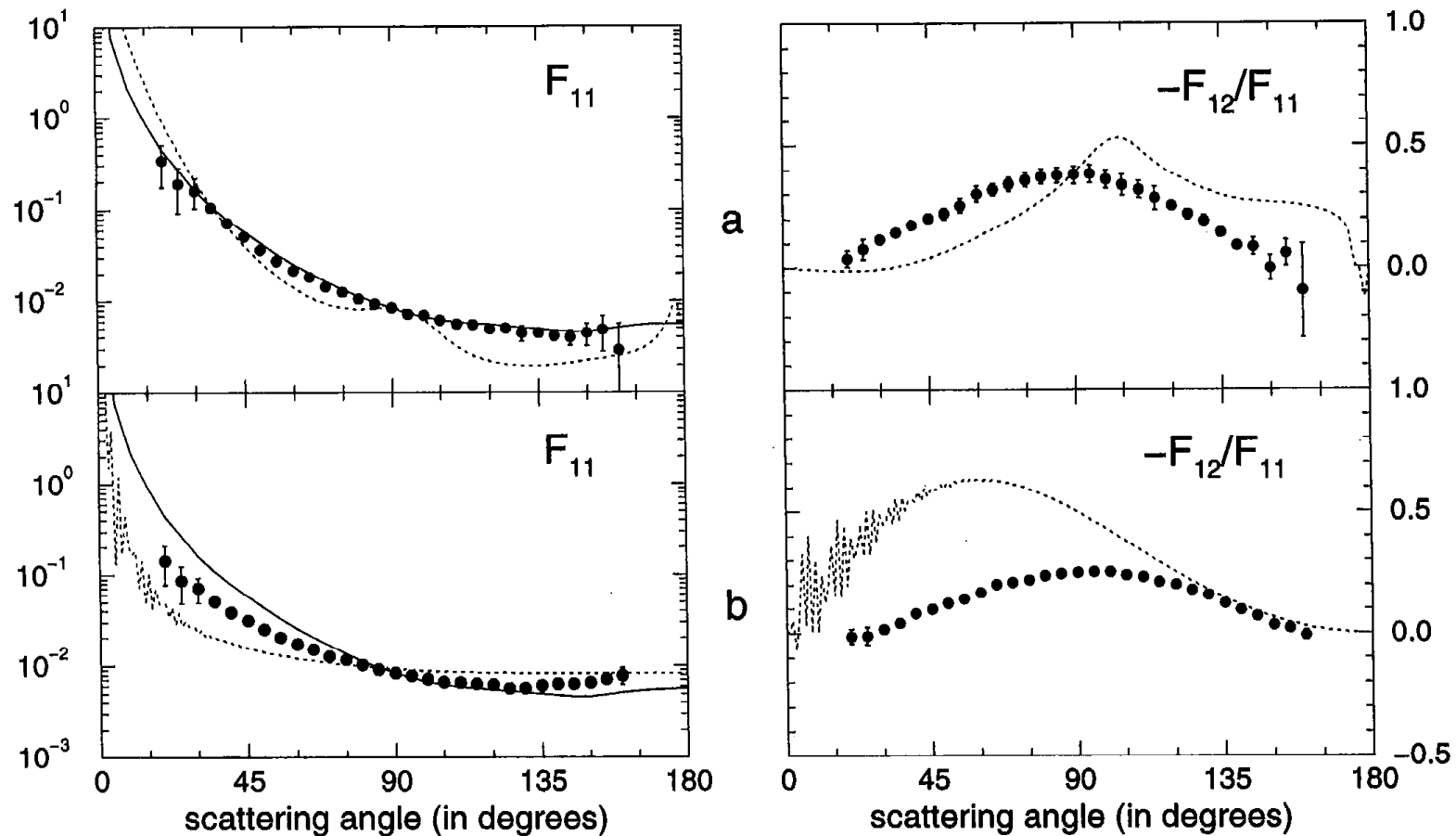


Fig. 9. Same as Fig. 6 for (a) Westerschelde silt with diameters ranging between 3 and 5  $\mu\text{m}$ , and (b) Westerschelde silt with diameters ranging between 5 and 12  $\mu\text{m}$ .

# Other ideas about polarization

- Ibrahim et al. 2012, Optics Express

## Retrieve $c/a$ with remote sensing

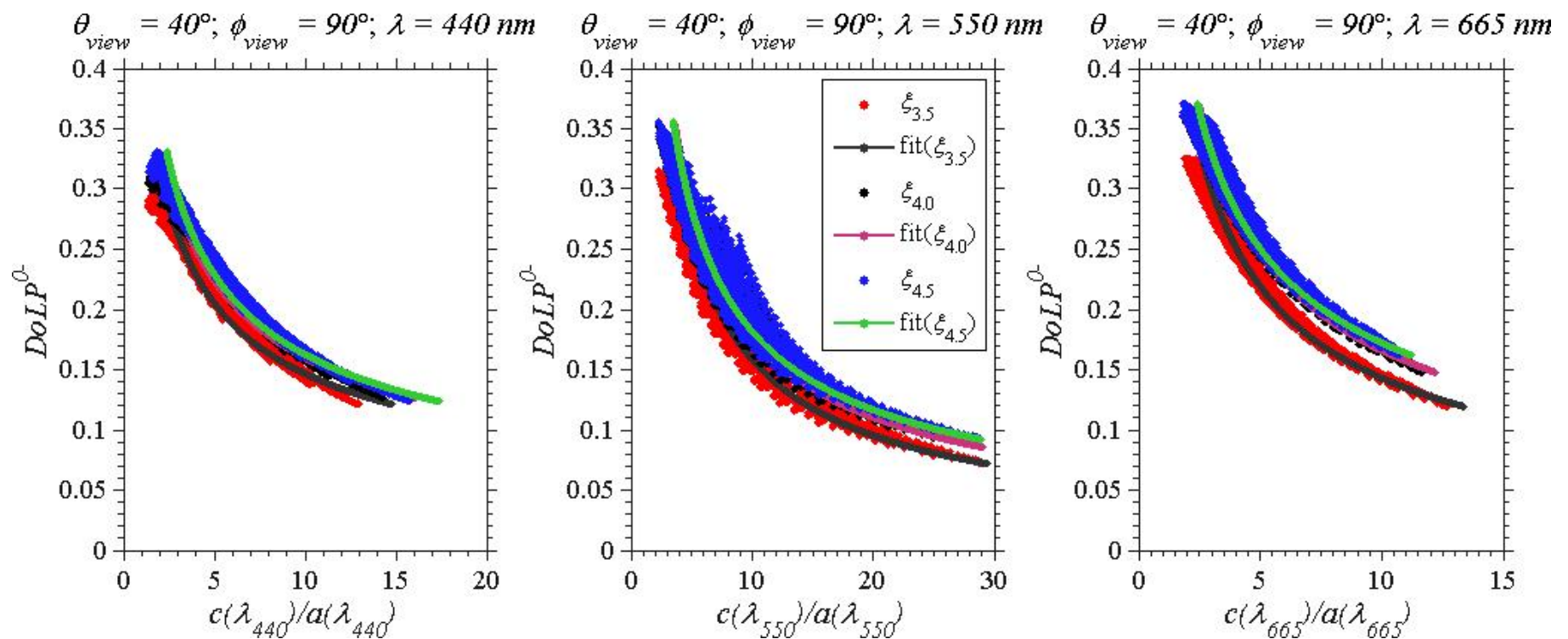


Fig. 8. Fitted relationship between DoLP at  $\theta_{view} = 40^\circ$  and  $\phi_{view} = 90^\circ$  and  $c/a$  ratio at three wavelengths for three different NAP slopes of the particle size distribution (PSD).

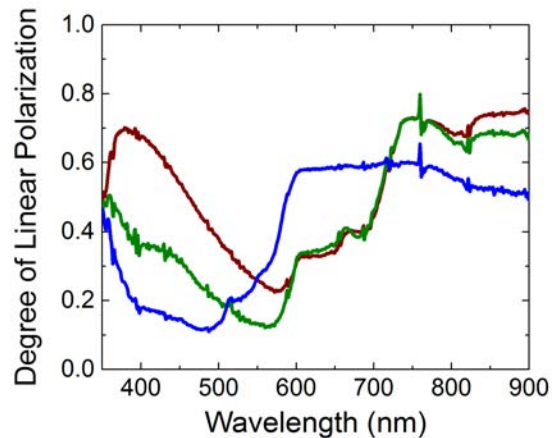
Your instrument may have a polarization sensitivity (old Rsr instruments, spectrometers without scrambling)

The satellite instrument is polarized, to greater or lesser extent.

Might be more information in the polarization signal, new sensors will be developed (one sank...APS on Glory, two will be with PACE) especially important for atmospheric aerosols.

# Discussion topic

From Polarimetry in the PACE mission, Science Team consensus Document, July 23, 2015.



View Angle, 50 degrees nadir  
Azimuth 90 degrees  
No mention of solar zenith angle

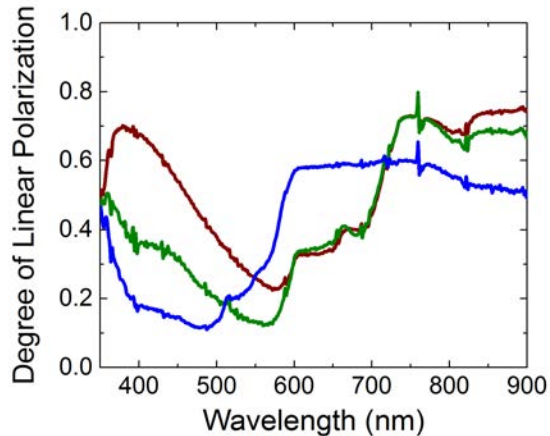
Blue curve clear water  
Green curve, productive water  
Brown curve High CDOM

Figure 7. Illustration of the degree of linear polarization (DoLP) measured above the surface of different water types in South Florida: Clear water (blue curve), biologically productive water (green curve), and productive water with high amounts of CDOM (brown curve). Measurements were made above the water surface with a hyperspectral polarimeter covering a wavelength range of 350-1100 nm (nominal, 380-900 nm useful), at a spectral resolution (FWHM) of 3 nm, and with a fine spectral sampling interval of 0.8 nm. The view angle is 50 degrees (near Brewster's angle) and oriented 90 degrees to the sun. The water varied from clear near the Keys to turbid in Florida Bay near the shoreline. These data are "raw" polarization data, in that no corrections for surface reflections have been applied. Also note that view angle is *critical* in these data. Measurements at 135 degrees to the sun often show very low DoLP. These are preliminary data. The main spectral polarimetric features are discernible. The high frequency fluctuations indicate that absolute spectral calibrations could be improved.



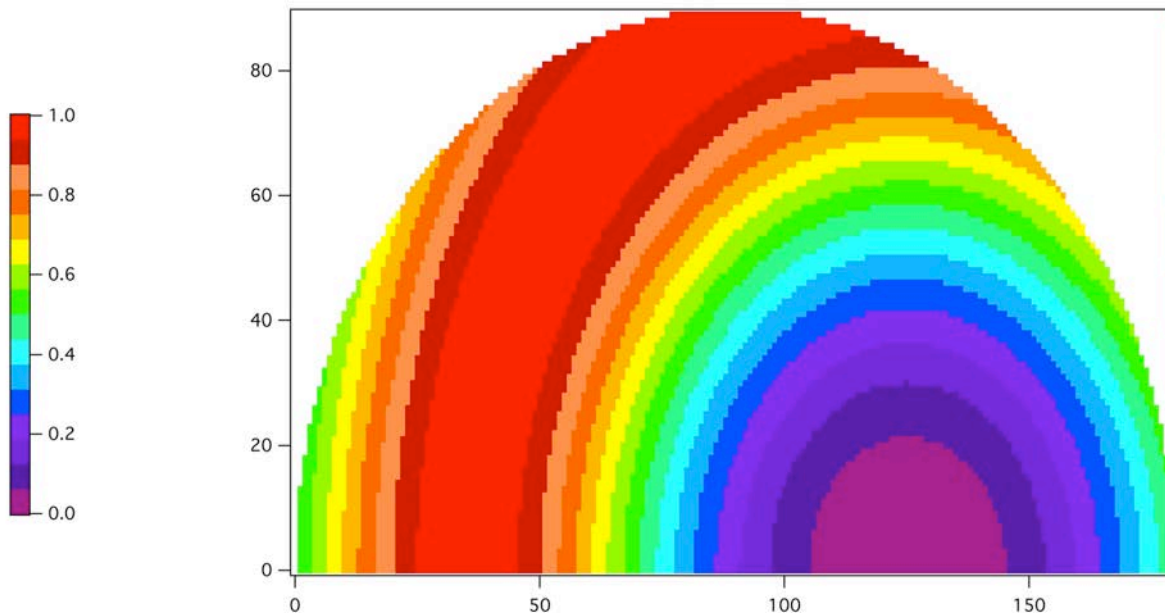
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Blue curve clear water  
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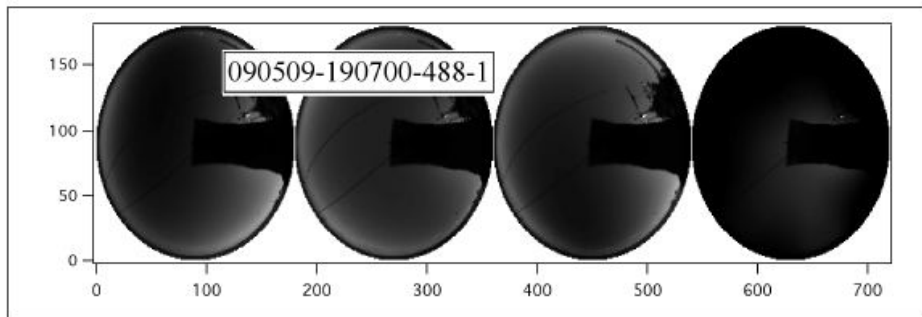
What does it mean to be at Brewster's angle in terms of DOP?

Is it surprising that high CDOM water has high DOP in blue?

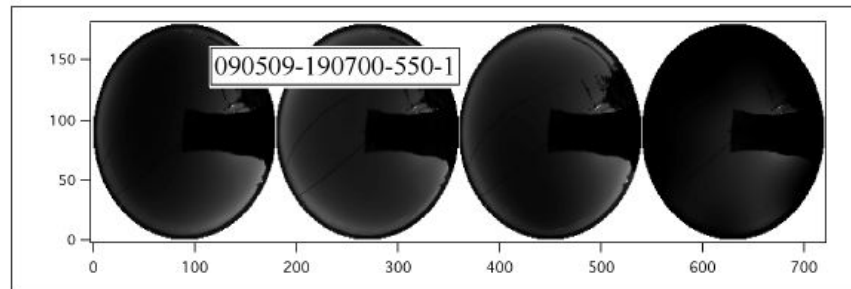
What about rest of spectrum?

Extra slides

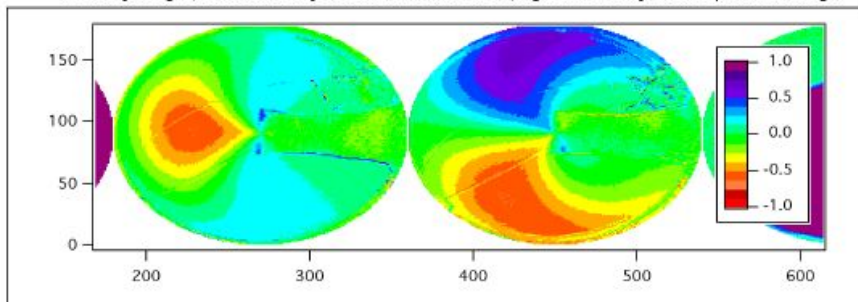
# Sky light radiance distribution.



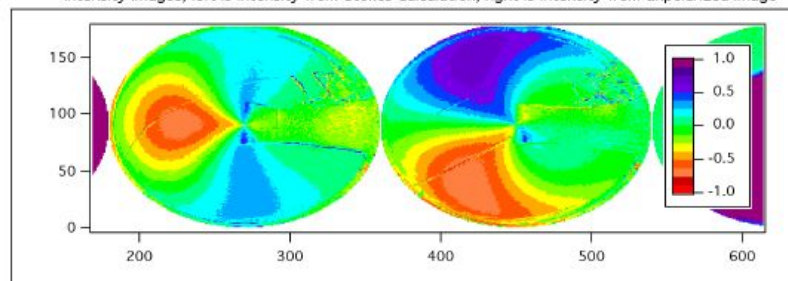
Intensity images, left is intensity from Stokes Calculation, right is intensity from unpolarized image



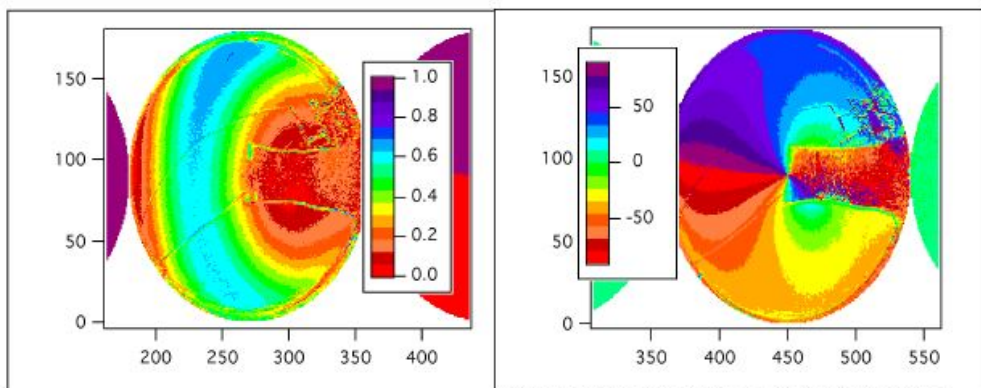
Intensity images, left is intensity from Stokes Calculation, right is intensity from unpolarized image



Normalized Stokes vectors Q (on left) and U (on right).

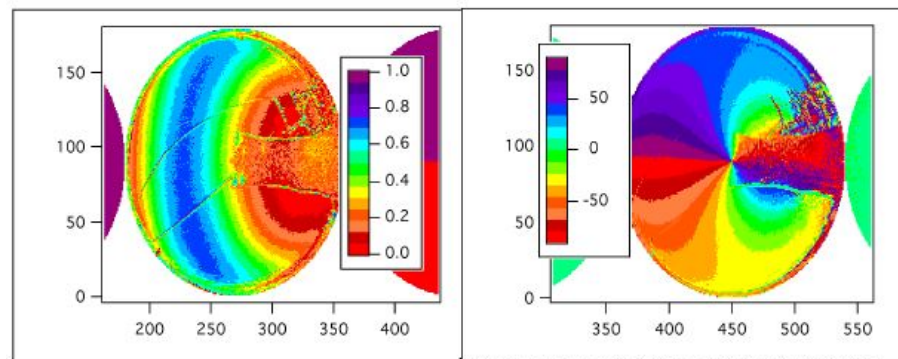


Normalized Stokes vectors Q (on left) and U (on right).



Degree of linear polarization

Frame of polarization, in degrees. Reference frame defined relative to plane with zenith and view direction. 0 deg is in the reference plane, positive angles rotate clockwise (looking along direction of propagation) towards the perpendicular to this plane.

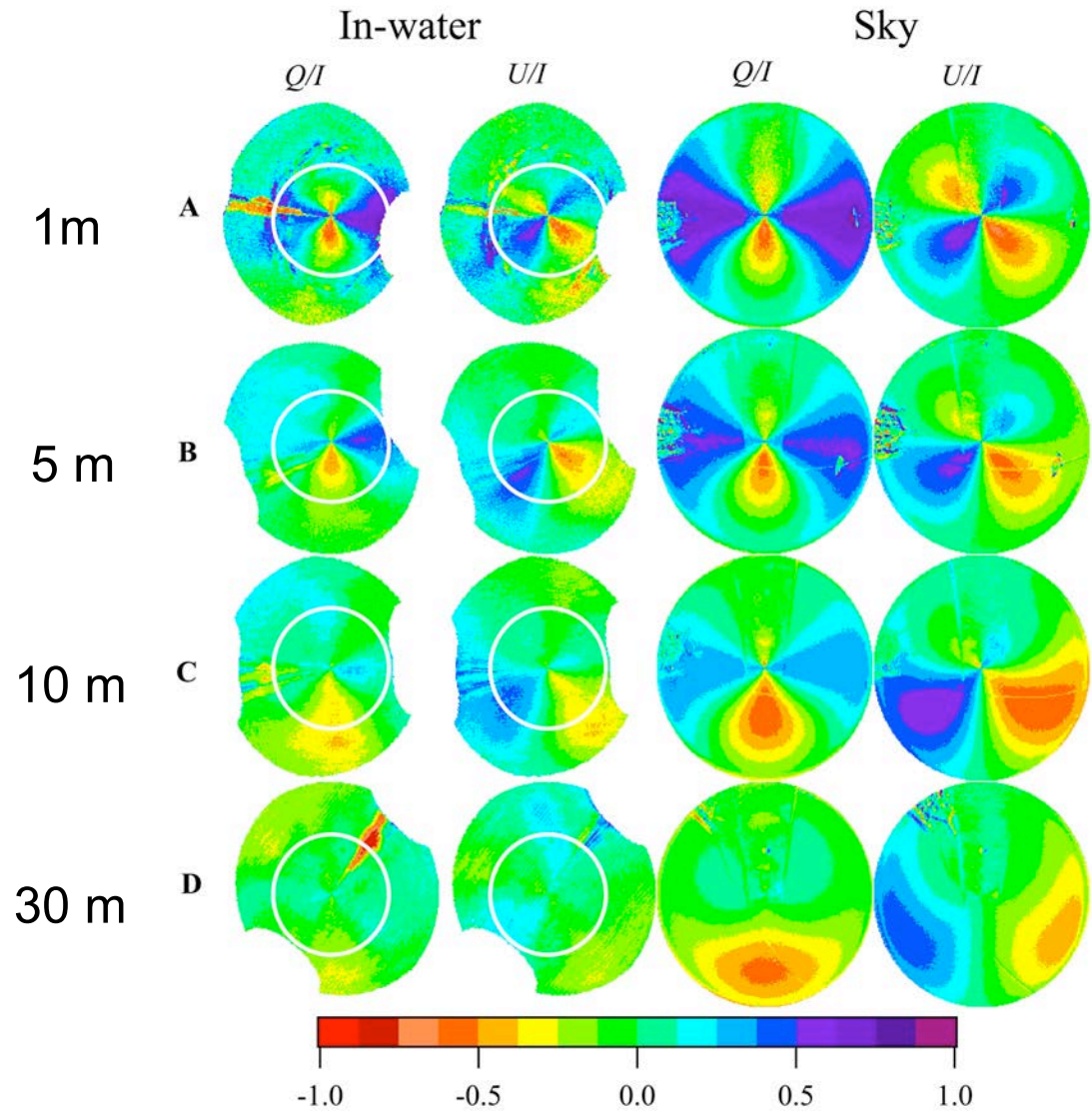


Degree of linear polarization

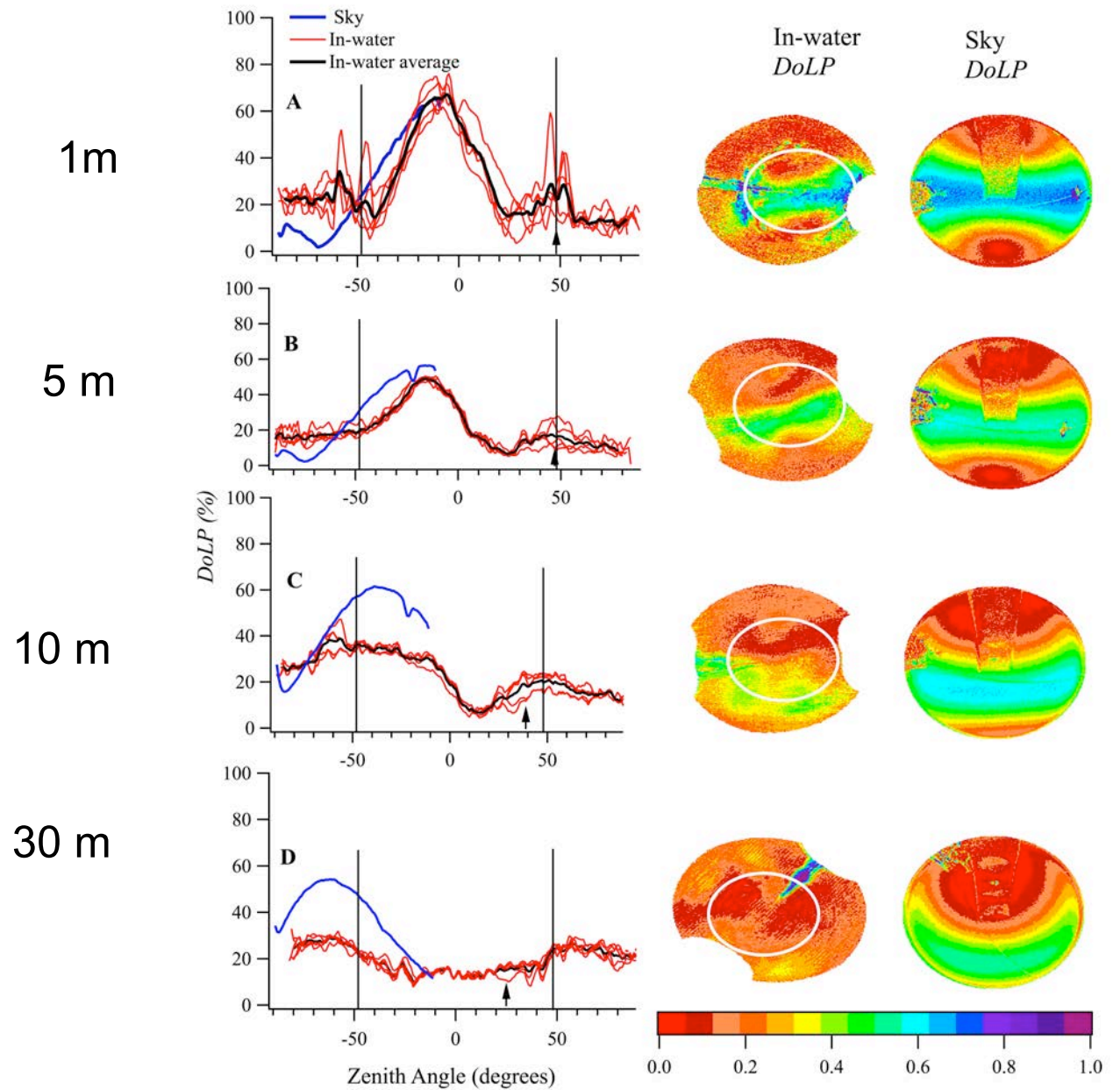
Frame of polarization, in degrees. Reference frame defined relative to plane with zenith and view direction. 0 deg is in the reference plane, positive angles rotate clockwise (looking along direction of propagation) towards the perpendicular to this plane.



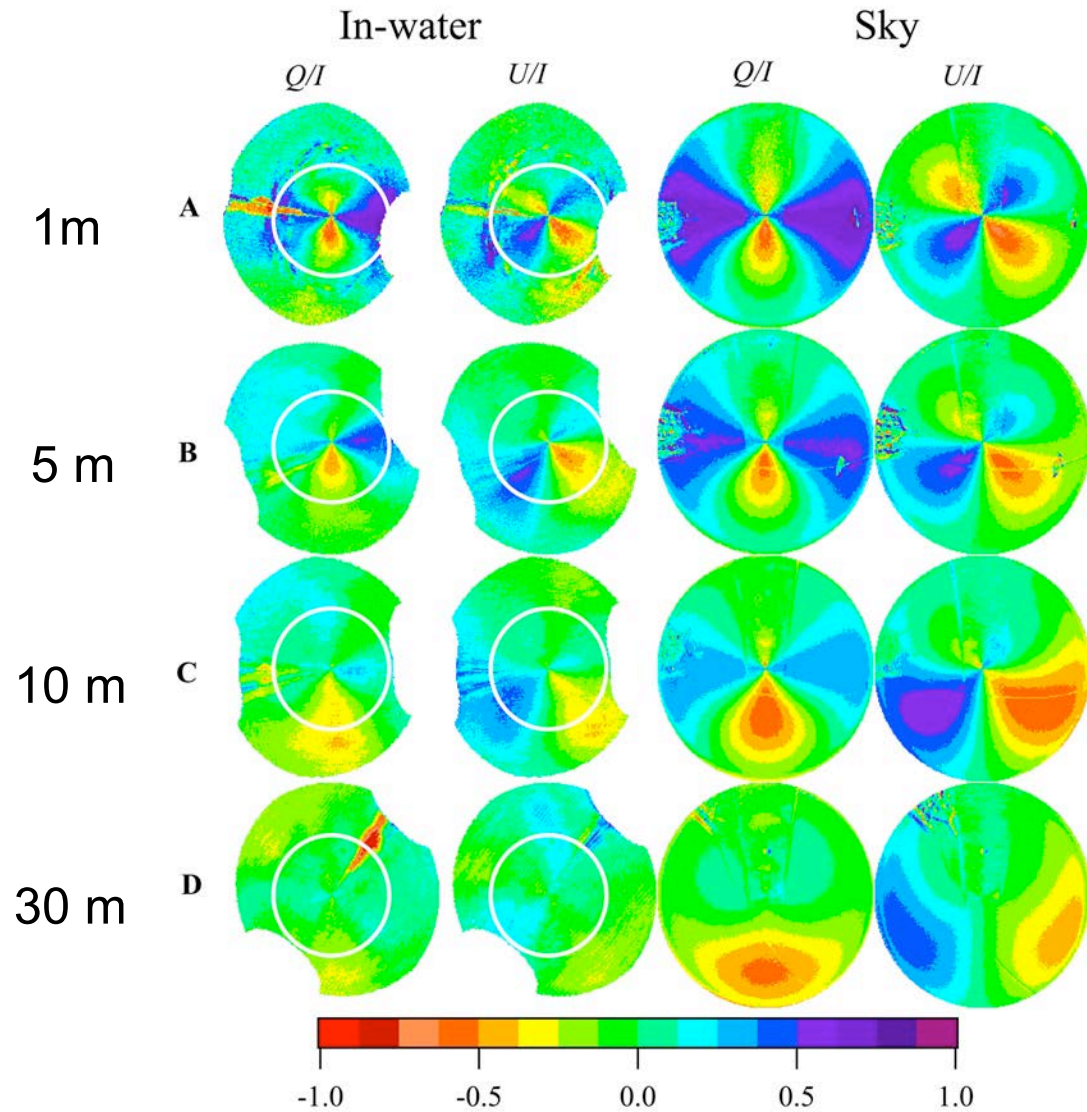
# Downwelling radiance distribution.



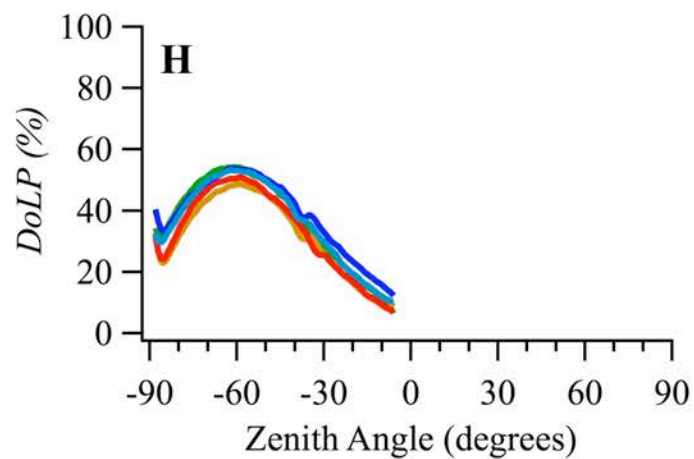
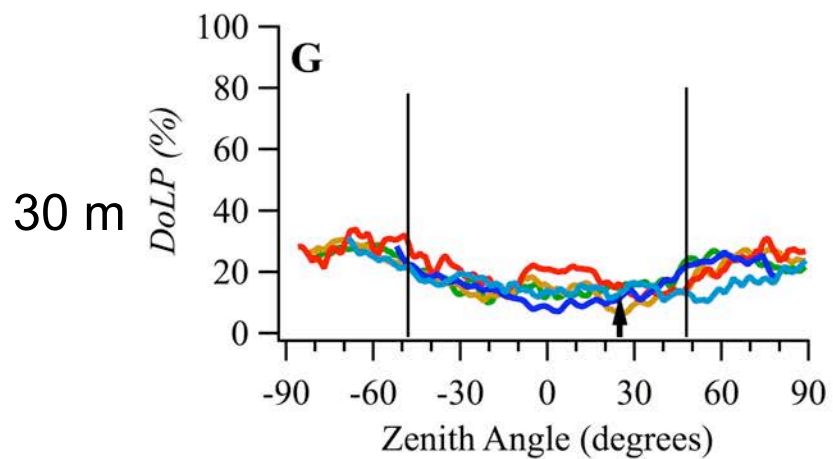
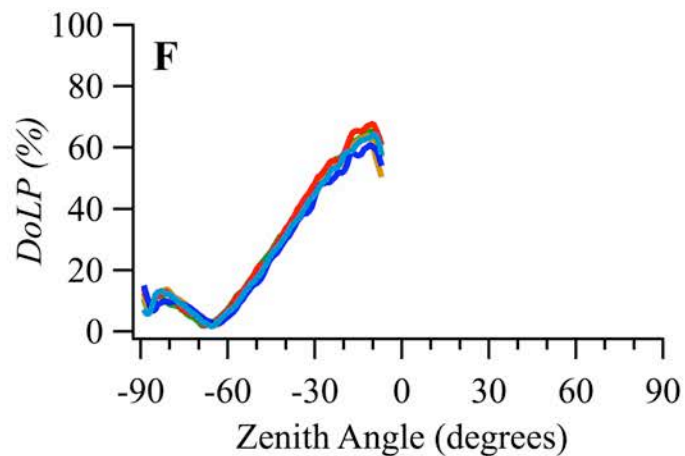
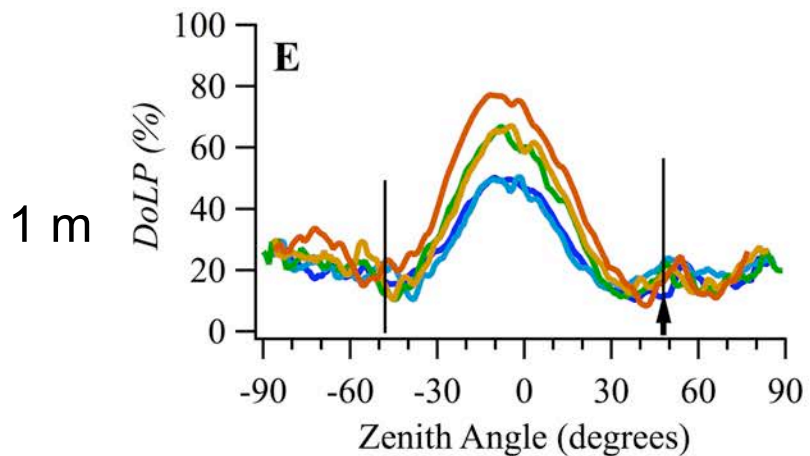
# Downwelling radiance distribution.



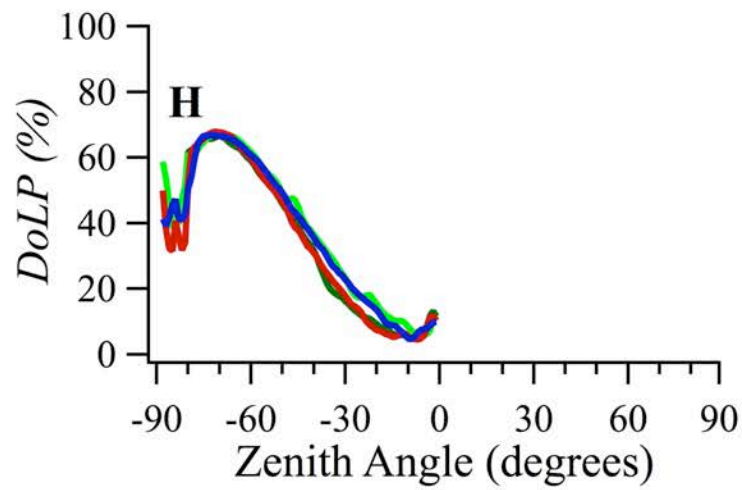
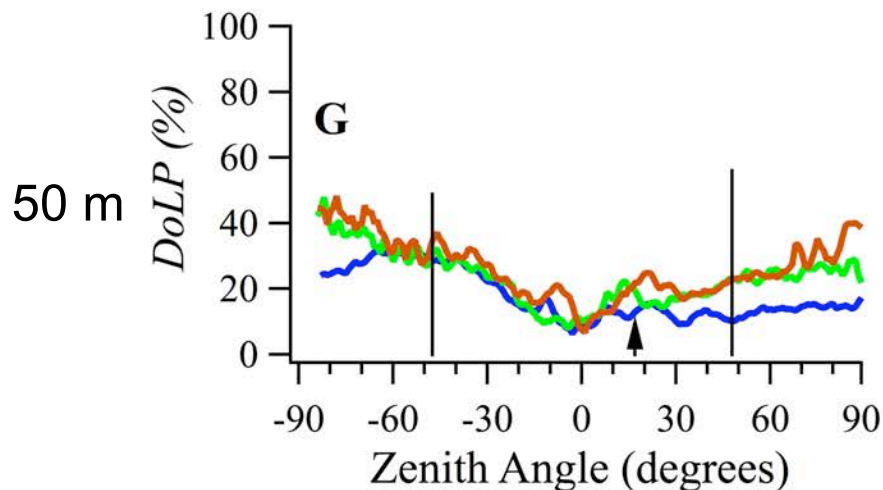
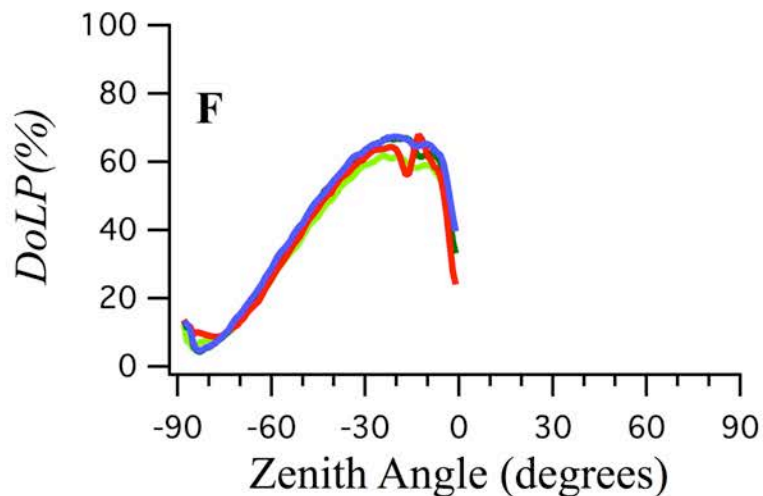
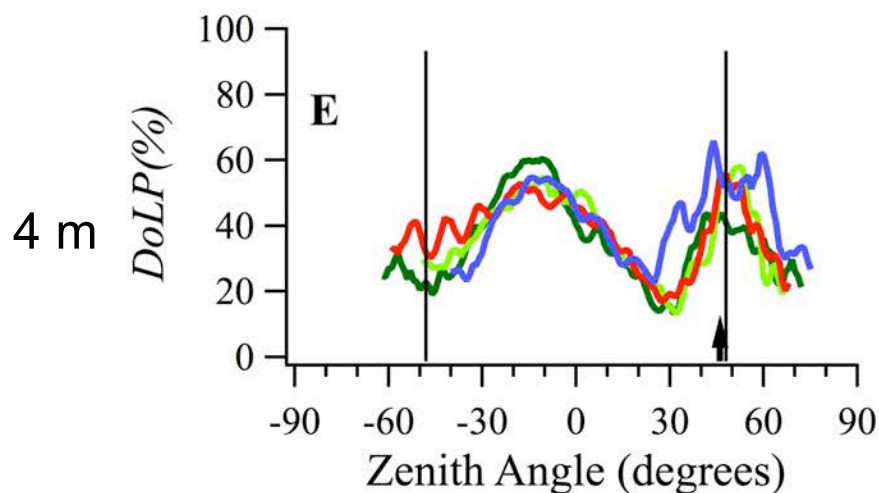
# Downwelling radiance distribution.



# Downwelling spectral variation, coastal



# Downwelling spectral variation, clear water





# Downwelling spectral variation, clear water

