Lectures on
Optical Oceanography and
Ocean Color Remote Sensing

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Surfaces

Schiller Coastal Studies Center
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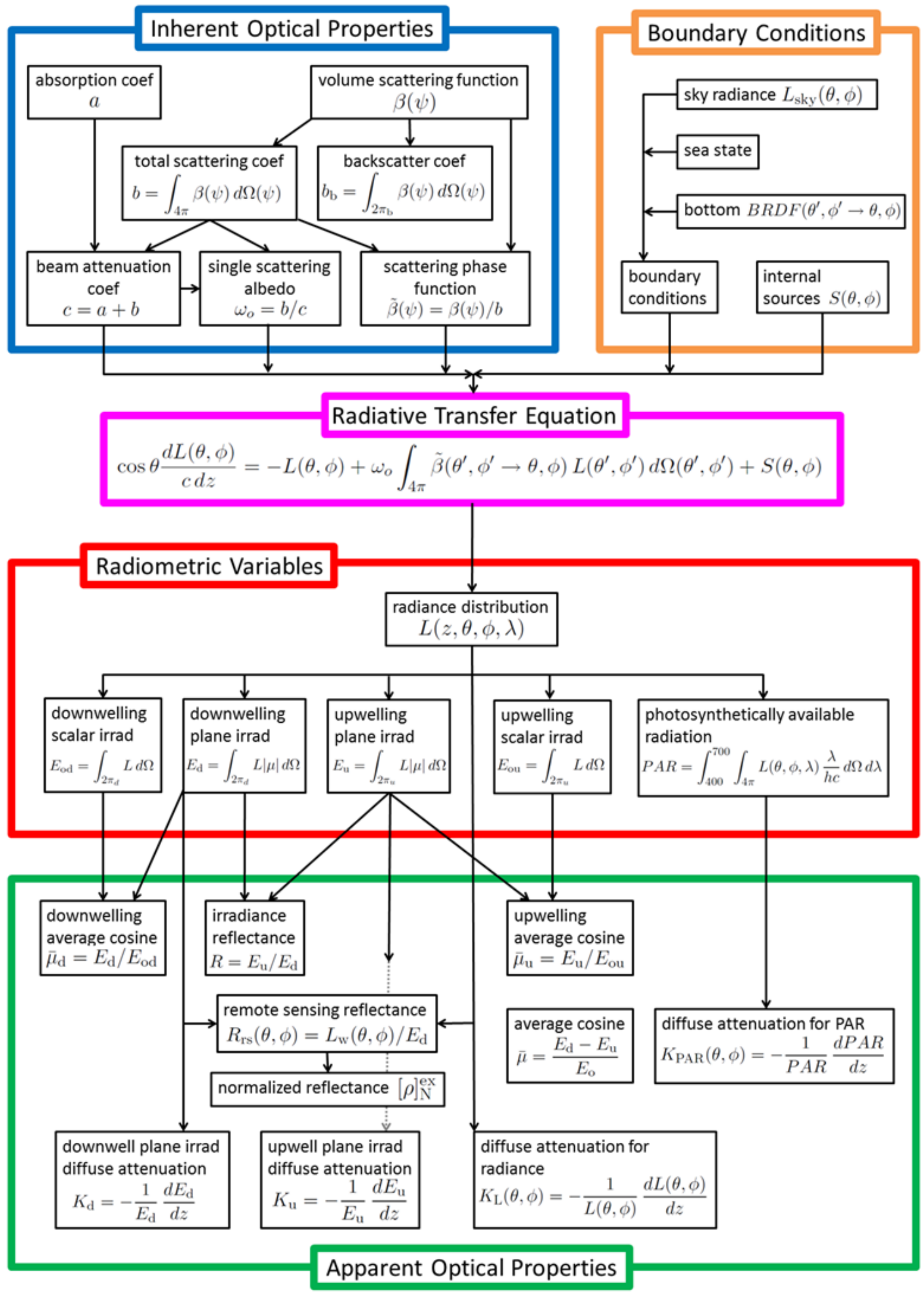
Surfaces

The sea surface reflects and transmits light.

In shallow water, the bottom reflects light back into the water column.

Reflectance by objects in the water affects their visibility.

Surfaces provide the boundary conditions for solving the RTE.



Surfaces

The bidirectional reflectance distribution function (BRDF): the most general way to describe reflectance by any surface

- Lambertian BRDF

The level air-water surface

- The n^2 law for radiance

Modeling random, wind-blown sea surfaces

- Cox-Munk slope model
- Surfaces based on wave spectra

The Bidirectional Reflectance Distribution Function (BRDF)

Recall: The fundamental IOPs, the absorption coefficient and the volume scattering function, tell you everything there is to know about how a volume of matter absorbs and scatters light.

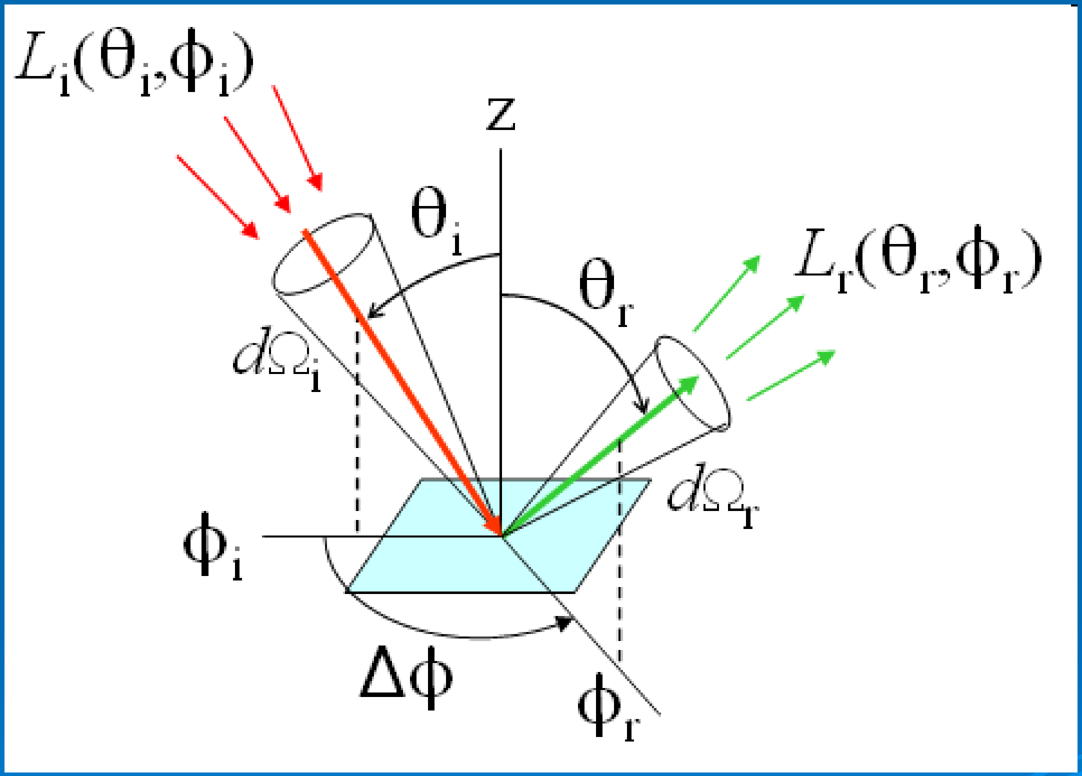
The BRDF is the IOP for surfaces (air-water surface, bottom sediment, a sea grass leaf, etc.)

The VSF describes how a volume scatters radiance from any incident direction into any scattered direction: $\mathrm{VSF}(\theta_i, \varphi_i \rightarrow \theta_s, \varphi_s, \lambda) = \mathrm{VSF}(\psi, \lambda)$

The BRDF describes how a surface reflects radiance from any incident direction into any reflected direction:
$\mathrm{BRDF}(\theta_i, \varphi_i \rightarrow \theta_r, \varphi_r, \lambda)$

BRDF Definition Geometry

The geometry for defining the BRDF(θ_i, φ_i, θ_r, φ_r, λ)



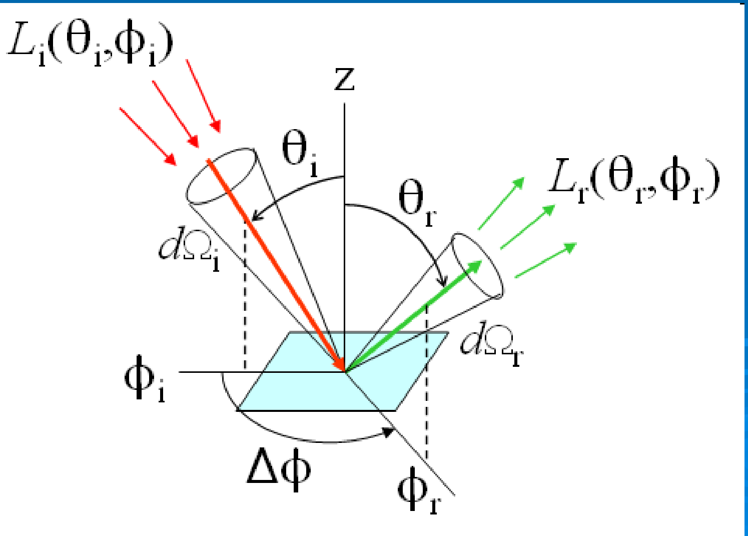
See https://www.oceanopticsbook.info/view/surfaces/the-brdf
Or OOB Section 8.6

BRDF Definition

How the BRDF is defined:

$$BRDF(\theta_i, \phi_i, \theta_r, \phi_r) \equiv \frac{dL_r(\theta_r, \phi_r)}{L_i(\theta_i, \phi_i)\ \cos\theta_i\ d\Omega_i(\theta_i, \phi_i)}$$

How it's measured: $= \dfrac{L_r(\theta_r, \phi_r)}{E_d(\theta_i, \phi_i)} \quad [\mathrm{sr}^{-1}]$



BRDF Usage

How the BRDF is used to compute the radiance reflected into a given direction by radiance incident from all directions (e.g., in HydroLight):

$$BRDF(\theta_i, \phi_i, \theta_r, \phi_r) \equiv \frac{dL_r(\theta_r, \phi_r)}{L_i(\theta_i, \phi_i)\ \cos\theta_i\, d\Omega_i(\theta_i, \phi_i)}$$

$$L_r(\theta_r, \phi_r) = \int_{2\pi_i} L_i(\theta_i, \phi_i)\, BRDF(\theta_i, \phi_i, \theta_r, \phi_r)\ \cos\theta_i\, d\Omega_i$$

$$\equiv \int_{2\pi_i} L_i(\theta_i, \phi_i)\, r(\theta_i, \phi_i, \theta_r, \phi_r)\, d\Omega_i\ .$$

in L&W

R_{rs} and the BRDF

Recall the way a BRDF is actually measured:

$$BRDF(\theta_i, \phi_i, \theta_r, \phi_r) = \frac{L_r(\theta_r, \phi_r)}{E_d(\theta_i, \phi_i)}$$

This looks similar to (and has the same units as) the remote-sensing reflectance:

$$R_{\mathrm{rs}}(\theta, \phi) = \frac{L_w(\theta, \phi)}{E_d}$$

They are not the same:

The BRDF has the incident irradiance E_d due to radiance in one particular direction.

R_{rs} has the irradiance E_d due to radiance in all downward directions.

Computing the BRDF in HydroLight

Use the option for the Sun in a black sky to get a collimated sky radiance onto the sea surface.

Define the IOPs and other inputs as always.

Run 1: Put the sun at the zenith, $(\theta, \varphi) = (0, 0)$. This gives one (θ_i, φ_i) incident direction for $E_d(\theta_i, \varphi_i)$ and all output directions for $L_r(\theta_r, \varphi_r)$.

Repeat for all other input directions (θ_i,φ_i)

Assemble the outputs from these runs to get

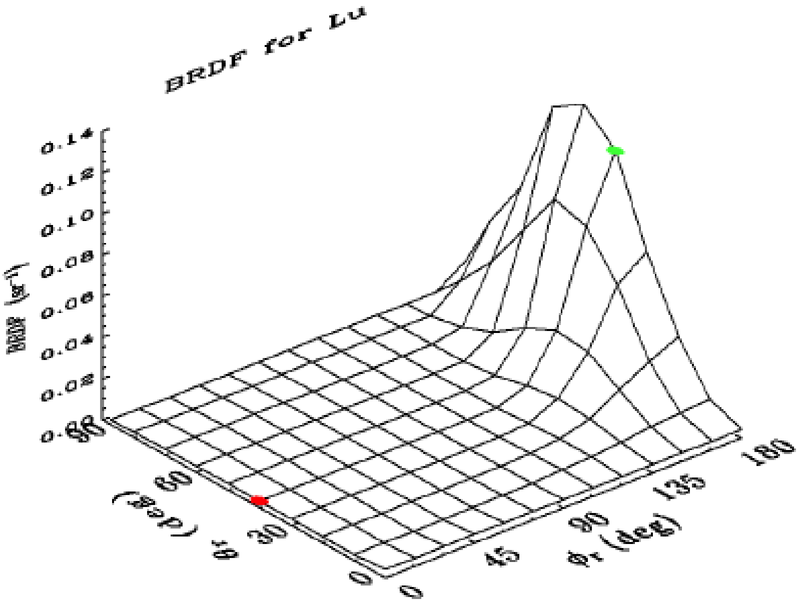
$$\mathrm{BRDF}(\theta_i, \varphi_i, \theta_r, \varphi_r) = L_r(\theta_r, \varphi_r) \,/\, E_d(\theta_i, \varphi_i)$$

Example Ocean BRDFs

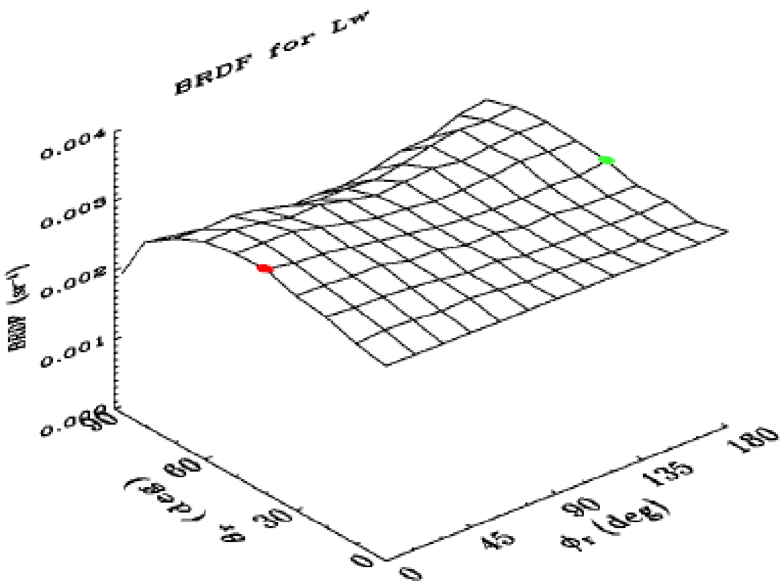
wind = 6 m/s
Case 1 water with Chl = 2 mg/m^3
infinitely deep water

The Lu BRDF is computed from the total upward radiance L_u just above the sea surface. This is the BRDF of the ocean including surface reflectance and water-leaving radiance.

The Lw BRDF is computed from just the water leaving radiance L_w. Note that this BRDF is not quite Lambertian.



BRDF for θ_s = 40.0, ϕ_s = 0.0, λ = 0.410 μm



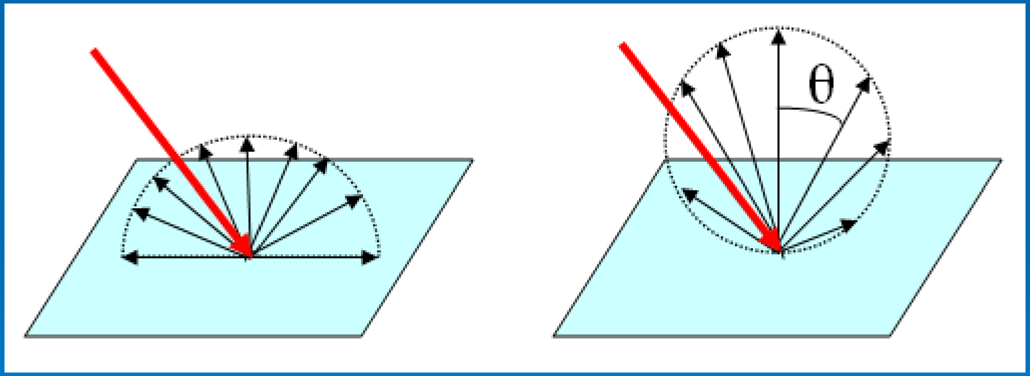
BRDF for θ_s = 40.0, ϕ_s = 0.0, λ = 0.410 μm

Lambertian BRDFs

You will sometimes see statements like

- A Lambertian surface reflects "light" equally into all directions. Lambertian surfaces are therefore also called isotropic/uniform/perfectly diffuse reflectors.

- A Lambertian surface reflects "light" with a cosine angular distribution. Lambertian surfaces are therefore also called cosine reflectors.



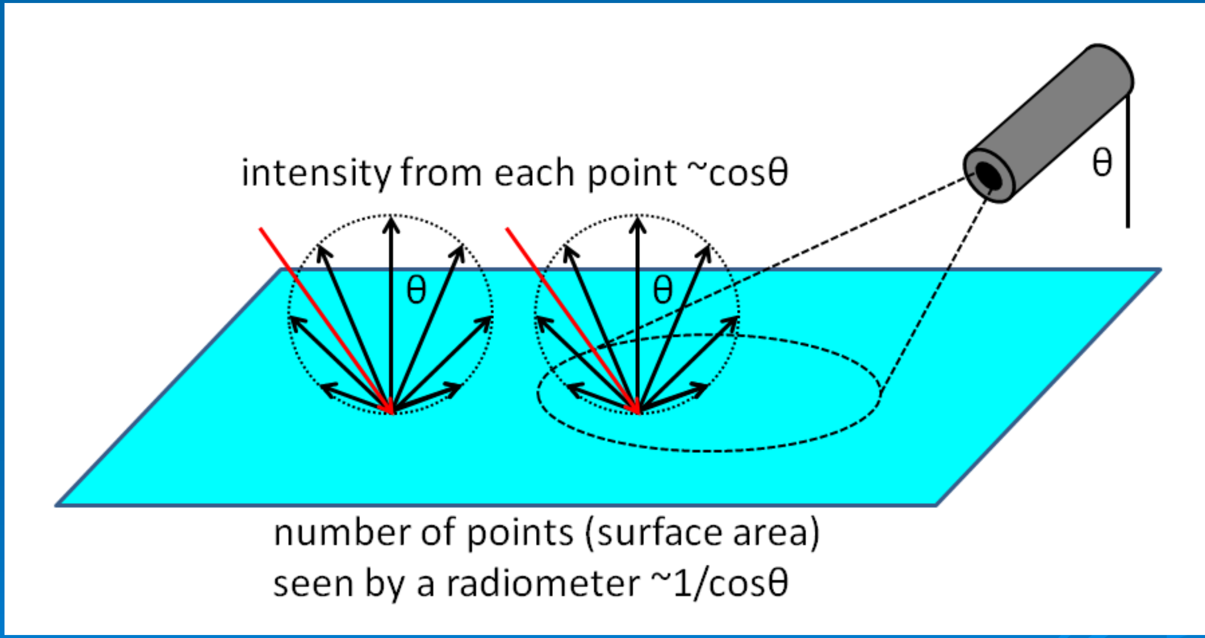
uniform reflectance cosine reflectance

Which definition is correct?

Lambertian BRDFs

The correct statements are

- Each point of a Lambertian surface reflects intensity in a cosine pattern
- A Lambertian surface reflects radiance equally in all directions



See https://www.oceanopticsbook.info/view/surfaces/lambertian-brdfs
Or OOB Section 8.8

Lambertian BRDFs

The BRDF of a Lambertian reflector is fully specified by its *reflectivity* ρ,

$$BRDF_{\text{Lamb}}(\theta_i, \phi_i, \theta_r, \phi_r) \;=\; \frac{\rho}{\pi}$$

ρ = 0 for a “black” surface
ρ = 1 for a “white” surface

The irradiance reflectance $R = E_u / E_d$ of any surface is

$$R \;=\; \frac{E_u}{E_d} \;=\; \frac{\iint_{2\pi_r} L_r(\theta_r, \phi_r)\, |\cos\theta_r|\, d\Omega_r}{\iint_{2\pi_i} L_i(\theta_i, \phi_i)\, |\cos\theta_i|\, d\Omega_i}$$

$$= \frac{\iint_{2\pi_r} \left[\iint_{2\pi_i} L_i(\theta_i, \phi_i)\, BRDF(\theta_i, \phi_i, \theta_r, \phi_r)\, |\cos\theta_i|\, d\Omega_i \right] |\cos\theta_r|\, d\Omega_r}{\int \int_{2\pi_i} L_i(\theta_i, \phi_i)\, |\cos\theta_i|\, d\Omega_i}$$

In general, *R* depends on both the surface (BRDF) and on the incident radiance.

For a Lambertian surface this reduces to

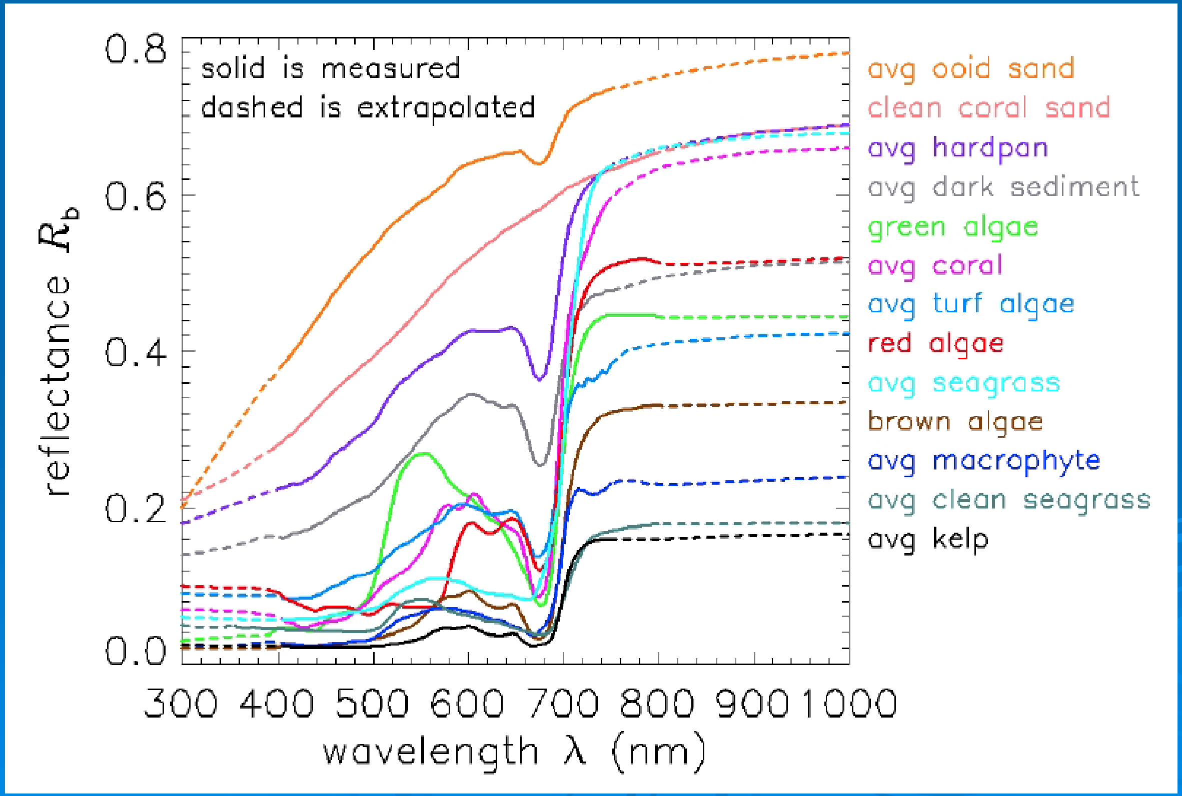
$$R \;=\; \frac{\frac{\rho}{\pi} \left[\iint_{2\pi_i} L_i(\theta_i, \phi_i)\, |\cos\theta_i|\, d\Omega_i \right] \iint_{2\pi_r} |\cos\theta_r|\, d\Omega_r}{\iint_{2\pi_i} L_i(\theta_i, \phi_i)\, |\cos\theta_i|\, d\Omega_i} \;=\; \rho$$

For a Lambertian suface, *R* depends only on the surface; $R = E_u / E_d = \rho$

HydroLight Bottom BRDFs

HydroLight does not require that the bottom BRDF be Lambertian (it is not for infinitely deep water, for example).

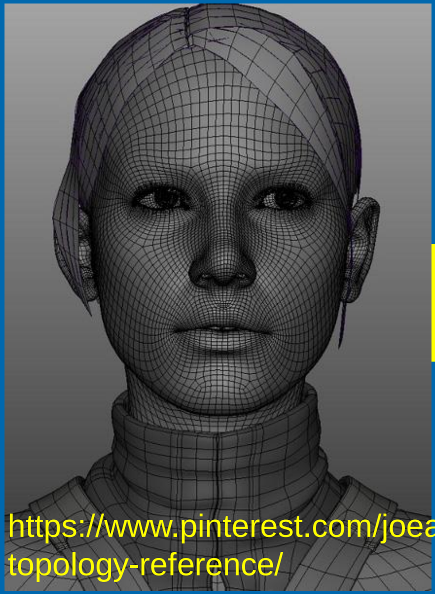
For finite depth bottoms, the default in HydroLight is to specifiy a bottom reflectance (really $\rho = E_u/E_d$), and H then assumes that the bottom is Lambertian.



Question:

Who knows more about BRDFs than anyone else?

Answer: The People Who Make Movies



https://www.pinterest.com/joeatilano4/topology-reference/

add BRDFs

https://developer.nvidia.com/gpugems/GPUGems/gpugems_ch03.html

hair and skin: 20 years of research with unlimited funding and unlimited computer power

Wind-speed dependent BRDFs of a water surface

https://support.solidangle.com/pages/viewpage.action?pageId=6455768

Monsters University (2013): > 100,000,000 hours of CPU time (24,000 cores running for almost 2 years) at Pixar. It's all Monte Carlo ray tracing and a library of BRDFs.

https://venturebeat.com/2013/04/24/the-making-of-pixars-latest-technological-marvel-monsters-university/

More Monte Carlo ray tracing and a library of BRDFs.

These Influencers Aren't Flesh and Blood, Yet Millions Follow Them

Balmain commissioned the former fashion photographer Cameron-James Wilson to create a "virtual army" of digital models, including, from left, Margot, Shudu and Zhi. Balmain

NY times, 18 June 2019

More Monte Carlo ray tracing and a library of BRDFs.

https://www.nytimes.com/interactive/2020/11/21/science/artificial-intelligence-fake-people-faces.html

Reflectance Terminology

In principle, a "reflectance" should have two adjectives: the first to describe the angular distribution of the source of the incident light and the second to describe the reflected light:

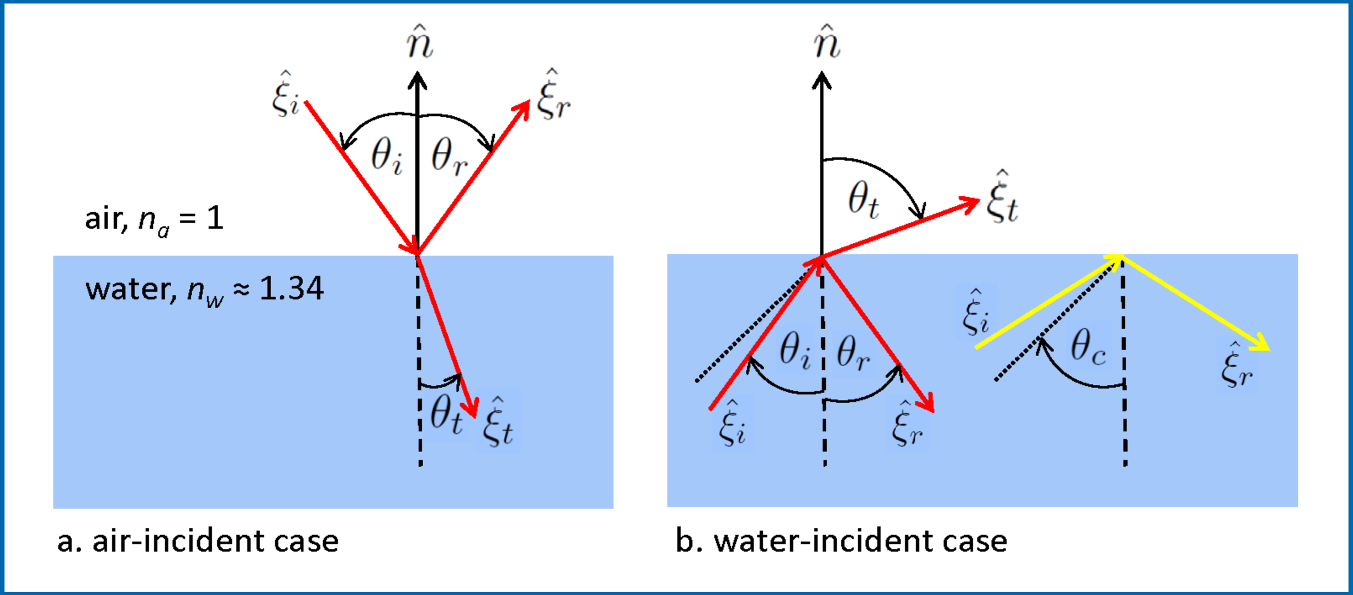
The BRDF is a BI-DIRECTIONAL reflectance: one particular incident radiance direction and one particular reflected radiance direction.

$R_{rs} = L_w / E_d$ is a HEMISPHERICAL-DIRECTIONAL reflectance: all directions in and one direction reflected.

$R = E_u / E_d$ is a HEMISPHERICAL-HEMISPHERICAL reflectance.

And so on for a dozen more measures of "reflectance" that you will see in the literature (e.g., Hapke, 1993, *Theory of Reflectance and Emittance Spectroscopy* defines 10 different reflectances and albedos, none of which corresponds to what oceanographers use.)

Level Air-water Surfaces



$$\theta_r = \theta_i$$

law of reflection

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

Snel's law (it really is Snel, not Snell)

$$\theta_t = \sin^{-1}\left(\frac{1}{n_w}\sin\theta_i\right)$$

for air-incident light ($n_1 = n_a = 1$; $n_2 = n_w$)

$$\theta_t = \sin^{-1}(n_w \sin\theta_i)$$

for water-incident light ($n_1 = n_w$; $n_2 = n_a = 1$)

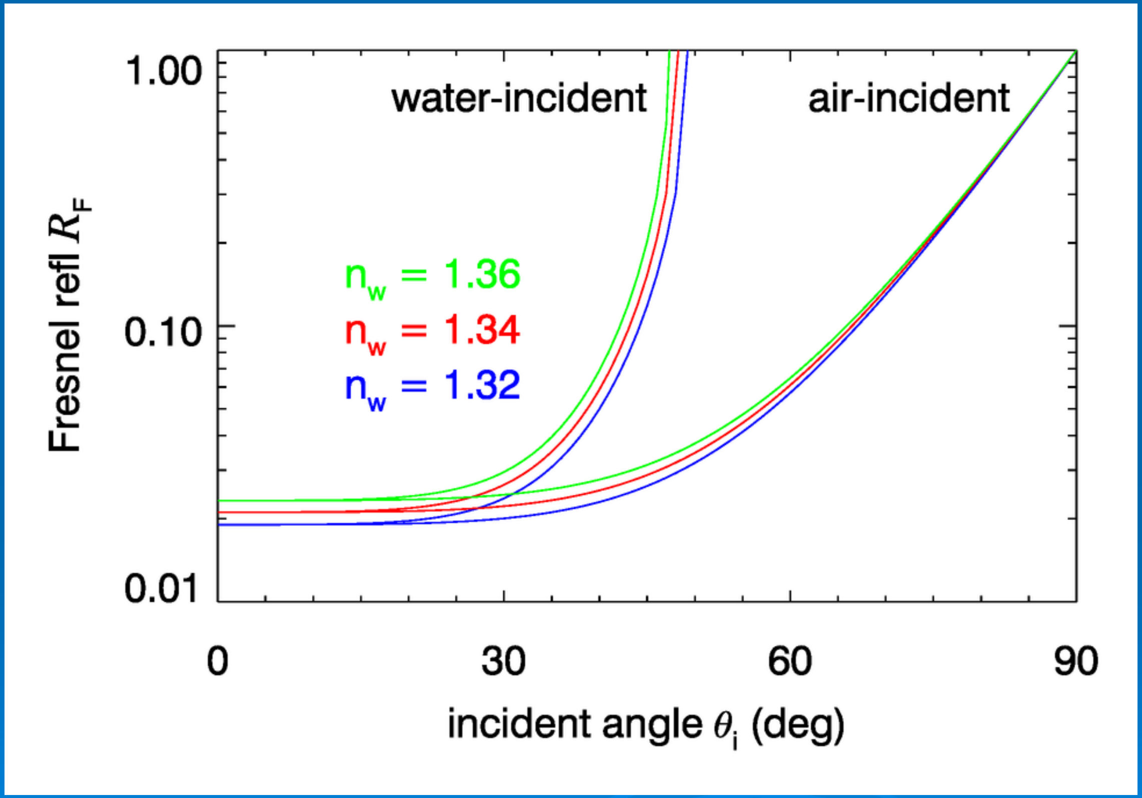
$$\theta_c = \sin^{-1}(1/n_w)$$

critical angle for total internal reflection

Reflection for Unpolarized Radiance

$$R_F(\theta_i) = \frac{1}{2}\left\{\left[\frac{\sin(\theta_i - \theta_t)}{\sin(\theta_i + \theta_t)}\right]^2 + \left[\frac{\tan(\theta_i - \theta_t)}{\tan(\theta_i + \theta_t)}\right]^2\right\}$$

Fresnel's Law

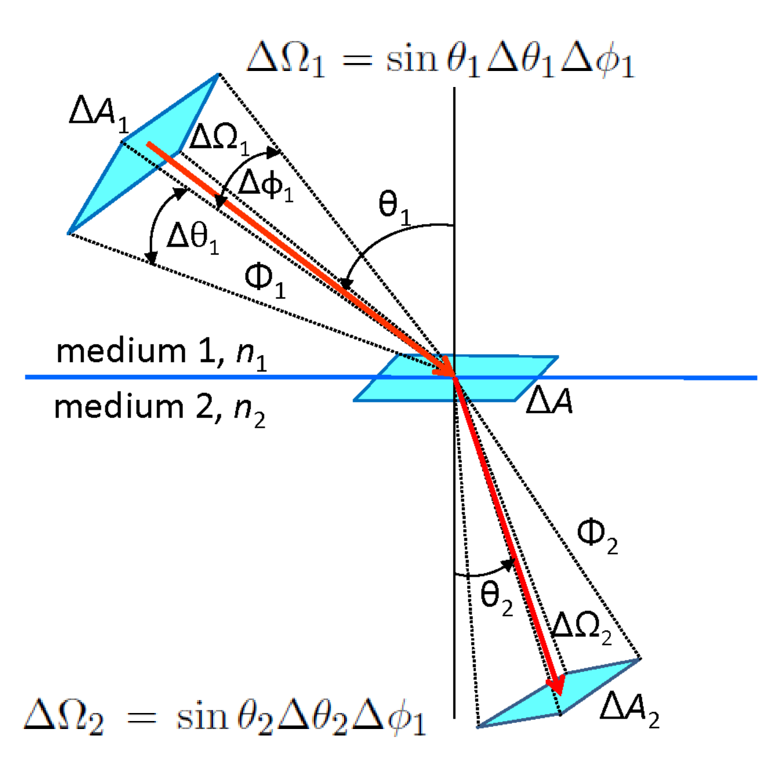


special case for normal incidence:

$$R_F(\theta_i = 0) = \left(\frac{n_w - 1}{n_w + 1}\right)^2$$

For the polarized reflection and transmission matrices, see OOB Sec. 8.3 or https://www.oceanopticsbook.info/view/surfaces/level-2/fresnel-equations-for-polarization

n^2 Law for Radiance



Example:
Normal incidence, $n_w = 1.34$, $T_F \approx 0.979$
The radiance just below a water surface is
$L_{\text{water}} = 0.979(1.34)^2 L_{\text{air}} \approx 1.76 L_{\text{air}}$

It's the law of conservation of energy, not the law of conservation of radiance. L/n^2 is conserved.

Squaring $n_1 \sin\theta_1 = n_2 \sin\theta_2$ and differentiating gives

$$n_1^2 \sin\theta_1 \cos\theta_1 \Delta\theta_1 = n_2^2 \sin\theta_2 \cos\theta_2 \Delta\theta_2$$

Multiplying by $\Delta\phi$ and rewriting in terms of solid angles
$\Delta\Omega = \sin\theta \Delta\theta \Delta\phi$
gives

$$n_1^2 \cos\theta_1 \Delta\Omega_1 = n_2^2 \cos\theta_2 \Delta\Omega_2$$

The radiances are defined by

$$L_1 = \frac{\Delta\Phi_1}{\Delta A_1 \Delta\Omega_1} \quad \text{and} \quad L_2 = \frac{\Delta\Phi_2}{\Delta A_2 \Delta\Omega_2}$$

Fresnel's equation gives the transmitted power as
$\Delta\Phi_2 = [1 - R_F(\theta_1)]\Delta\Phi_1 = T_F \Delta\Phi_1$
The areas are related by
$\Delta A_1 = \Delta A \cos\theta_1$ and $\Delta A_2 = \Delta A \cos\theta_2$
Thus the ratios of the incident and transmitted radiances are

$$\frac{L_2}{L_1} = \frac{\Delta\Phi_2}{\Delta\Phi_1}\frac{\Delta A_1 \Delta\Omega_1}{\Delta A_2 \Delta\Omega_2} = T_F \frac{\cos\theta_1 \Delta\Omega_1}{\cos\theta_2 \Delta\Omega_2} = T_F \frac{n_2^2}{n_1^2}$$

or

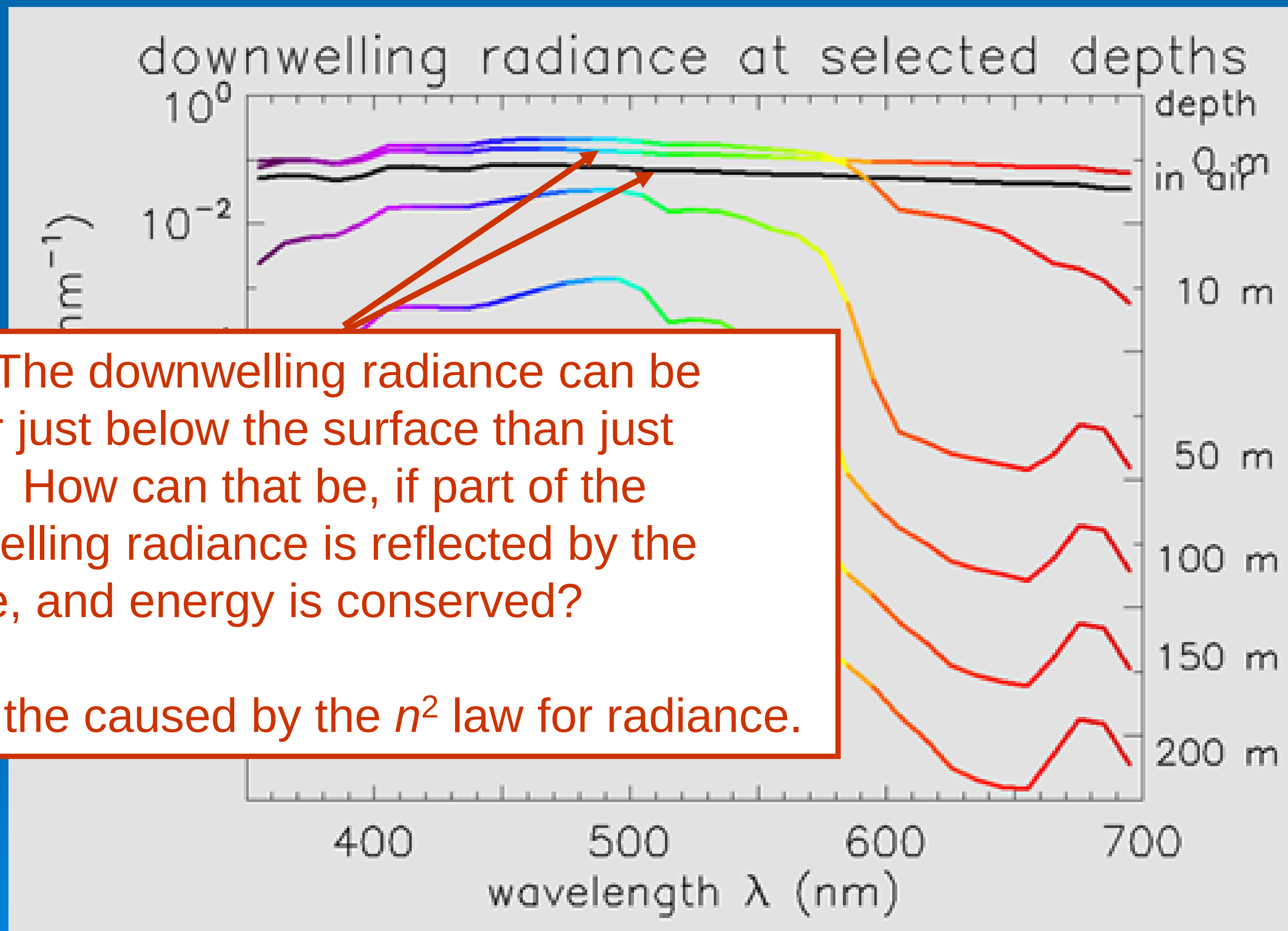
$$\frac{L_2}{n_2^2} = T_F \frac{L_1}{n_1^2}$$

The n^2 law for radiance

which is the n^2 law for radiance

Recall from the Radiometry Lecture...

We can now answer the question about how radiance can be greater below the surface than above.

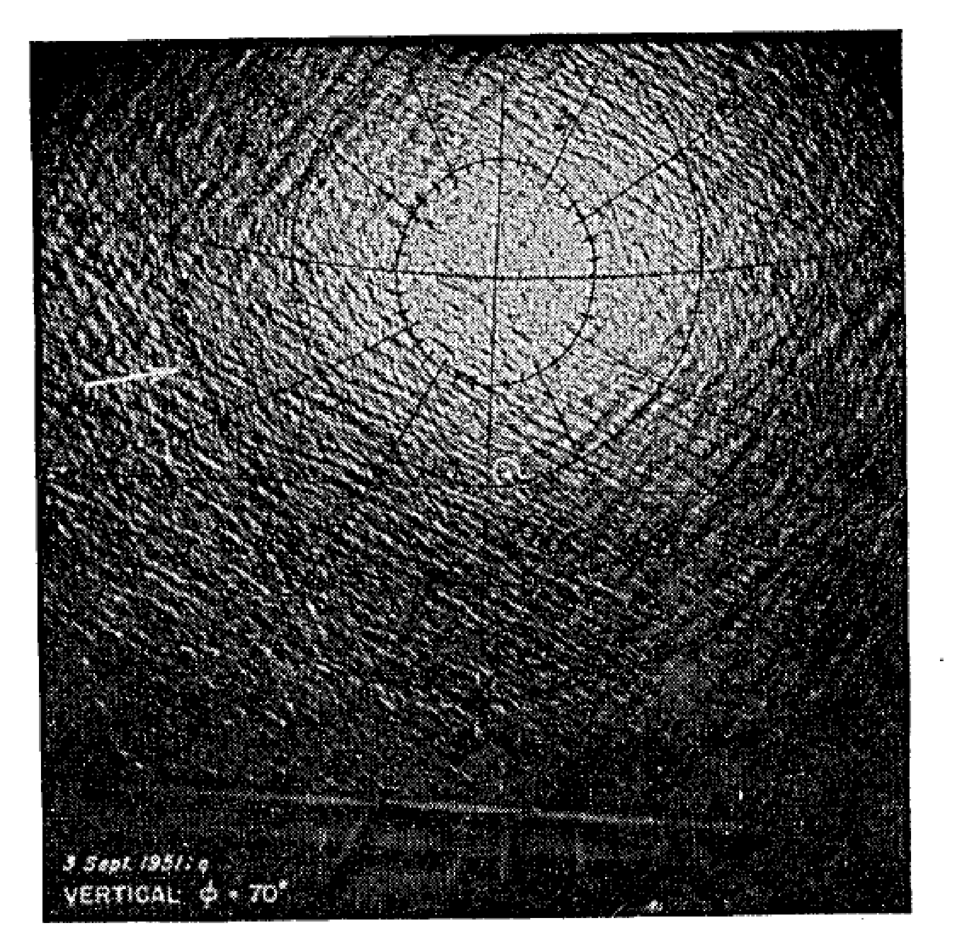


Note: The downwelling radiance can be greater just below the surface than just above. How can that be, if part of the downwelling radiance is reflected by the surface, and energy is conserved?

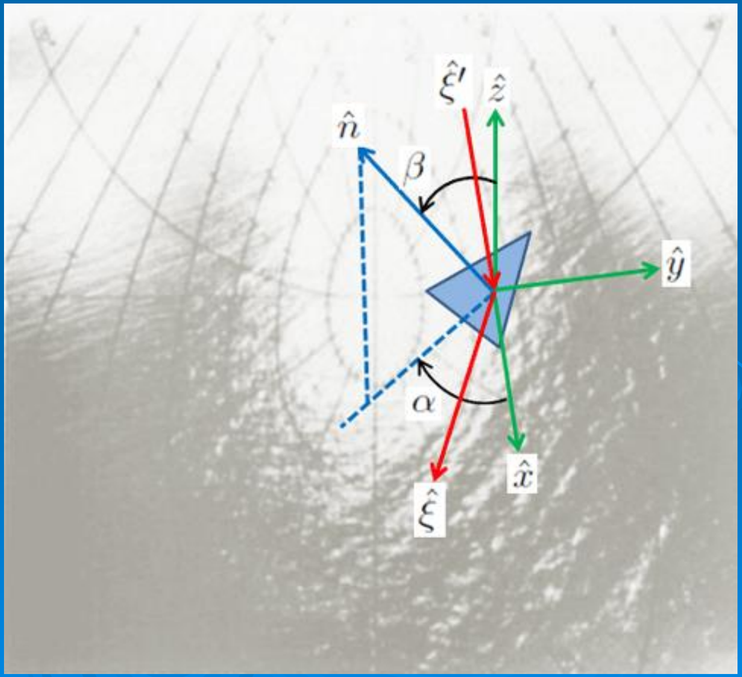
This is the caused by the n^2 law for radiance.

Random Sea Surfaces: Cox-Munk

Cox and Munk (1954) took photographs of sea surface glitter patterns from an airplane and determined what the surface slopes gave the glitter patterns.



wind = 8.6 m/s



Cox and Munk (1954)

Cox-Munk Wave slope-Wind speed Model

σ_c^2 is the slope variance in the cross-wind direction
σ_u^2 is the slope variance in the along-wind direction
$\sigma^2 = \sigma_c^2 + \sigma_u^2$ is the total slope variance

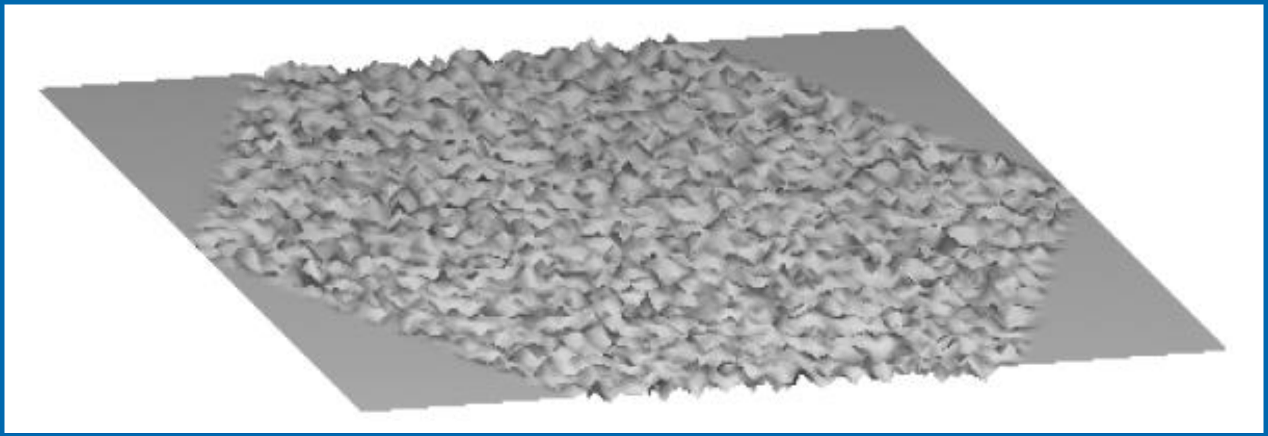
$$\sigma_c^2 = 0.003 + 1.92 \times 10^{-3} W \pm 0.002$$
$$\sigma_u^2 = 0.000 + 3.16 \times 10^{-3} W \pm 0.004$$
$$\sigma_c^2 + \sigma_u^2 = 0.003 + 5.12 \times 10^{-3} W \pm 0.004$$

These equations are one option in HydroLight to generate random realizations of sea surfaces. See Light and Water §4.3 for the math.

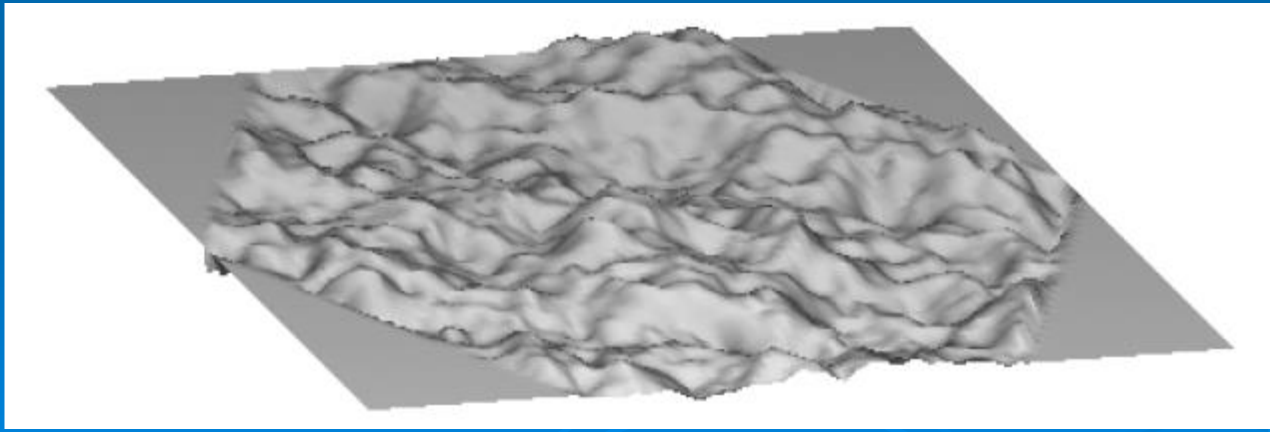
The Cox-Munk equations correctly reproduce the sea surface *slopes*, which is what matters most for optics.

They do NOT reproduce the surface *heights*, which is what matters most for the visual appearance of the water surface.

Random Sea Surface Realizations



a surface generated using Cox-Munk statistics. The slope statistics are correct, but not the wave elevations



a surface that reproduces both wave slopes and wave elevations

Sea Surface Generation Based on Variance Spectra

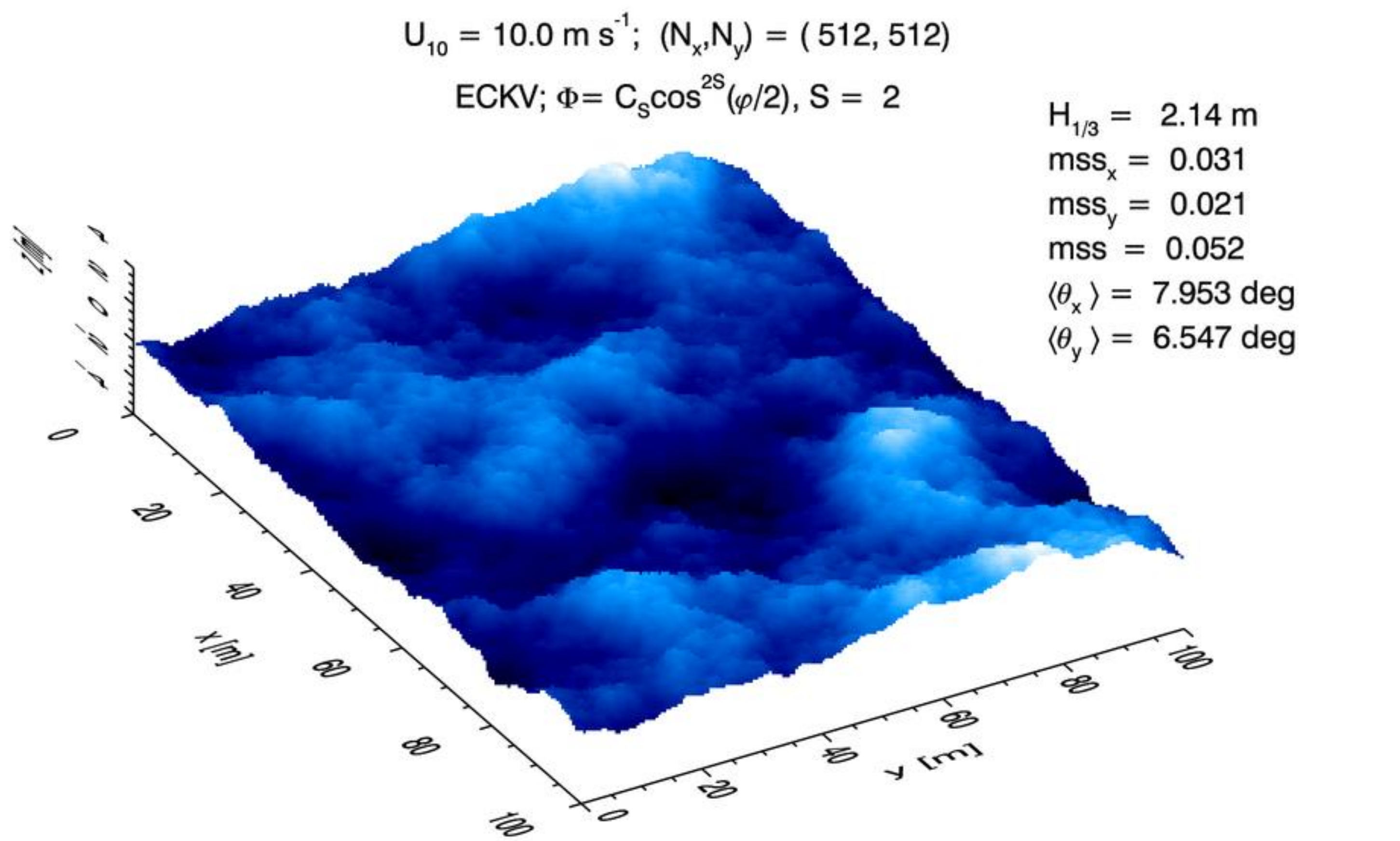
A better way to generate random sea surfaces is to use wave elevation variance spectra ("power spectra") as the starting point.

Fourier transforms can then be used to generate sea surfaces that reproduce both the wave slopes and the elevations.

How this is done is very complicated (a full day class covering Fourier transforms, wave spectra, and how to combine them)

See my Tutorial *Modeling Sea Surfaces,* the web book pages starting at
https://oceanopticsbook.info/view/surfaces/level-2/modeling-sea-surfaces
or OOB Appendices A, B, C, & D

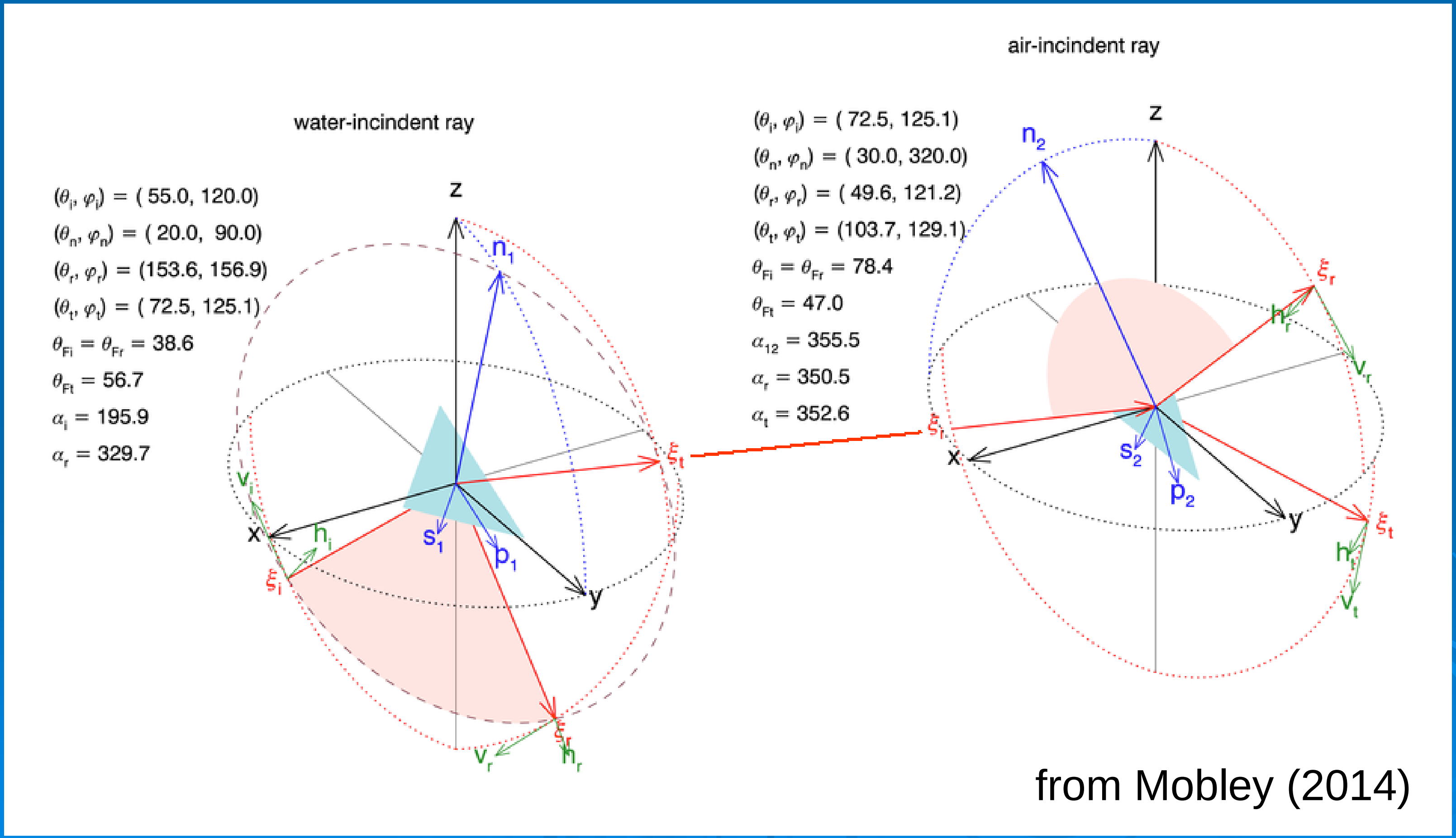
Sea Surface Generation Based on Variance Spectra



slope variable	DFT value	Cox-Munk formula	Cox-Munk value
mss_x	0.031	$0.0316U$	0.032
mss_y	0.021	$0.0192U$	0.019
mss	0.052	$0.001(3+5.12U)$	0.054

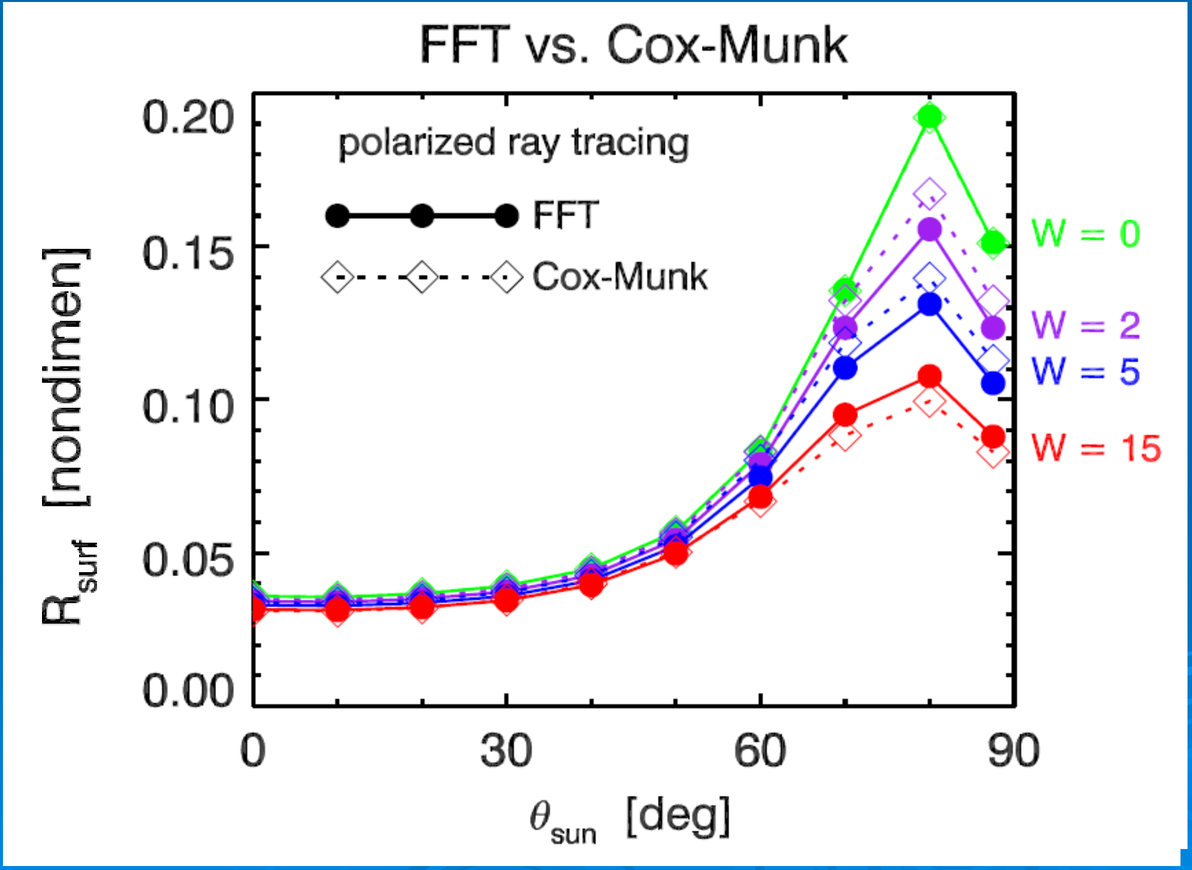
Surfaces Are Used for Monte Carlo Ray Tracing to Compute Surface Optical Properties

Very complicated math; see Mobley (2014) HydroPol Math Doc, Appendix B

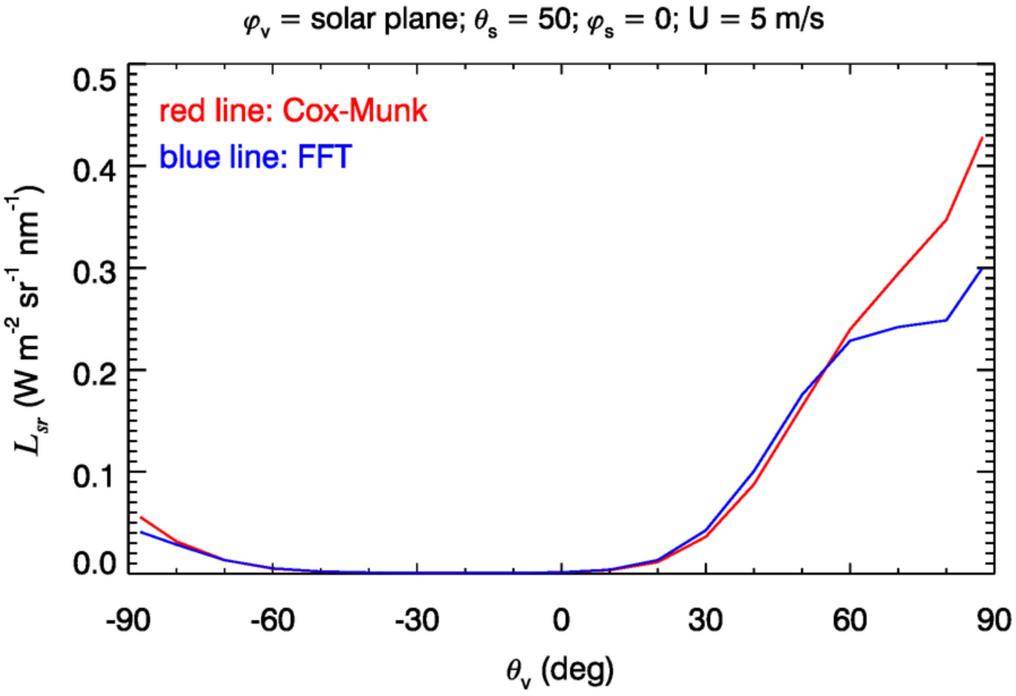


Cox-Munk vs Fully Resolved Surfaces

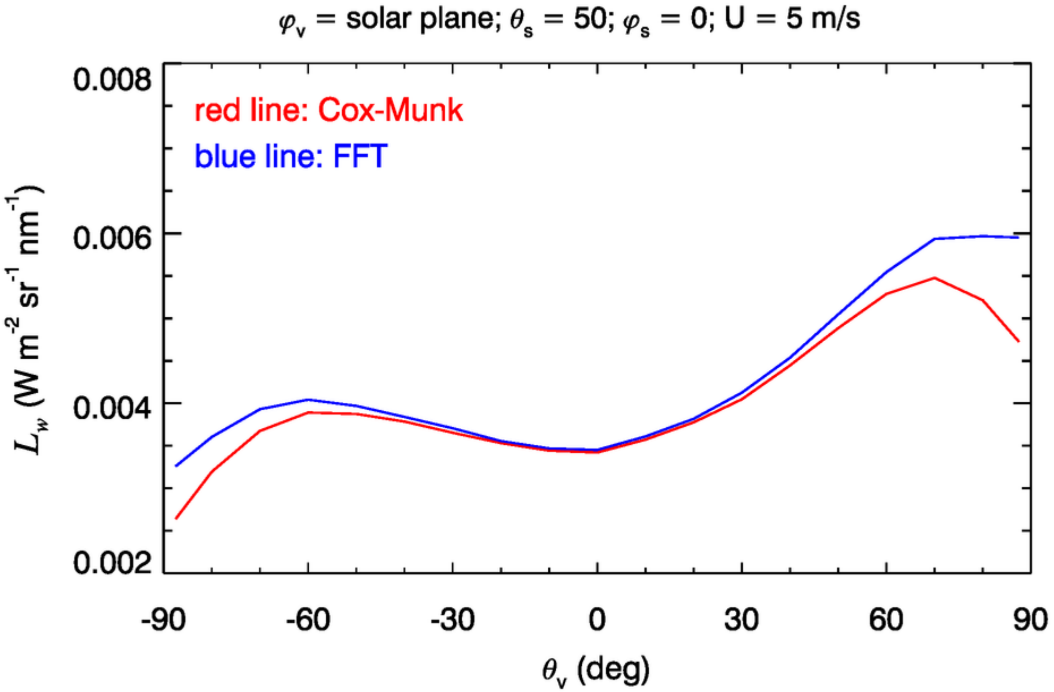
For sea-surface *energy reflectance* calculations, Cox-Munk irradiance reflectances $R = E_u/E_d$ are close to those computed using more realistic surfaces. This *is not true for radiance reflectance in particular directions* (glitter patterns can be different).



Mobley (2015)

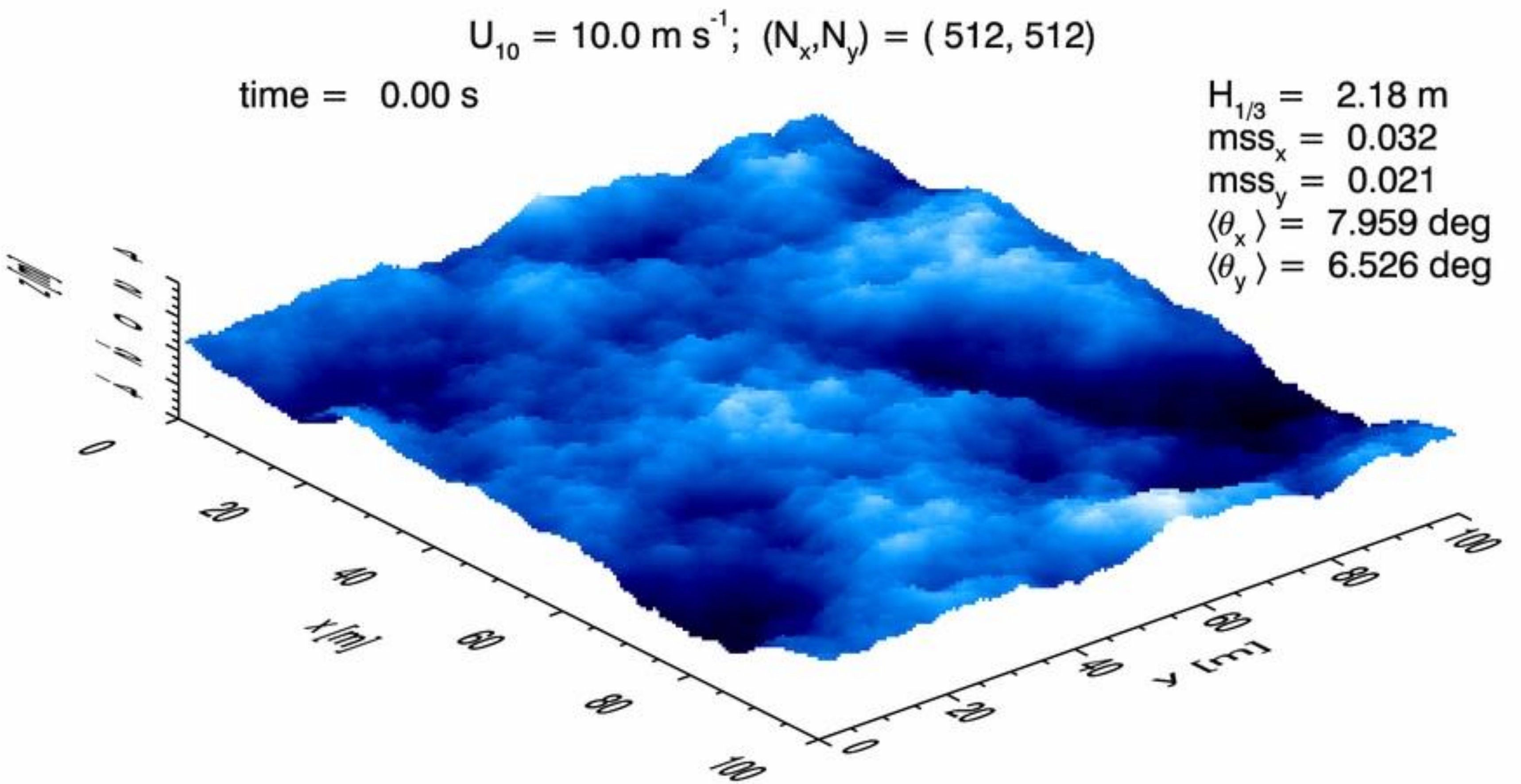


Cox-Munk does very well in most cases because the slopes are correct, except for near-grazing angles

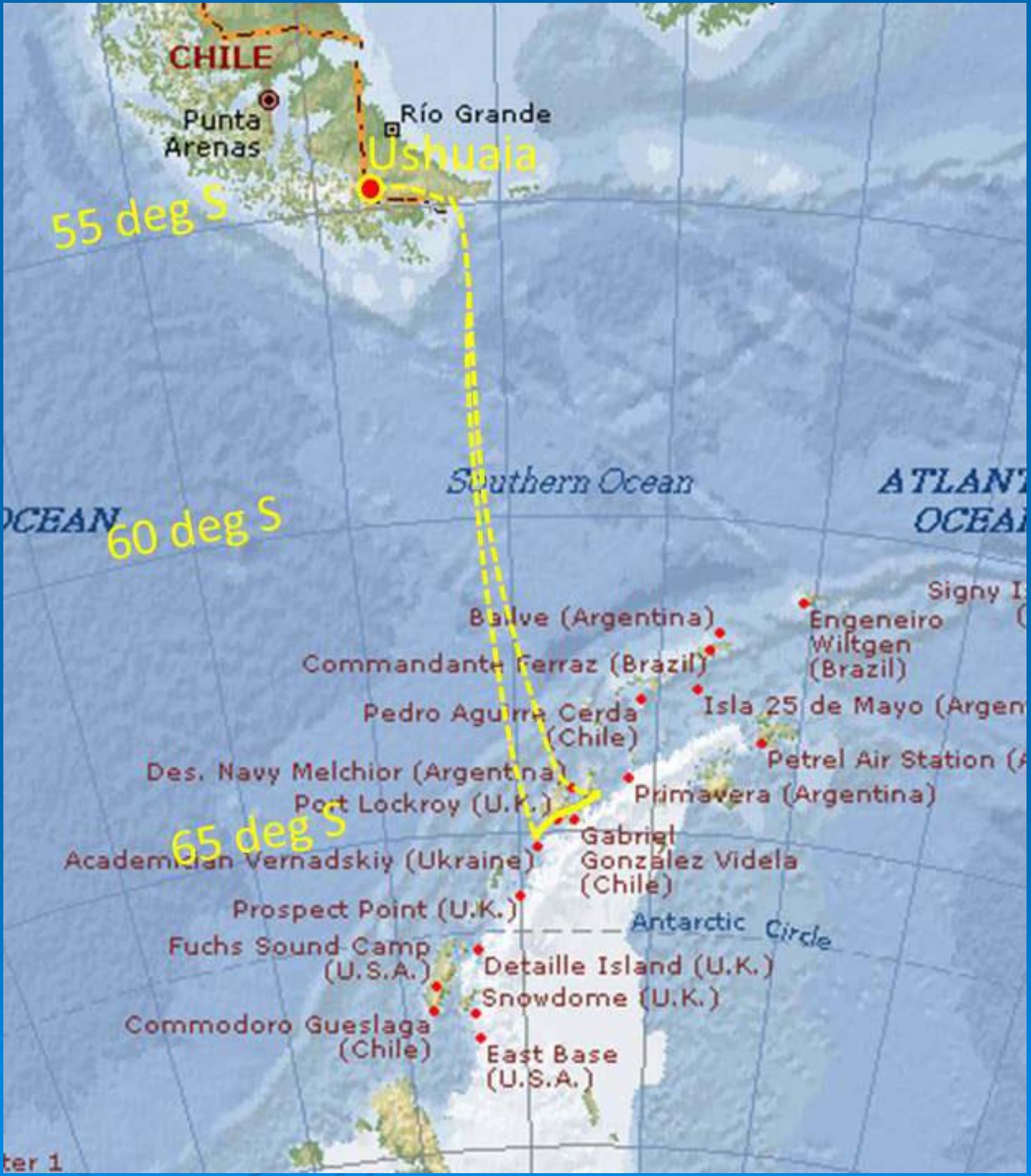


Note: the water-leaving radiance L_w is NOT isotropic

Time-dependent Sea Surface Generated from a Wave Variance Spectrum and FFTs



There is only one really good way to learn about sea surfaces: cross the Drake Passage in a 20 m sailboat.



Which will give a more authentic and exciting Antarctic experience?

Emerald Princess:	951 feet;	113,561 tons;	crew of 1200;	3114 passengers
Icebird:	60 feet;	39 tons;	crew of 2;	7 passengers

The best way to learn about waves