

2021 Summer Course
on Optical Oceanography and Ocean
Color Remote Sensing

Curtis Mobley

Photometry: Visibility and Color

Delivered at the Bowdoin College
Schiller Coastal Studies Center

Today: A Quick Survey of Photometry

Radiometry: what instruments measure (energy units)

Photometry: what eyes see (physiological sensation)

Photometry enables us to study “visibility” and “color” as seen by humans.

- **Visibility:** Derive some basic laws used to determine if we can or cannot see something
 - Contrast
 - Secchi depth
 - Point and beam spread functions
- **Color:** See how the physiological concept of “color” is quantified
 - CIE chromaticity diagram
 - Forel-Ule index



Photometry: Light Measures for the Human Eye

Radiometry is based on measurements of spectral radiance as made by instruments that measure light as a function of wavelength, one wavelength at a time, using energy units.

When the human eye is the sensor, all wavelengths are seen at once, but the eye is not equally sensitive to all wavelengths. To understand what humans see, we must use broadband radiometric variables weighted by the relative response of the eye at different wavelengths.

The “human eye equivalents” of radiometric variables are called photometric variables.

Every radiometric quantity, IOP, and AOP has a photometric equivalent.

The photometric equivalent of radiance is called luminance, and corresponds to what we think of as “brightness”.

Camouflage

Underwater photo of a coral head. What animal do you see?



<https://www.youtube.com/watch?v=eS-USrwuUfA>

Who Cares About Underwater Visibility?

Alexander the Great
~330 BCE

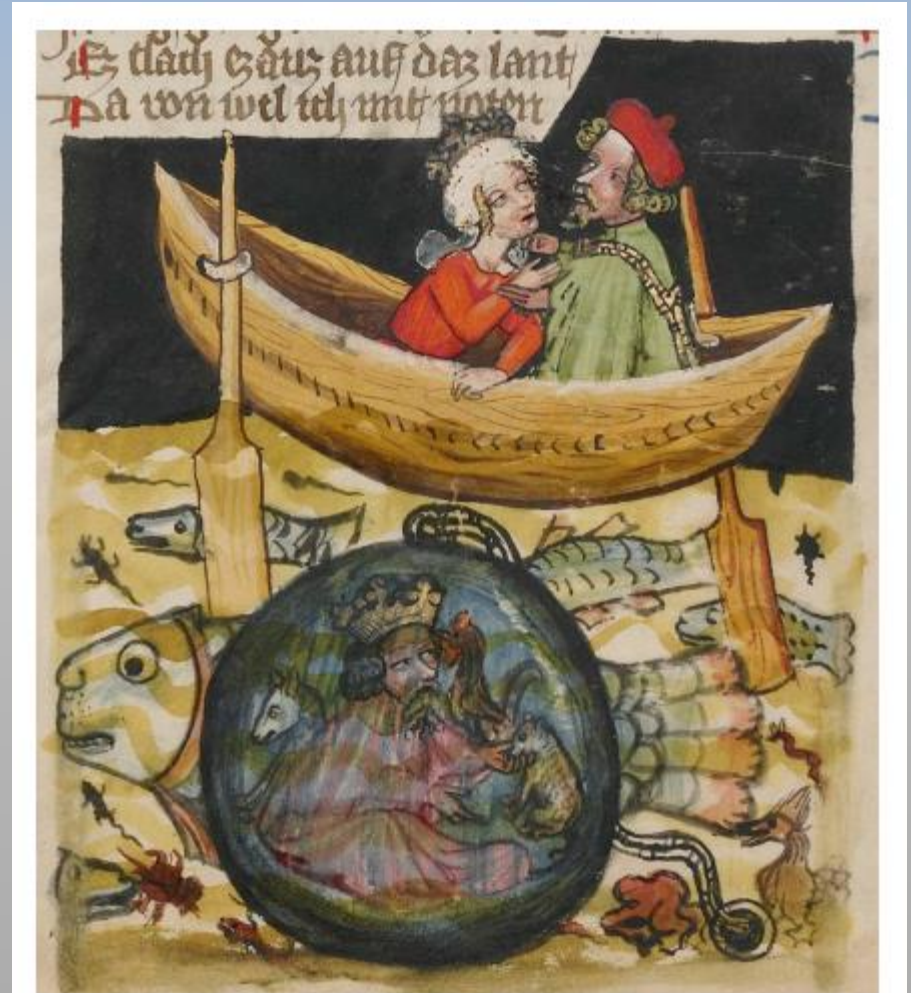


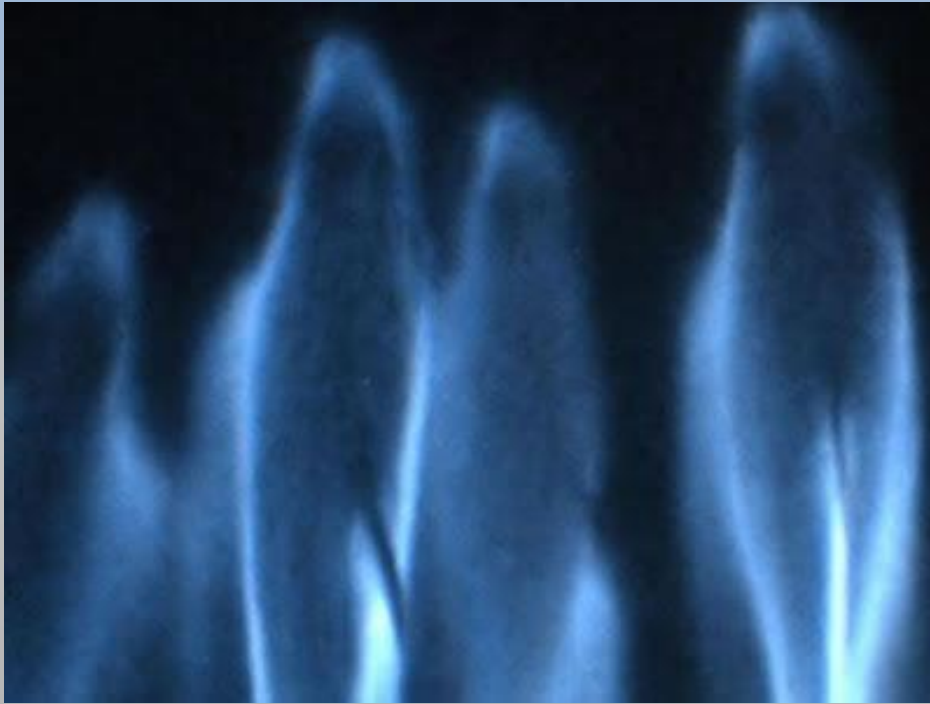
Fig. 6. Tempera painting from the 15th century depicting Alexander the Great in a clear diving bell that has been cast adrift by his wife and her lover. (Image courtesy of the J. Paul Getty Museum.)

Who Cares About Underwater Visibility?

Every Navy in the world



Who Cares About Underwater Visibility?



Dolphins, fishermen, divers, swimmers, marine archaeologists, and the Creature from the Black Lagoon



Liu Bolin: The Invisible Man



Grocery Shopping in China



WWII Razzle-Dazzle Camouflage



Merchant vessel dazzle-painted as seen through a submarine periscope.

WWII Razzle-Dazzle Camouflage

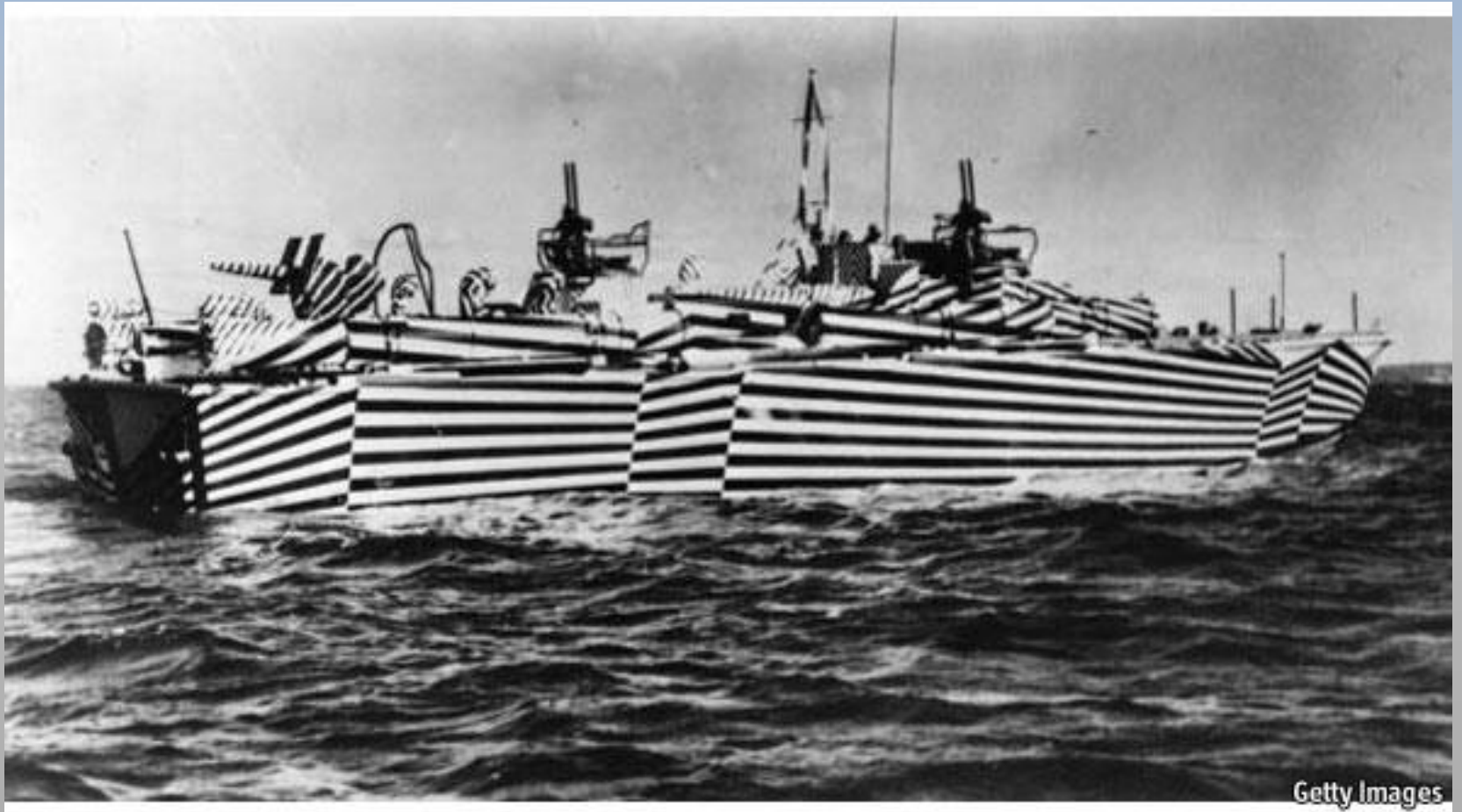


Merchant vessel dazzle-painted as seen through a submarine periscope.



The same vessel on identical course painted grey,

Razzle-Dazzle Ship Camouflage



Getty Images

Levels of Visibility

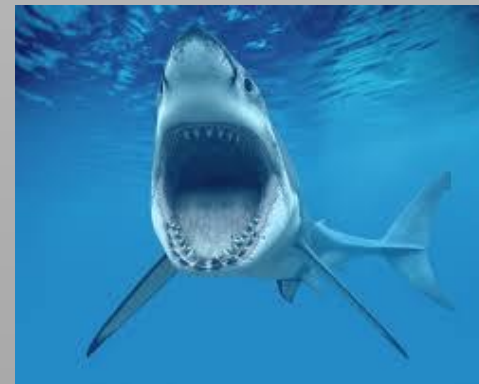
Detection: I can see that there is something in the water, but I can't tell what it is.



Classification: I can see what general type of thing it is: it's a fish, not a scuba diver.



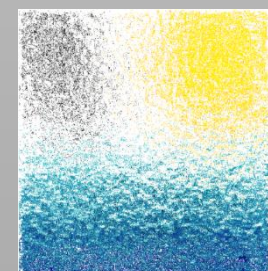
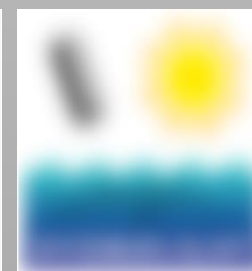
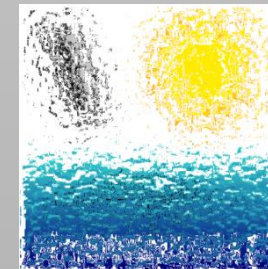
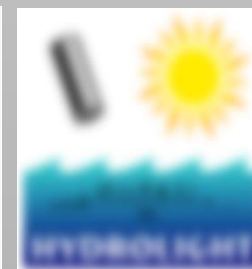
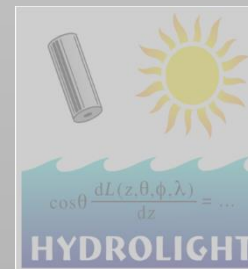
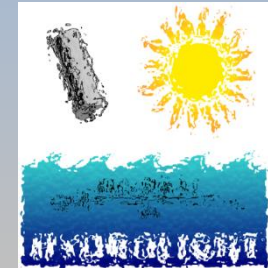
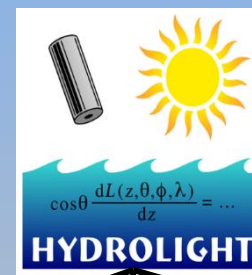
Identification: I can see exactly what it is: it's a hungry great white shark.



Environmental Effects on Visibility

- Absorption
- Scattering by particles
- Scattering by turbulence

affect visibility in different ways

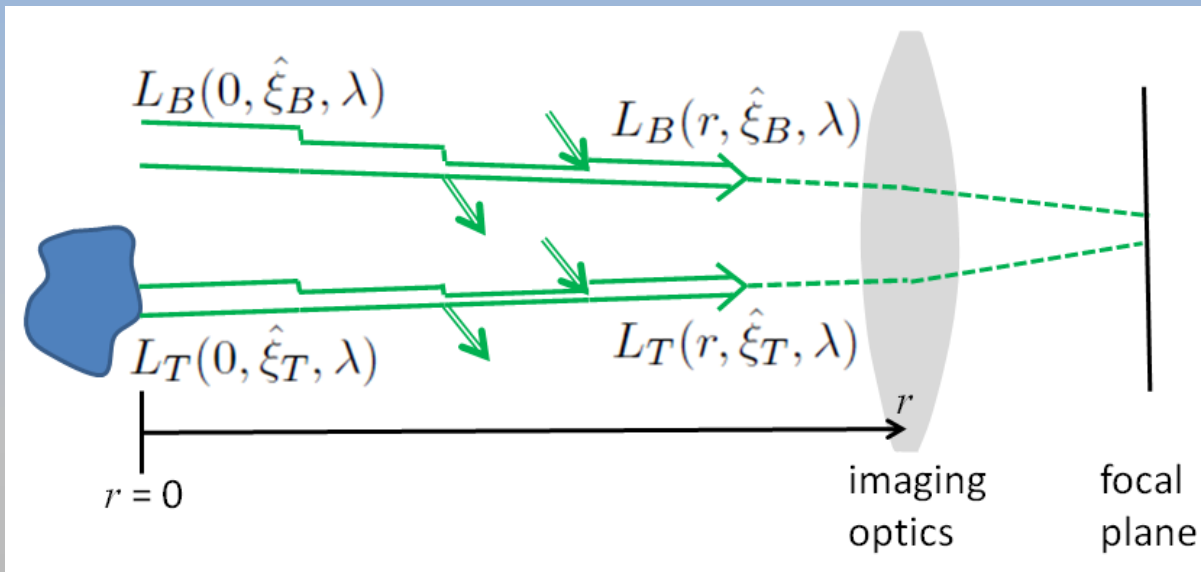


absorption:
image stays
crisp but
fades out

particles:
image steady
but blurred

turbulence:
image
twinkles
or breaks up

The Radiance Difference Law



Assume that

- A small object (the target) illuminated by ambient daylight
- The radiance leaving the target does not significantly affect the ambient radiance that is present in the absence of the target.
- The two directions $\hat{\xi}_T$ and $\hat{\xi}_B$ are almost parallel, $\hat{\xi}_T \approx \hat{\xi}_B = \hat{\xi}$, so that the absorbing and scattering losses and additions to each beam are the same between the target area and the imaging system.

The Radiance Difference Law

The RTE

$$\begin{aligned}\frac{dL(r, \hat{\xi}, \lambda)}{dr} &= -c(r, \lambda) L(r, \hat{\xi}, \lambda) + \int_{4\pi} L(r, \hat{\xi}', \lambda) \beta(\hat{\xi}' \rightarrow \hat{\xi}, \lambda) d\Omega(\hat{\xi}') \\ &\equiv -c(r, \lambda) L(r, \hat{\xi}, \lambda) + L^*(r, \hat{\xi}, \lambda)\end{aligned}$$

governs the changes along path $\hat{\xi}$ in the initial radiances for both the background and the target. Because the ambient radiance distribution and the IOPs are assumed to be the same for each path, the path radiance term L^* , which describes scattering into the beam, will be the same for both background and target radiances. Thus the two radiances obey

$$\frac{dL_T(r, \hat{\xi}, \lambda)}{dr} = -c(r, \lambda) L_T(r, \hat{\xi}, \lambda) + L^*(r, \hat{\xi}, \lambda) \quad (1)$$

$$\frac{dL_B(r, \hat{\xi}, \lambda)}{dr} = -c(r, \lambda) L_B(r, \hat{\xi}, \lambda) + L^*(r, \hat{\xi}, \lambda) \quad (2)$$

The Radiance Difference Law

Subtracting Eq. (2) from Eq. (1) gives

$$\frac{d[L_T(r, \hat{\xi}, \lambda) - L_B(r, \hat{\xi}, \lambda)]}{dr} = -c(r, \lambda) [L_T(r, \hat{\xi}, \lambda) - L_B(r, \hat{\xi}, \lambda)]$$

This equation has the solution

$$[L_T(r, \hat{\xi}, \lambda) - L_B(r, \hat{\xi}, \lambda)] = [L_T(0, \hat{\xi}, \lambda) - L_B(0, \hat{\xi}, \lambda)] \exp \left[- \int_0^r c(r', \lambda) dr' \right]$$

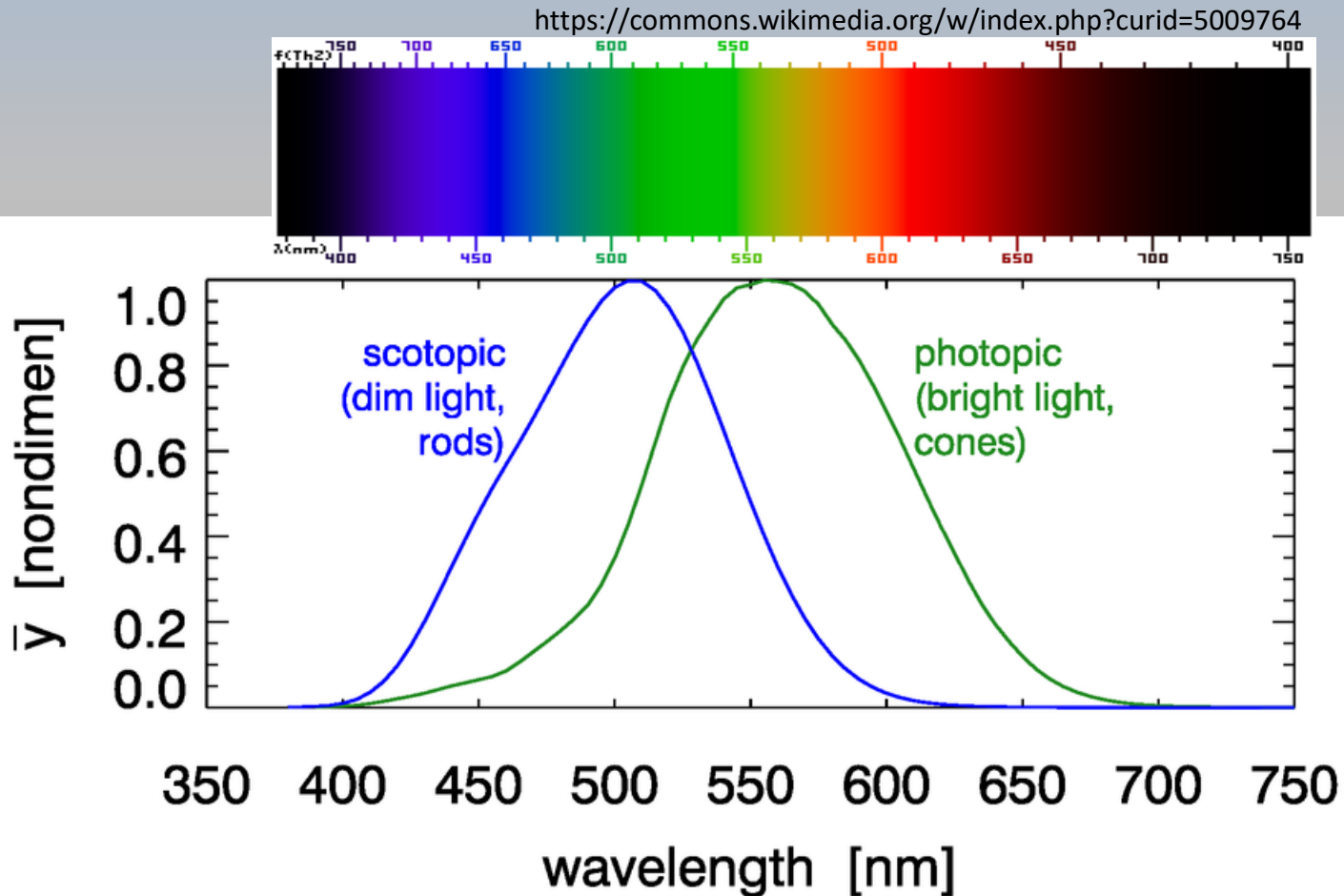
Usually $c(r, \lambda)$ can be assumed constant along the viewing path (a few tens of meters at most), in which case this solution reduces to

$$[L_T(r, \hat{\xi}, \lambda) - L_B(r, \hat{\xi}, \lambda)] = [L_T(0, \hat{\xi}, \lambda) - L_B(0, \hat{\xi}, \lambda)] \exp[-c(\lambda) r]$$

Even though each radiance L_T and L_B individually depends on all IOPs, their difference depends only on the beam attenuation c . This is known as the radiance difference law (e.g., Preisendorfer, 1986).

Luminosity Functions

The *photopic luminosity function* shows the relative sensitivity of a normal human eye to light of different wavelengths, for bright-light (daytime) illumination. The *scotopic luminosity function* shows the sensitivity for dim (nighttime) illumination.



Radiometry to Photometry

Radiometric variables are in energy units of watts = Joules/sec.

Photometric or “vision” variables are in units of lumens, which is a measure of “brightness” as seen by the human eye.

$$L_v \equiv K_m \int_0^\infty L(\lambda) \bar{y}(\lambda) d\lambda \quad [\text{lumen m}^{-2} \text{sr}^{-1}]$$

$$\frac{\text{lumen}}{\text{m}^2 \text{sr}}$$

$$683 \frac{\text{lumen}}{\text{W}}$$

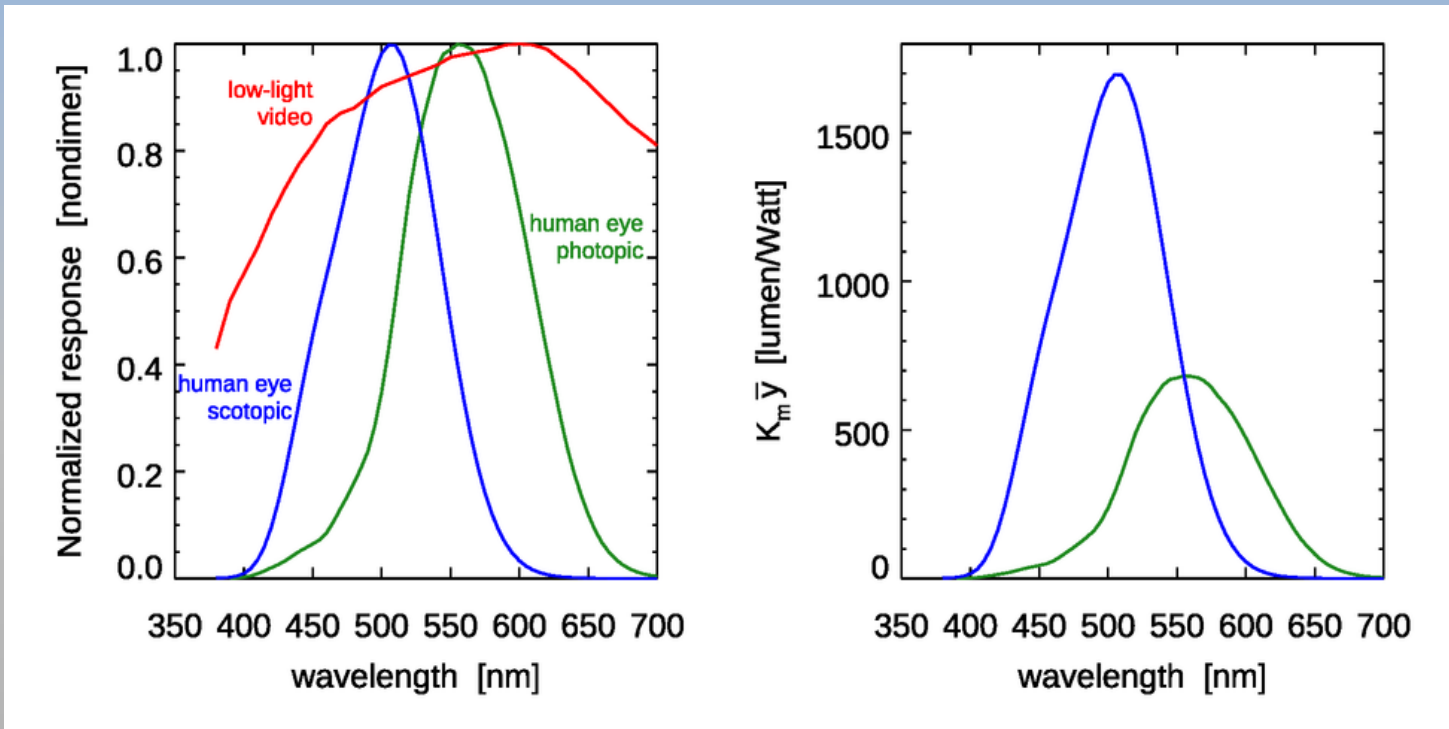
$$\frac{\text{W}}{\text{m}^2 \text{sr nm}}$$

photopic luminosity function (nondimen)

The power-to-brightness conversion factor K_m is called the *maximum luminous efficacy*. Its value traces back to a “standard candle.” (For scotopic, $K_m = 1700 \text{ lm/W}$)

SI fundamental unit of brightness: 1 Candela (cd) = 1 lumen/steradian
(an ordinary candle is about 1 Candela)

Brightness vs Power



Note: even though a laser might emit a lot of power at 350 nm, and can blind you if the exposure is long enough, you cannot see the ultraviolet laser light (so $y_{bar}(\lambda) = 0$). The laser is powerful, but not visually bright.

Typical Luminances

The sun is roughly a black body at $T = 5782$ K. The corresponding luminance is

$$L_v^{\text{sun}} = K_m \int_0^\infty L_{BB}(T = 5782 \text{ K}, \lambda) \bar{y}(\lambda) d\lambda = 1.86 \times 10^9 \text{ lm m}^{-2} \text{ sr}^{-1}$$

$$L_{BB}(\lambda) = \frac{2hc^2}{\lambda^5} \frac{1}{e^{hc/\lambda kT} - 1}$$

from the web book page
on black body radiation

Source	Luminance ($\text{lm m}^{-2} \text{ sr}^{-1} = \text{cd m}^{-2}$)
solar disk, above the atmosphere	2×10^9
solar disk, at Earth's surface, Sun near the zenith	1×10^9
melting platinum at 2042 K	$\equiv 6 \times 10^5$
60 W frosted light bulb	1×10^5
sunlit snow surface	1×10^4
full Moon's disk	6×10^3
clear blue sky, directions away from the Sun	3×10^3
heavy overcast, zenith direction	1×10^3
twilight sky	3
clear sky, moonlit night	3×10^{-2}
overcast sky, moonless night	3×10^{-5}

from the OOB

Table 11.1: Typical luminances L_v .

Lighting Facts Per Bulb

Brightness **820 lumens**

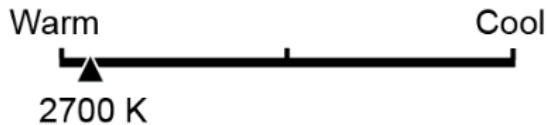
Estimated Yearly Energy Cost \$7.23

Based on 3 hrs/day, 11¢/kWh
Cost depends on rates and use

Life

Based on 3 hrs/day **1.4 years**

Light Appearance



Energy Used **60 watts**

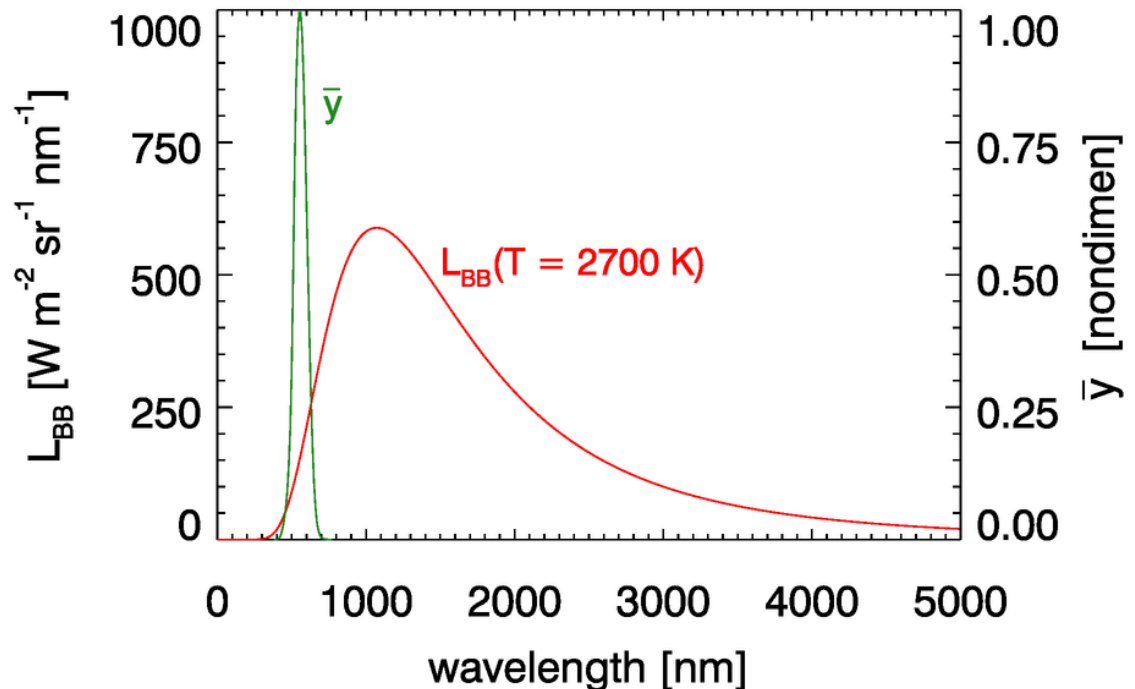
Lightbulb Inefficiency

60 W x 683 lumens/W = 40,980 lumens

Actually getting 820 lm, so just 2% of energy used is converted to what you can actually see. The rest is heat.

$$L_{BB}(\lambda) = \frac{2hc^2}{\lambda^5} \frac{1}{e^{hc/\lambda kT} - 1}$$

from the web book page
on black body radiation



Iluminance

Just as irradiances can be computed from radiances, illuminances can be computed from luminances by equations of the same form as for radiometric variables. For example

$$E_{dv} = \int_{2\pi_d} L_v(\theta, \phi) |\cos \theta| \sin \theta d\theta d\phi$$

and also

$$R_v = \frac{E_{uv}}{E_{dv}}$$

$$K_{dv}(z) = -\frac{d \ln E_{dv}(z)}{dz}$$

and so on...

from the OOB

Source	Illuminance (lm m ⁻² = lux)
Sun at the zenith, clear sky	1 × 10 ⁵
Sun at 60 deg zenith angle, clear sky	5 × 10 ⁴
overcast day	1000
well-lit room	300-500
very dark, heavily overcast day	100
full Moon at 60 deg zenith angle, clear sky	0.2
starlight, moonless night, clear sky	4 × 10 ⁻³
moonless night, heavy overcast, in a thick forest	10 ⁻⁴

Table 11.2: Typical Illuminances E_{dv} .

Radiometric IOPs become Photometric AOPs

The photometric equivalents of IOPs are computed by equations of the form

$$c_v(\hat{\xi}) = \frac{\int_0^\infty L(\hat{\xi}, \lambda) \bar{y}(\lambda) c(\lambda) d\lambda}{\int_0^\infty L(\hat{\xi}, \lambda) \bar{y}(\lambda) d\lambda} \quad [\text{m}^{-1}]$$

The photopic beam attenuation

The photopic beam attenuation c_v depends on the viewing direction, even though beam c does not, because of the weighing by the ambient radiance.

c_v is therefore an AOP, not an IOP. There are no IOPs for photometric variables because their photometric equivalents are computed by equations like that for c_v .

However, in many (most?) cases, c_v is within a few percent of $c(555 \text{ nm})$, and is almost independent of the ambient radiance and viewing direction, as should be the case for a “good” AOP.

The Luminance Difference Law

We can repeat the previous derivation of the radiance difference law using luminances and photopic IOP equivalents and get the luminance difference law:

$$[L_{Tv}(r, \hat{\xi}) - L_{Bv}(r, \hat{\xi})] = [L_{Tv}(0, \hat{\xi}) - L_{Bv}(0, \hat{\xi})] \exp[-c_v(\hat{\xi}) r]$$

This is the basis for classical visibility theory for the human eye.

Inherent and Apparent Contrast

The visual contrast of the target viewed against the background as seen from zero distance is the *inherent contrast*, defined as

$$C(0, \hat{\xi}) \equiv \frac{L_{Tv}(0, \hat{\xi}) - L_{Bv}(0, \hat{\xi})}{L_{Bv}(0, \hat{\xi})}$$

The visual contrast of the target viewed against the background as seen from distance r is the *apparent contrast*, defined as

$$C(r, \hat{\xi}) \equiv \frac{L_{Tv}(r, \hat{\xi}) - L_{Bv}(r, \hat{\xi})}{L_{Bv}(r, \hat{\xi})}$$

If the target is darker (brighter) than the background, $C(0)$ is negative (positive). The apparent contrast keeps the same sign as $C(0)$ as r increases and $C(r)$ approaches zero.

For a black background, $L_{Bv}(0) = 0$, the inherent contrast becomes infinite. For a black target, $L_{Tv}(0) = 0$, and $C(0) = -1$

The Law of Contrast Reduction

The luminance difference law allows the apparent contrast to be rewritten as

$$\begin{aligned} C(r, \hat{\xi}) &= \frac{L_{Tv}(0, \hat{\xi}) - L_{Bv}(0, \hat{\xi})}{L_{Bv}(0, \hat{\xi})} \frac{L_{Bv}(0, \hat{\xi})}{L_{Bv}(r, \hat{\xi})} \exp[-c_v(\hat{\xi}) r] \\ &= C(0, \hat{\xi}) \frac{L_{Bv}(0, \hat{\xi})}{L_{Bv}(r, \hat{\xi})} \exp[-c_v(\hat{\xi}) r] \end{aligned}$$

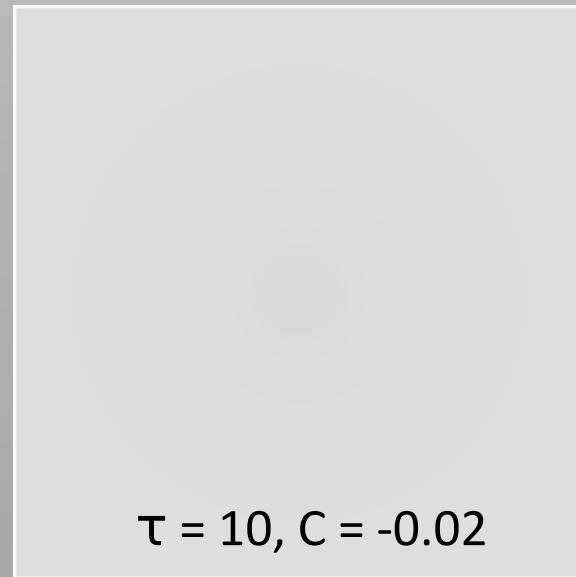
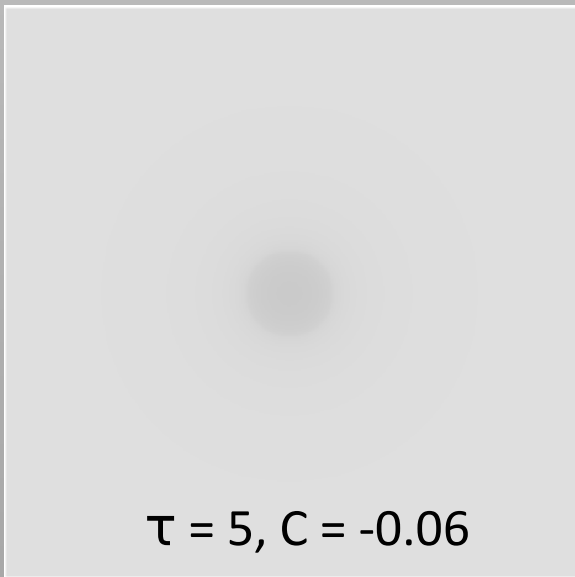
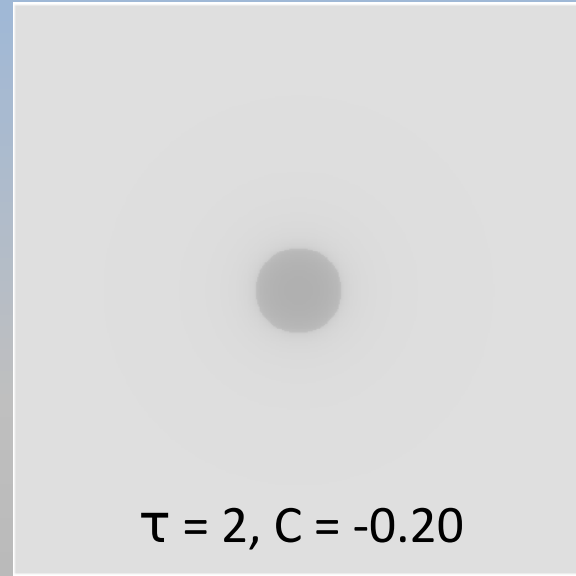
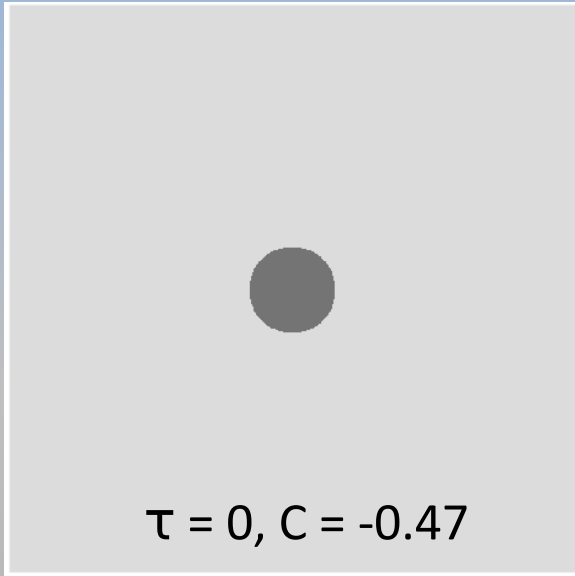
If $L_{Bv}(0, \hat{\xi}) \approx L_{Bv}(r, \hat{\xi})$, (e.g., horizontal viewing) this reduces to

$$C(r, \hat{\xi}) = C(0, \hat{\xi}) \exp[-c_v(\hat{\xi}) r]$$

Either of the last two equations is called the ***law of contrast reduction*** (or contrast transmittance).

Apparent Contrast

Objects Become Invisible When $|C| \lesssim 0.05$ to 0.02



Koschmieder's Law

For horizontal viewing and a black target ($C(0) = -1$), experience shows that the target can be detected at a visual range VR corresponding to $C(r = VR)/C(0) \approx 0.02$ (0.015 to 0.05 in the literature). Then

$$C(r, \hat{\xi}) = C(0, \hat{\xi}) \exp[-c_v(\hat{\xi}) r]$$

can be solved for this minimum contrast to get

$$VR(\hat{\xi}) = -\frac{\ln(0.02)}{c_v(\hat{\xi})} \approx \frac{3.9}{c_v(\hat{\xi})} \approx \frac{3.9}{c(550)}$$

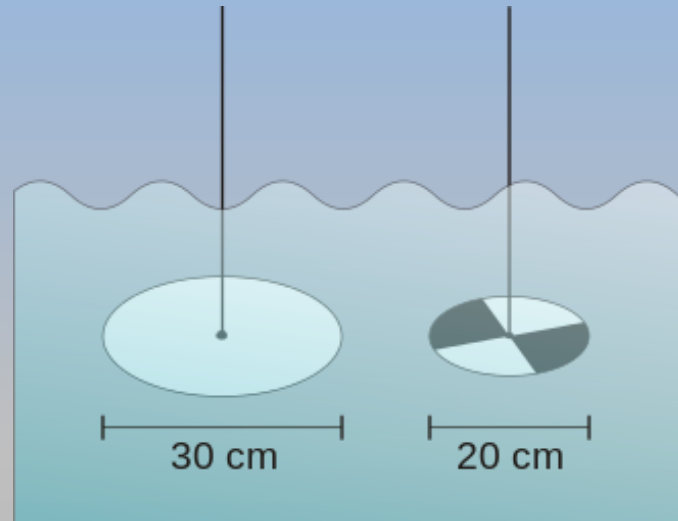
This result, originally developed for horizontal viewing of dark targets in the atmosphere, is known as Koschmieder's law.

Note: for a black background (e.g., viewing a distant lighthouse at night), $C(0) = \infty$, and a different analysis is required (Allard's Law)

The Secchi Depth

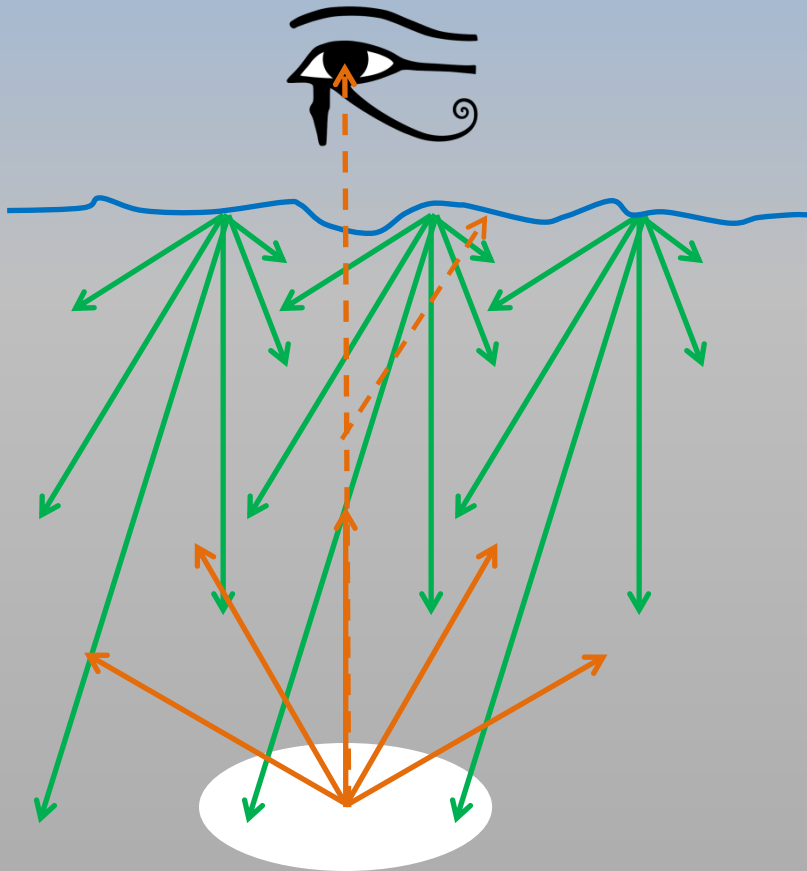


Fr. Angelo Secchi, S.J., 1818-1878
Astronomer (esp. solar and spectroscopy)



“Classic” Secchi Disk Theory

(Reviewed by Preisendorfer, L&O, 1986)



Light onto the disk is described by downwelling plane illuminance $E_{dv}(z)$, which attenuates by $K_{dv}(z)$

Angularly small white disk is a lambertian reflector with $R_v = E_{uv}/E_{dv}$ (typically 0.85)

Reflected light from the disk to the eye is luminance $L_v(z, \xi)$, which attenuates by $c_v(z, \xi)$

Must include loss of *contrast* due to surface roughness

Classic Secchi Disk Theory

Consider only the case of looking straight down, and drop the direction arguments in luminances and contrasts, e.g. $L_{Bv}(z, \hat{\xi}) = L_{Bv}(z)$.

$$E_{dv}(z) = E_{dv}(0) \exp[-\langle K_{dv} \rangle_z z] \quad \langle \dots \rangle_z \text{ is average over } 0 \text{ to } z \quad (1)$$

$$L_{Tv}(z) = E_{dv}(z) R_{Tv} / \pi \quad \text{the target is a lambertian reflector} \quad (2)$$

$$R_{Bv}(z) = \frac{E_{uv}(z)}{E_{dv}(z)} \quad \text{illum. refl. of background water} \quad (3)$$

$$L_{Bv}(z) = E_{dv}(z) R_{Bv}(z) / \pi \quad \text{assumes water is a lambertian refl.} \quad (4)$$

$$C_{in}(z) = \frac{L_{Tv}(z) - L_{Bv}(z)}{L_{Bv}(z)} \quad \text{inherent contrast at depth } z \quad (5)$$

$$= \frac{R_{Tv} - R_{Bv}(z)}{R_{Bv}(z)} \quad \text{by (2) and (4) into (5)} \quad (6)$$

$$C_{ap}(0) = \frac{L_{Tv}(0) - L_{Bv}(0)}{L_{Bv}(0)} \quad \text{apparent contr. just below surface} \quad (7)$$

$$L_{Tv}(0) - L_{Bv}(0) = [L_{Tv}(z) - L_{Bv}(z)] \exp[-\langle c_v \rangle_z z] \quad \text{the luminance difference law} \quad (8)$$

$$C_{ap}(0) = \frac{[L_{Tv}(z) - L_{Bv}(z)]}{L_{Bv}(0)} \exp[-\langle c_v \rangle_z z] \quad \text{by (8) into (7)} \quad (9)$$

$$= \frac{R_{Tv} - R_{Bv}(z)}{R_{Bv}(0)} \frac{E_{dv}(z)}{E_{dv}(0)} \exp[-\langle c_v \rangle_z z] \quad \text{by (2) and (4) into (9)} \quad (10)$$

$$C_{ap}(0) = C_{in}(z) \exp[-(\langle K_{dv} \rangle_z + \langle c_v \rangle_z) z] \quad \text{by (1) and (6), assuming } R_{Bv}(0) = R_{Bv}(z)$$

Classic Secchi Disk Theory

$$C_{ap}(0) = C_{in}(z) \exp[-(\langle K_{dv} \rangle_z + \langle c_v \rangle_z) z]$$

gives the apparent contrast of the Secchi disk as seen from just below the water surface. For viewing from above the surface, we must account for loss of contrast caused by the water surface. This loss is due both to refraction by waves and to reflected sky light.

$$C_{ap}(\text{air}) = \mathcal{T} C_{ap}(0) = \mathcal{T} C_{in}(z) \exp[-(\langle K_{dv} \rangle_z + \langle c_v \rangle_z) z]$$

Note that \mathcal{T} is a transmission of *contrast*, not of luminance or illuminance.

The Secchi depth z_{SD} is the depth at which the apparent contrast in air falls below a threshold contrast C_T . Solving for z_{SD} when $C_{ap}(\text{air}) = C_T$ gives

$$z_{SD} = \frac{\ln \left[\frac{\mathcal{T} C_{in}(z)}{C_T} \right]}{\langle K_{dv} \rangle_{z_{SD}} + \langle c_v \rangle_{z_{SD}}} \equiv \frac{\Gamma}{\langle K_{dv} \rangle_{z_{SD}} + \langle c_v \rangle_{z_{SD}}}$$

Studies with human observers show that C_T depends on the angular subtense of the disk and on the ambient luminance (P86, Table 1). The values of Γ vary from about 6 to 9 for a disk with $R_{Tv} = 0.85$, depending on the water reflectance R_{Bv} (0.015 to 0.1; P86, Table 2). HydroLight uses $\Gamma = 8$ as its default.

Note that the last equation must be solved iteratively because $\langle K_{dv} \rangle_{z_{SD}}$ and $\langle c_v \rangle_{z_{SD}}$ are averages over the (unknown) Secchi depth z_{SD} .

Secchi Disk Theory Revised

To evaluate

$$z_{SD} = \frac{\Gamma}{\langle K_{dv} \rangle_{z_{SD}} + \langle c_v \rangle_{z_{SD}}}$$

you must know the photopic $K_{dv}(z)$ and $c_v(z)$, which in turn come from $E_d(z, \lambda)$ and $c(z, \lambda)$.

Lee et al. (2015) re-examined the classic theory:

- The disk needs not be angularly small and can perturb the ambient light field seen near the edge of the disk.
- Visibility is not based on target vs background *luminance* differences at the sharp edge of the disk, but on differences in target and background *reflectances*
- Visibility is determined by the wavelength where the disk is most visible (can change with depth and between water bodies), rather than on broadband photopic variables.

The end result is to

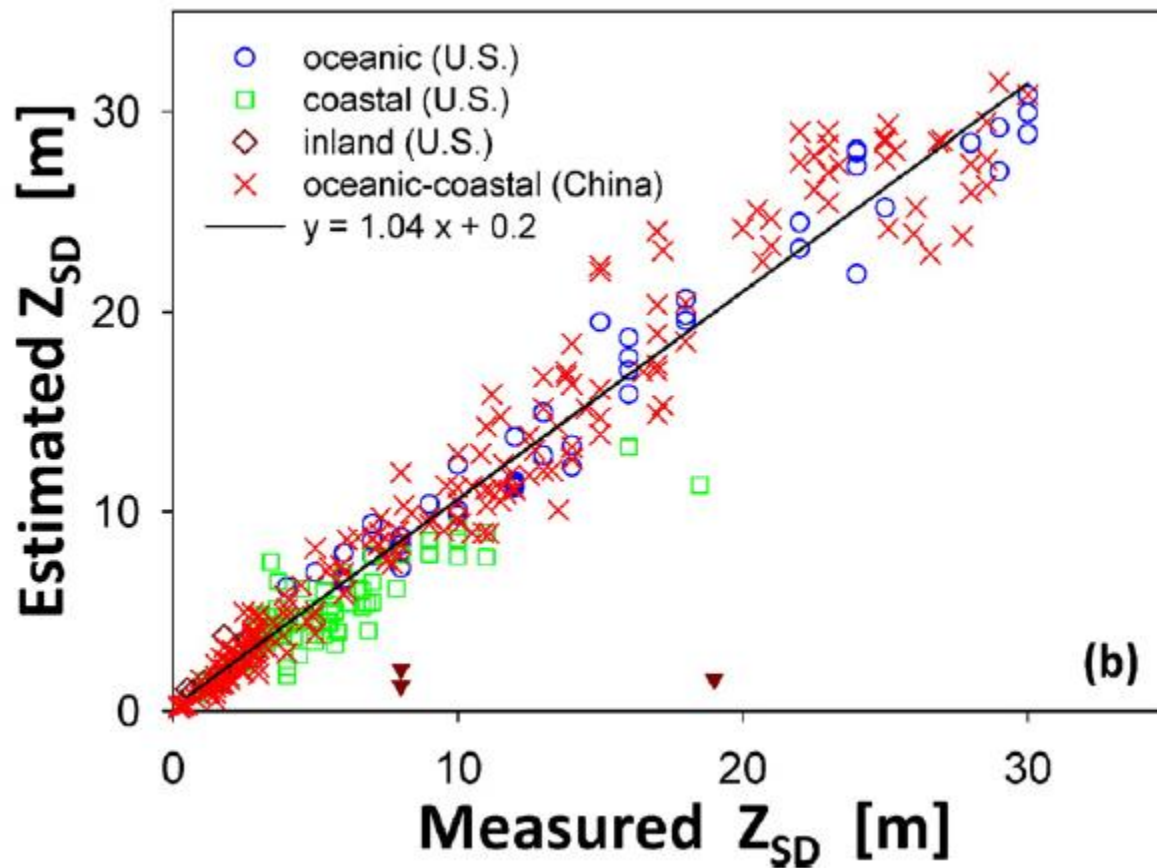
- Replace the photopic $K_{dv}(z)$ with $K_d(z, \lambda_o)$, where λ_o is the wavelength at which $K_d(z, \lambda)$ is a minimum
- Replace the photopic $c_v(z)$ with $1.5K_d(z, \lambda_o)$

The end result is a formula of the form

$$z_{SD} = \frac{\gamma}{2.5K_d(z, \lambda_o)}$$

This formula has the great virtue that $K_d(z, \lambda_o)$ can be estimated from satellite imagery.

Secchi Depth by the Lee Formula



Based on SeaBASS data with z_{SD} and R_{rs} at 443, 488, 532, 555, 665 nm.

a and b_b estimated from R_{rs}

K_d estimated from a , b_b , and SZA of 30 deg

Still to be done: Comparison of classic and Lee results. Need to measure $E_d(z, \lambda)$, $c(z, \lambda)$, z_{SD} , and disc properties, for a wide range of waters.

Underwater Visibility by Zaneveld and Pegau Formula

(Optics Express, 2003)

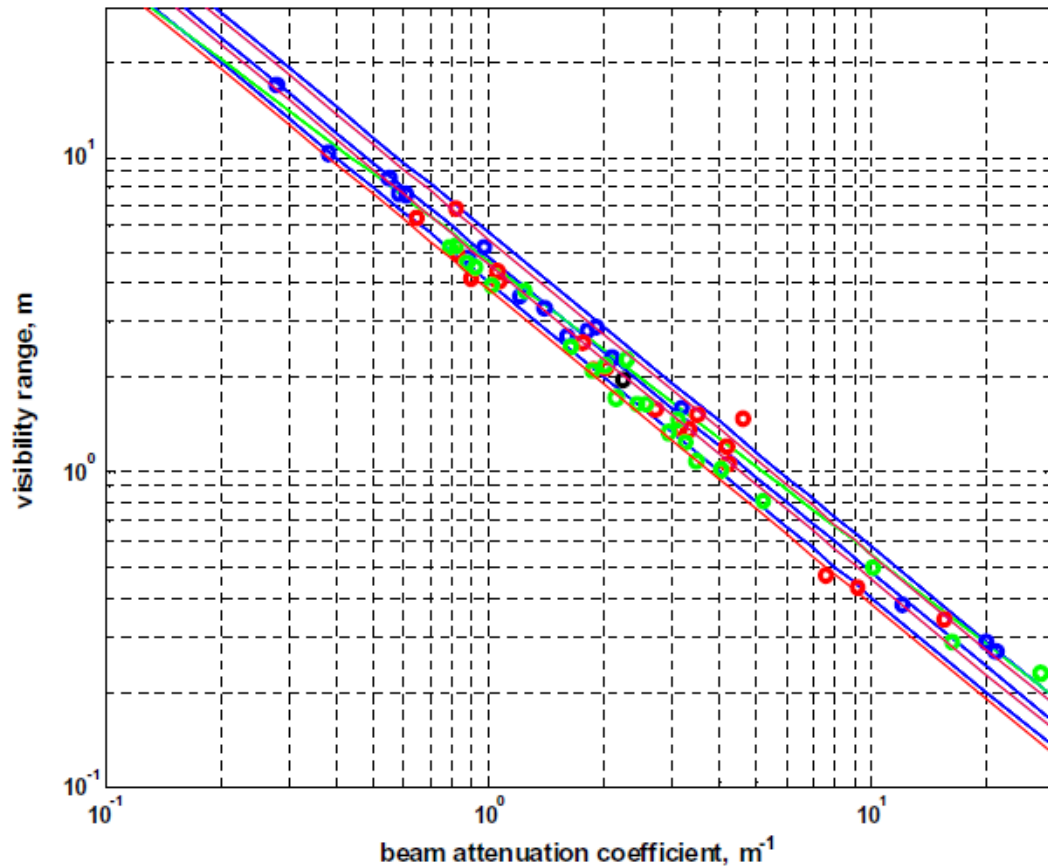


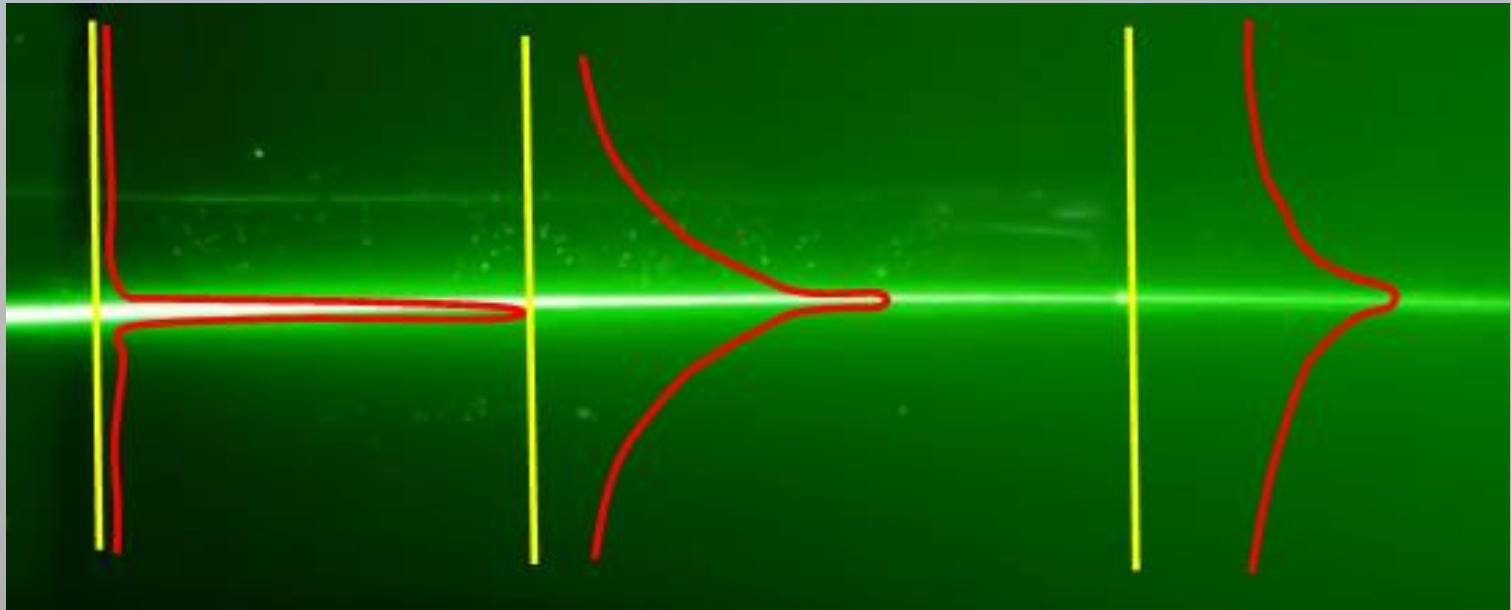
Fig. 3. Horizontal visibility of a 200 mm diameter black target. Blue points, Davies-Colley, "green" c-meter; red points, Zaneveld, $c(532)*0.9+0.081$; black point, Twardowski $c_{pg}(532)*0.9+0.081$; green points, Pegau, $c_{pg}(532)*0.9+0.081$; blue lines vis. range = $y = 4.8/\alpha$ and $\pm 20\%$ lines; green line vis. range = $y = (5.207 - 0.368 \ln y)/\alpha$; red lines vis. range = $y = 4.55/\alpha$ and $\pm 20\%$ lines; $r^2 = 0.985$.

Re-examined visibility of a black target underwater.

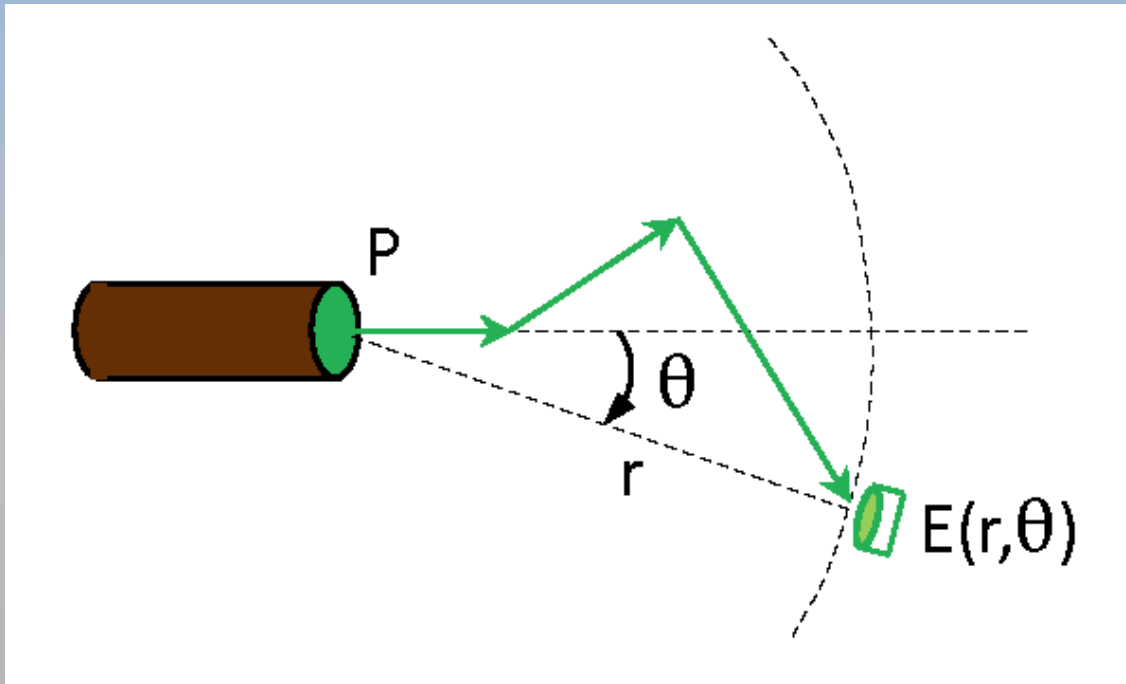
When viewed horizontally,
Visual Range =
 $4.8/[c(532)*0.9 + 0.081] \pm 10\%$

The Beam Spread Function (BSF)

The VSF describes a *single* scattering event. The BSF accounts for all (multiple) scattering and absorption between the source and the detector.



The Beam Spread Function (BSF)



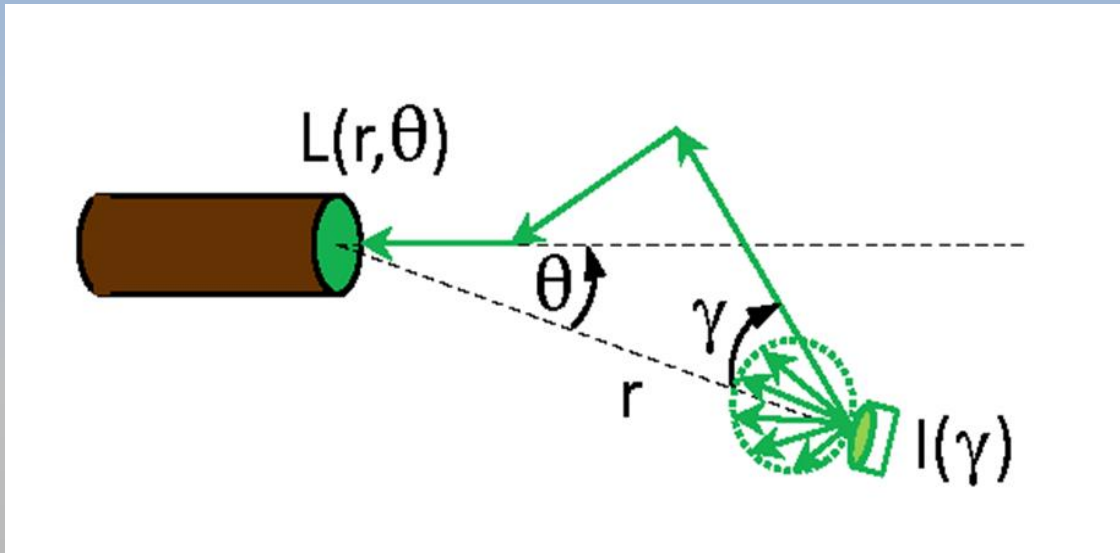
Collimated source
emitting spectral
power P

Cosine detector
measuring spectral
plane irradiance $E(r, \theta)$

The BSF is the detected irradiance normalized by the emitter power:

$$BSF(r, \theta) \equiv \frac{E(r, \theta)}{P} \quad \frac{\text{W m}^{-2} \text{ nm}^{-1}}{\text{W nm}^{-1}} = \text{m}^{-2}$$

The Point Spread Function (PSF)



Cosine source
emitting spectral
intensity $I(\gamma)$

Collimated detector
measuring spectral
radiance $L(r, \theta)$

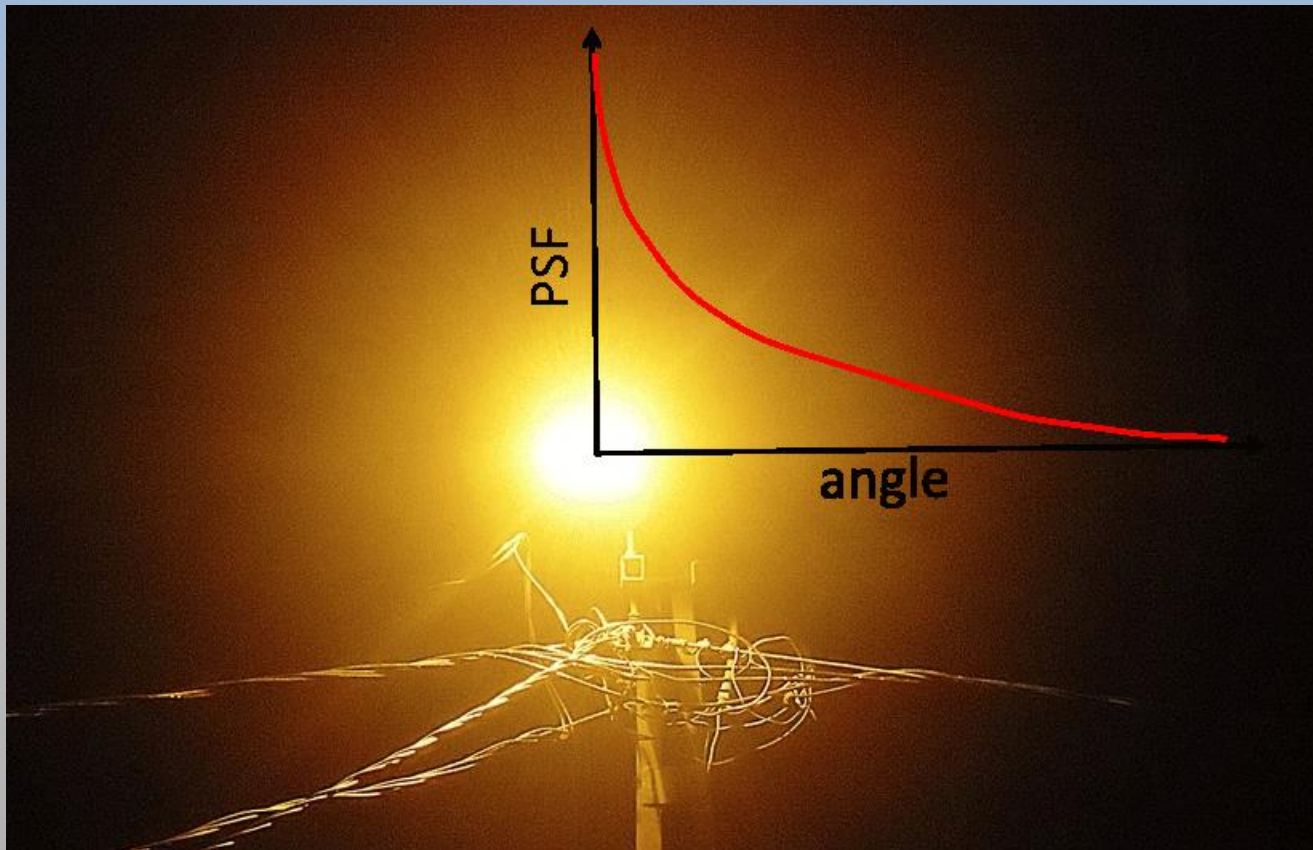
$$I(\gamma) \equiv \begin{cases} \frac{P}{\pi} \cos(\gamma) & 0 \leq \gamma \leq \pi/2 \\ 0 & \pi/2 < \gamma \leq \pi \end{cases} \quad \text{W sr}^{-1} \text{ nm}^{-1}$$

$$P = \int_{4\pi} I(\gamma) d\Omega$$

The PSF is the detected radiance normalized by the max emitted intensity:

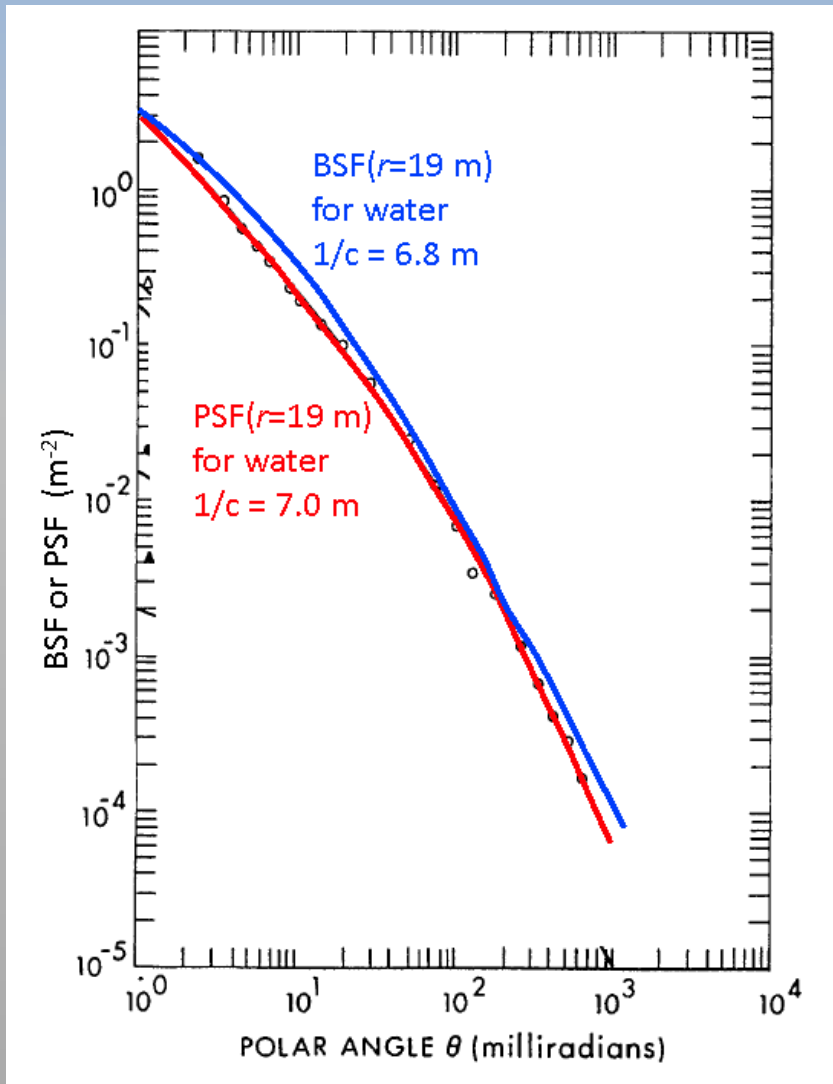
$$PSF(r, \theta) \equiv \frac{L(r, \theta)}{P/\pi} \quad \frac{\text{W m}^{-2} \text{ sr}^{-1} \text{ nm}^{-1}}{\text{W sr}^{-1} \text{ nm}^{-1}} = \text{m}^{-2}$$

The Point Spread Function (PSF)



The glow around a distant streetlight is essentially the PSF

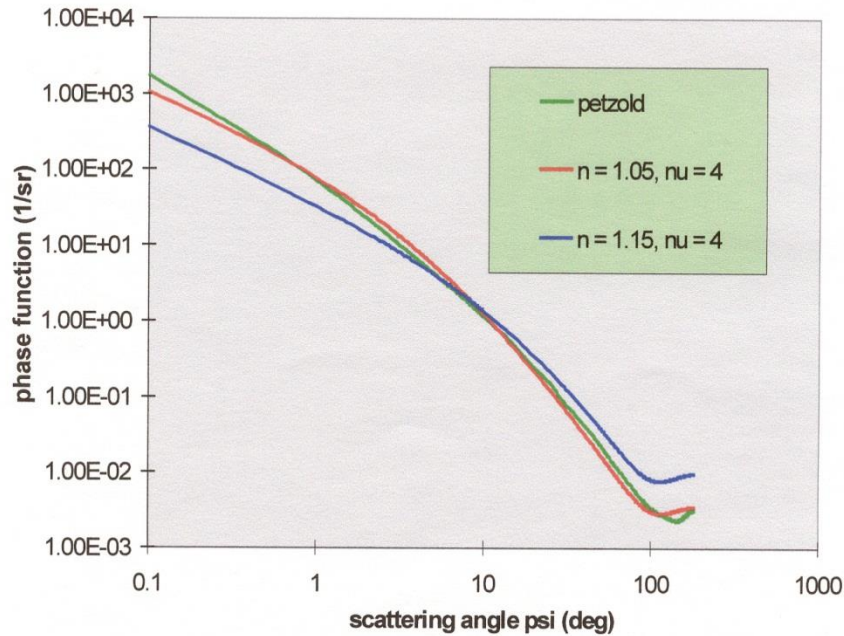
BSF = PSF by EM Reciprocity



Equality of PSF and BSF
finally proved by Howard
Gordon (AO 1994)

redrawn from Mertens and Replogle JOSA (1977)

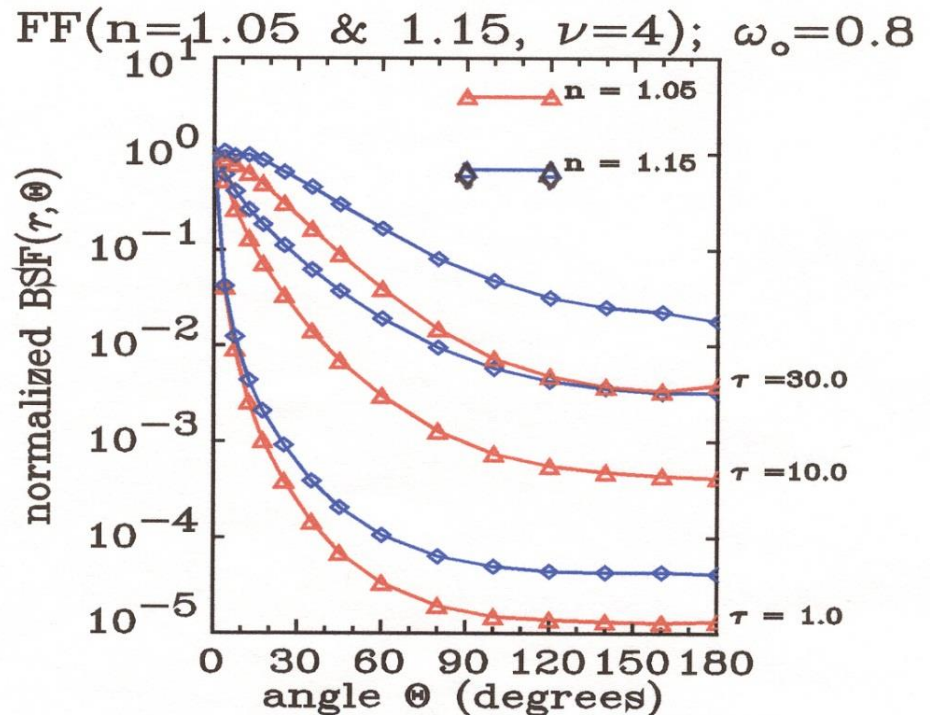
Fournier-Forand “biological” and “mineral” phase functions



These PSFs were computed by Monte Carlo simulations

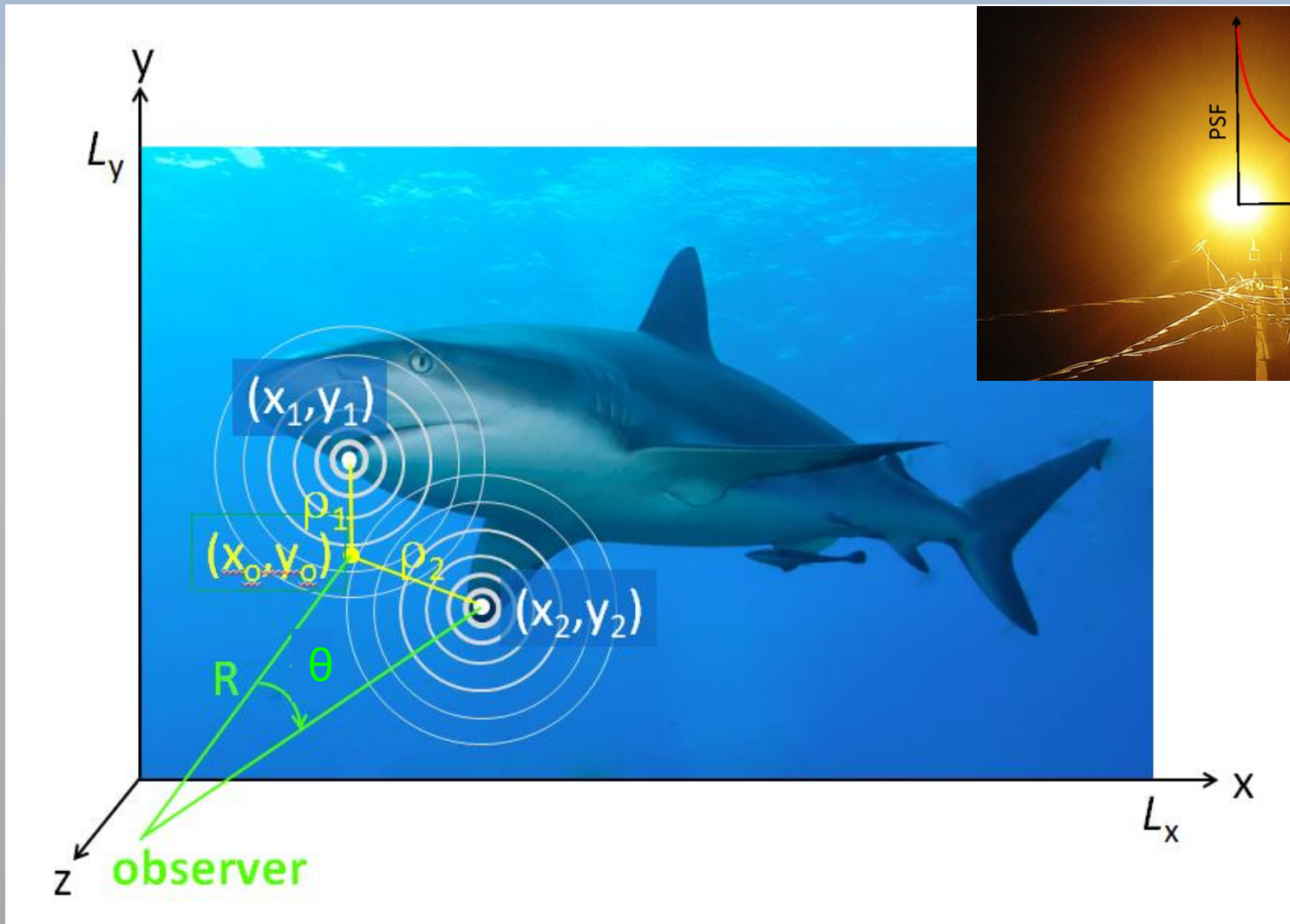
The PSF Depends on IOPs and Distance

Effect of phase function on BSF; “biological” vs. “mineral”



The PSF is the Key to Image Analysis

When viewing point (or direction) (x_o, y_o) , light is scattered into that direction from all other points in the scene according to the PSF value



The PSF is the Key to Image Analysis

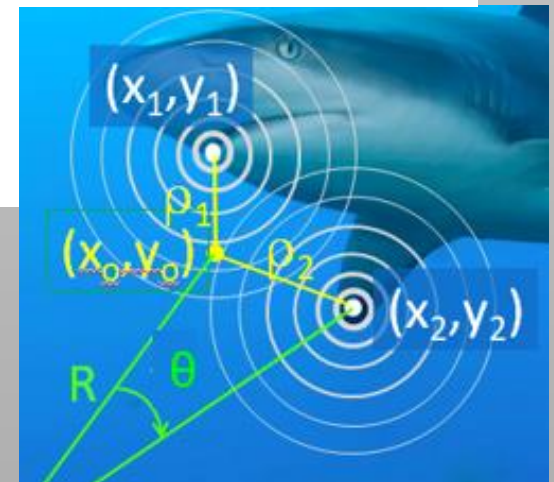
Let $S(x, y, 0)$ be the pattern of bright and dark values (e.g., 0 to 255 at a given wavelength) at each point of a scene, when seen from zero distance (or through a vacuum). Then when looking at point (x_o, y_o) of the scene from a distance (range) R , the value of $S(x_o, y_o, R)$ is

$$S(x_o, y_o, R) = \int_{\text{all } x} \int_{\text{all } y} S(x_o, y_o, 0) PSF(x_o - x, y_o - y, R) dx dy$$

$$\rho = \sqrt{(x_o - x)^2 + (y_o - y)^2}, \quad \theta = \tan^{-1} \left(\frac{\rho}{R} \right)$$

$$PSF(x_o - x, y_o - y, R) = PSF(R, \theta)$$

The PSF completely characterizes the effect of the environment on an image. If you know the PSF, you can predict what any image will look like.



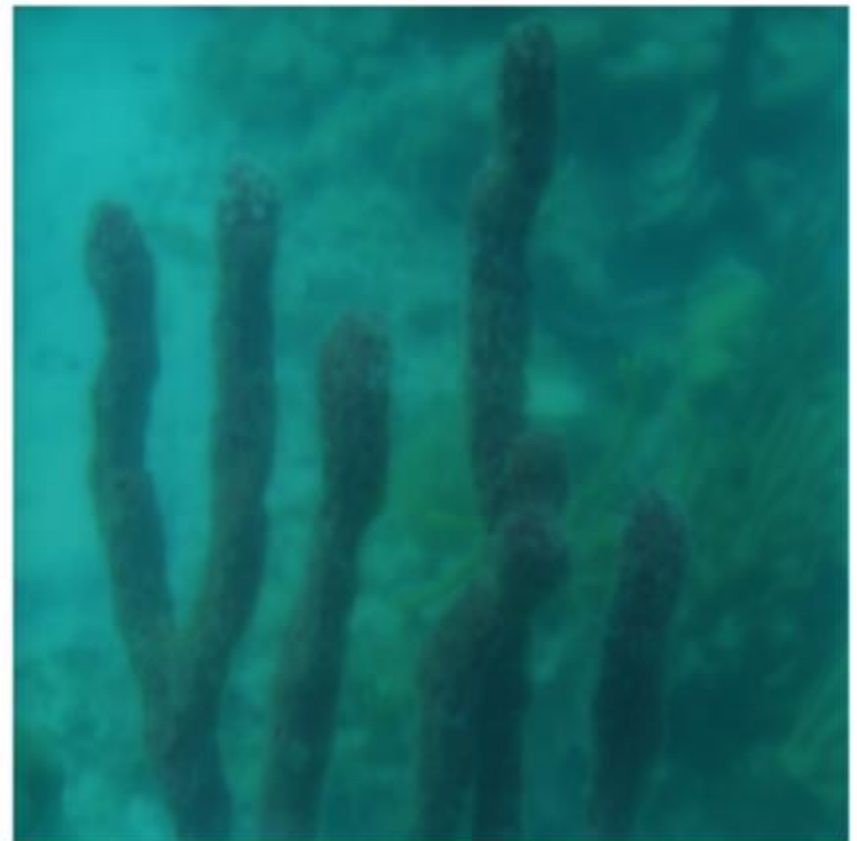
The PSF Depends on Wavelength

$$\text{Chl} = 0.5 \text{ mg m}^{-3}, L_x = L_y = 1 \text{ m}$$

R = 0



R = 5 m



Terminology

Integrals of the form $h(y) = \int f(x) g(x - y) dx$ are called *convolution* integrals. They have wonderful mathematical properties, which you will learn about in your Fourier Transforms class or in OOB Appendices A & F.

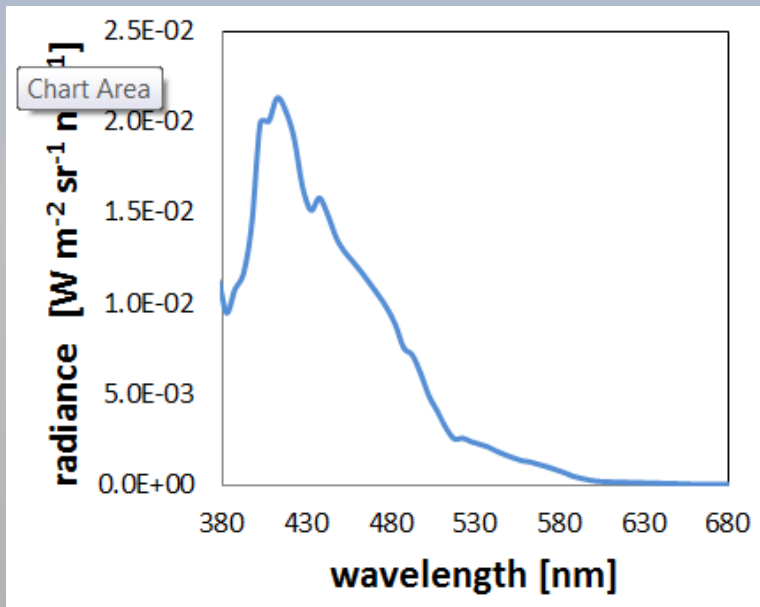
Note that a convolution of $f(x)$ and $g(x)$ gives a *function*.

Integrals of the form $h = \int f(x) g(x) dx$ are NOT convolutions. They are just the integral of $f(x)$ weighted by $g(x)$, and the result is a *number*.

Color

Which gives you a better idea of the color of an iceberg?

A calibrated radiance spectrum



Color

Which gives you a better idea of the color of an iceberg?

A calibrated radiance spectrum

or

a color photo seen by eye?

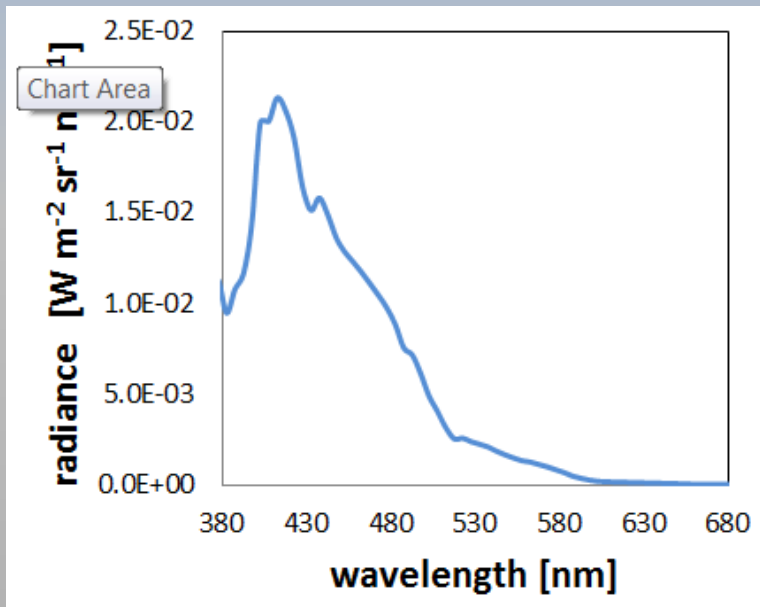
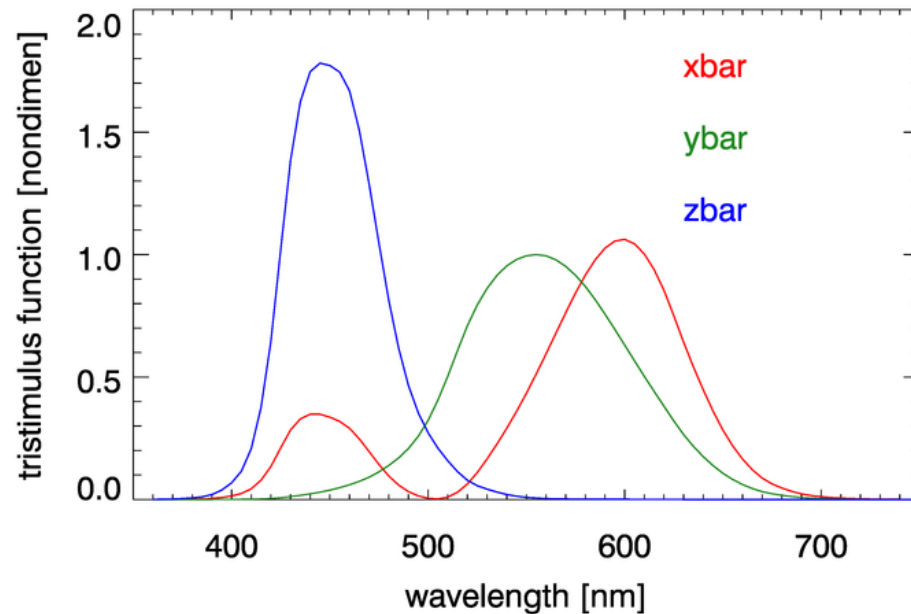


photo by Curtis Mobley, Errera Channel, Antarctica

How do we relate the spectrum to the physiological phenomenon of color?

CIE 1931 Tristimulus Functions

CIE = Commission Internationale de l'Eclairage



$$X = K_m \int_0^{\infty} F(\lambda) \bar{x}(\lambda) d\lambda$$
$$Y = K_m \int_0^{\infty} F(\lambda) \bar{y}(\lambda) d\lambda$$
$$Z = K_m \int_0^{\infty} F(\lambda) \bar{z}(\lambda) d\lambda$$

$F(\lambda)$ can be radiance, irradiance, reflectance, etc.

CIE chromaticity coordinates:

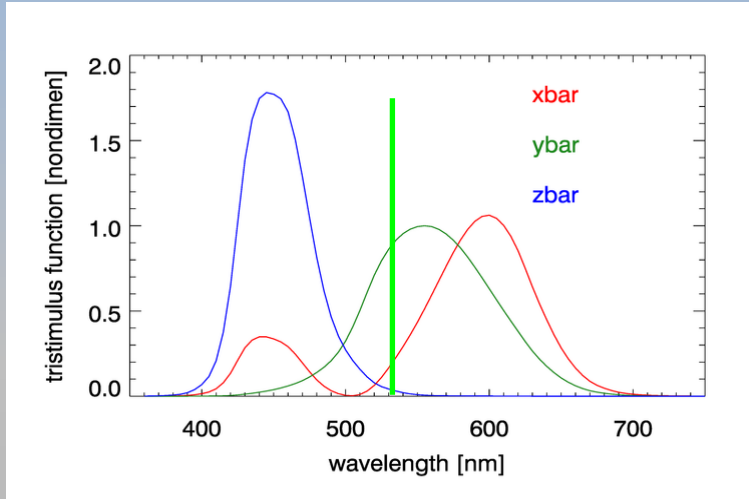
$$x = \frac{X}{X + Y + Z}$$
$$y = \frac{Y}{X + Y + Z}$$
$$z = \frac{Z}{X + Y + Z}$$

Empirically derived functions that roughly describe the spectral sensitivity of the 3 types of cone cells in normal (European, male) human eyes.

Note that $x + y + z = 1$

CIE 1931 Tristimulus Functions

CIE = Commission Internationale de l'Eclairage



The area under each tristimulus curve is the same, so for a white spectrum, $F = \text{constant}$, we get $x = 1/3$, $y = 1/3$, $z = 1/3$

For a 532 nm laser:

$$X = K_m 0.19, Y = K_m 0.95, Z = K_m 0.04$$

$$x = 0.16, y = 0.81, z = 0.03$$

How about a 300 nm laser?

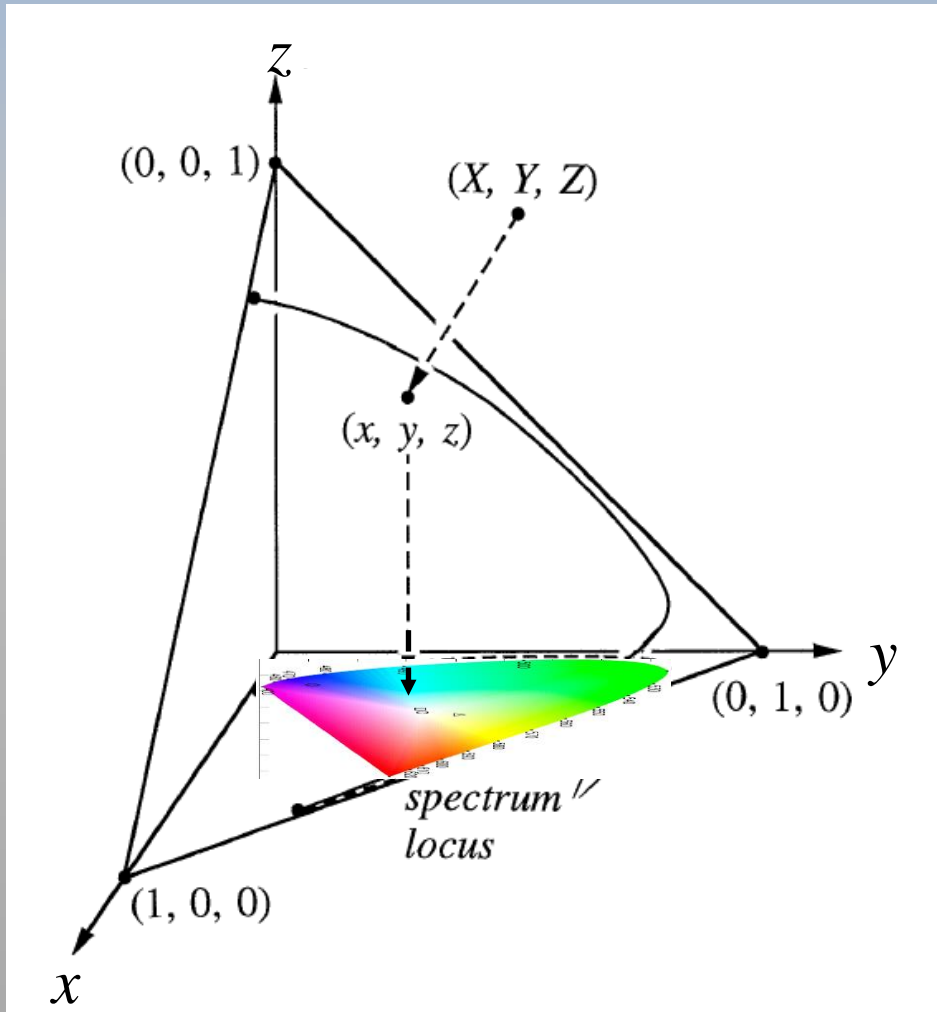
$$X = K_m \int_0^{\infty} F(\lambda) \bar{x}(\lambda) d\lambda$$
$$Y = K_m \int_0^{\infty} F(\lambda) \bar{y}(\lambda) d\lambda$$
$$Z = K_m \int_0^{\infty} F(\lambda) \bar{z}(\lambda) d\lambda$$

$F(\lambda)$ can be radiance, irradiance, reflectance, etc.

CIE chromaticity coordinates:

$$x = \frac{X}{X + Y + Z}$$
$$y = \frac{Y}{X + Y + Z}$$
$$z = \frac{Z}{X + Y + Z}$$

The Chromaticity Plane and Diagram



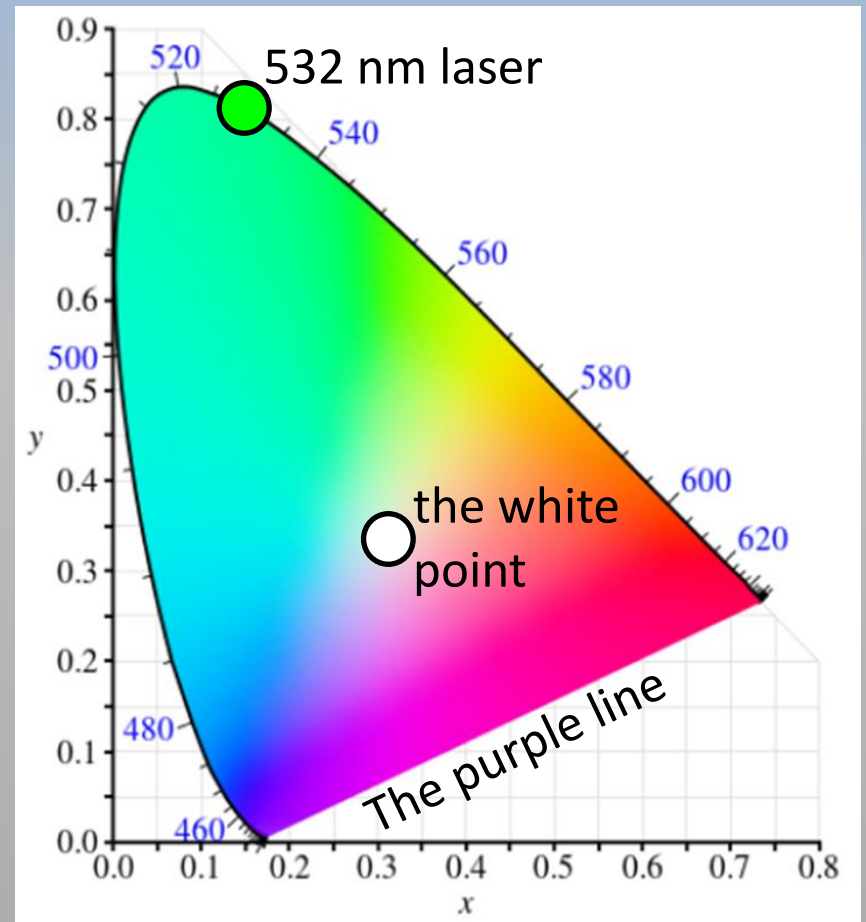
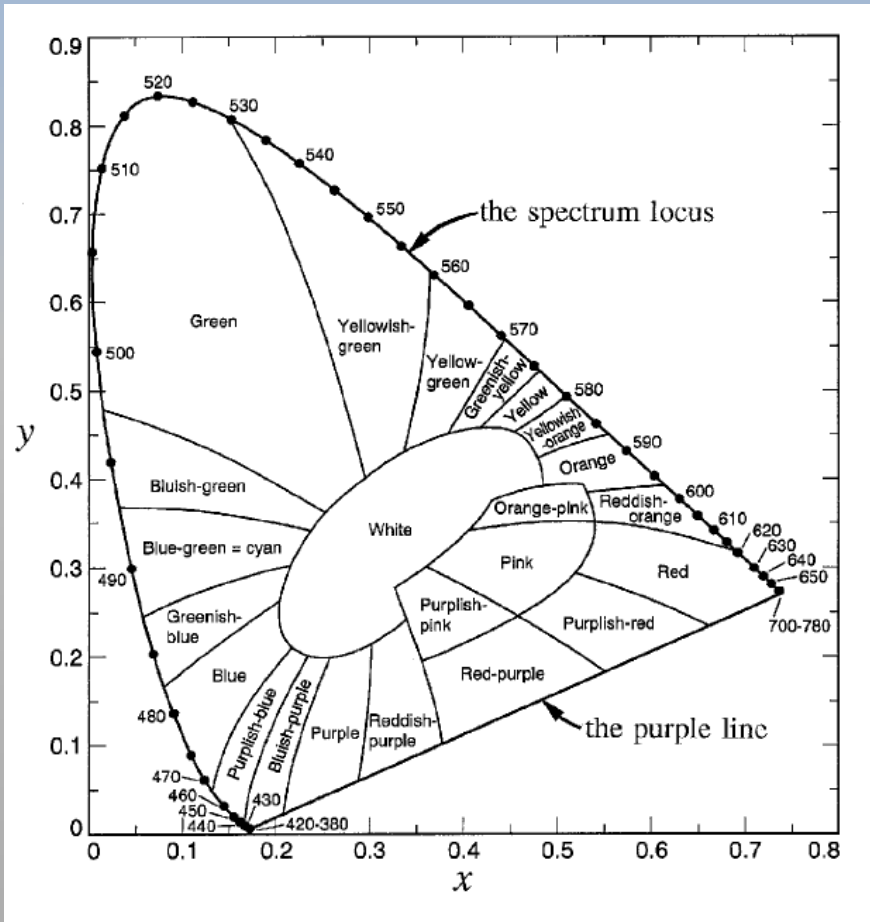
Any spectrum $F(\lambda)$ gives a point (x, y, z) on the plane
 $x + y + z = 1$.

Usually plot just (x, y) , and call it a chromaticity diagram.

Then “color in” the (x, y) plot to indicate the visual colors represented by each (x, y) point.

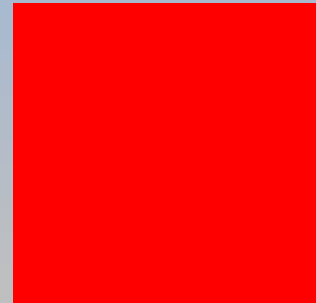
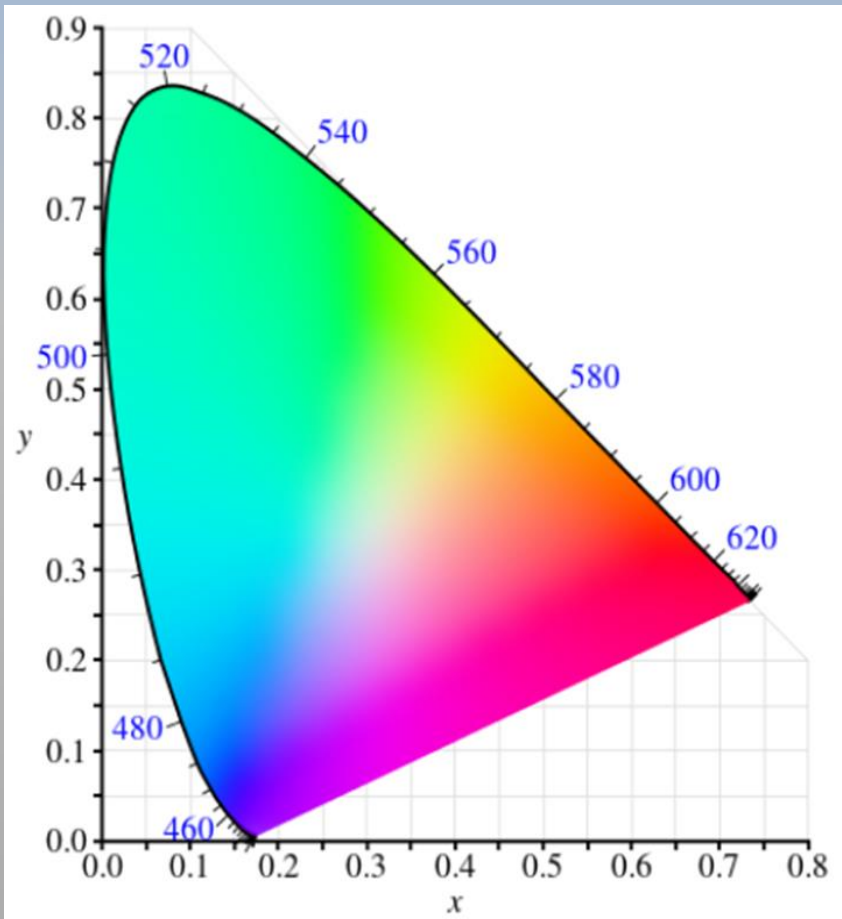
The Chromaticity Diagram

Can represent any color that the normal human eye can see.

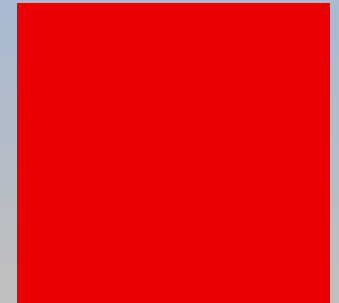


The Chromaticity Diagram

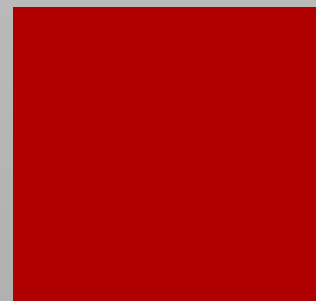
Where is brown, for example?



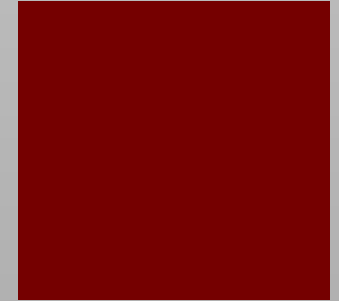
RGB = (255,0,0)



(200,0,0)



(150,0,0)

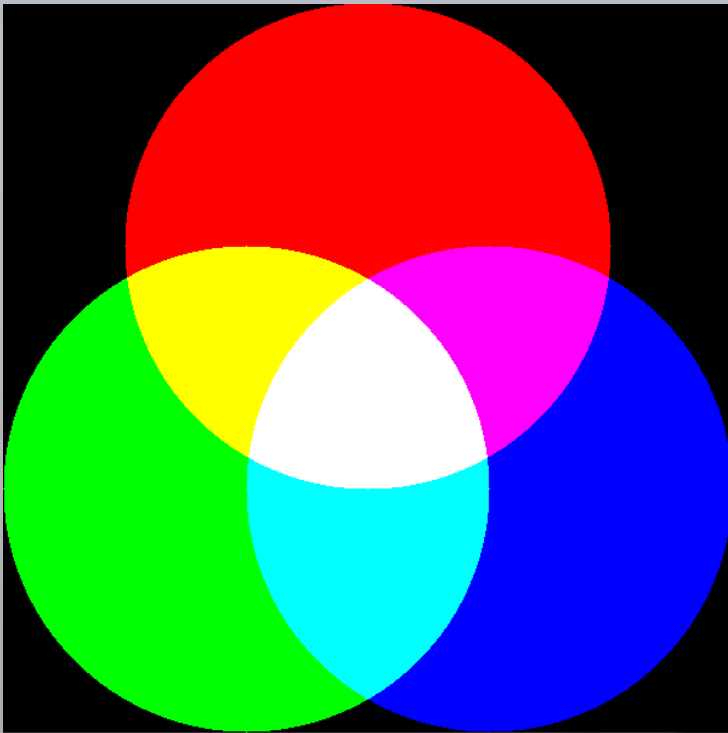


(100,0,0)

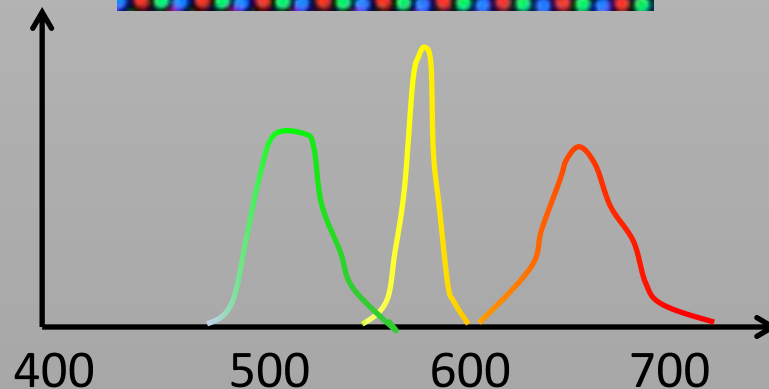
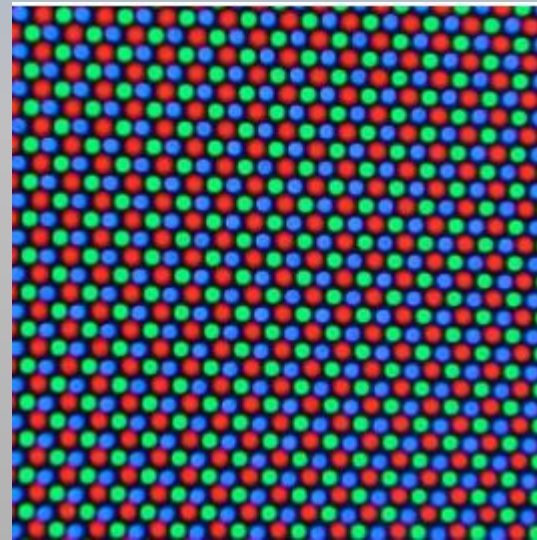
and gray is just white that isn't very bright

Metameric Spectra

A given spectrum $F(\lambda)$ gives a unique CIE color (x,y) . However, many different spectra $F(\lambda)$ can give the same (x,y) . Two different spectra giving the same (x,y) are called *metameric*, and are *perceived* as the same color. This is why an RGB display can simulate almost any color as seen by the human eye and brain.

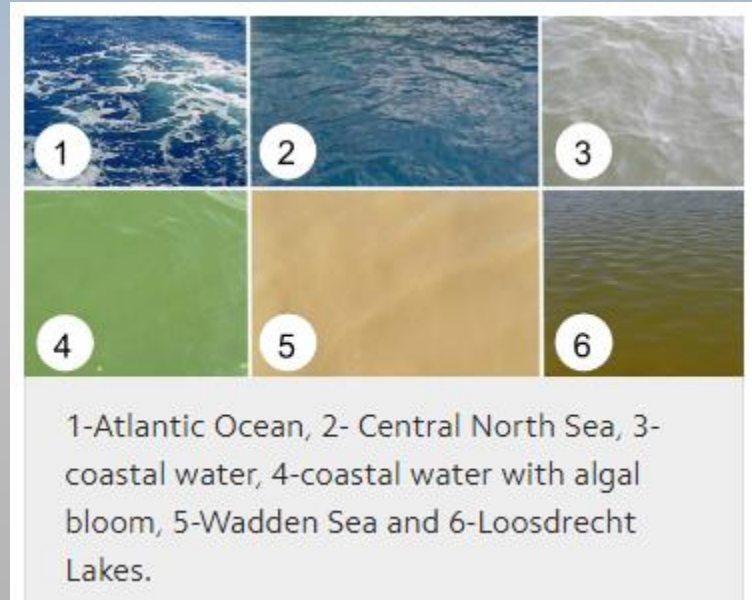


Red + Green = Yellow
Red + Blue = Purple



The Forel-Ule Color Indices

Tubes of 21 well-defined solutions, which you match by eye to the water color.

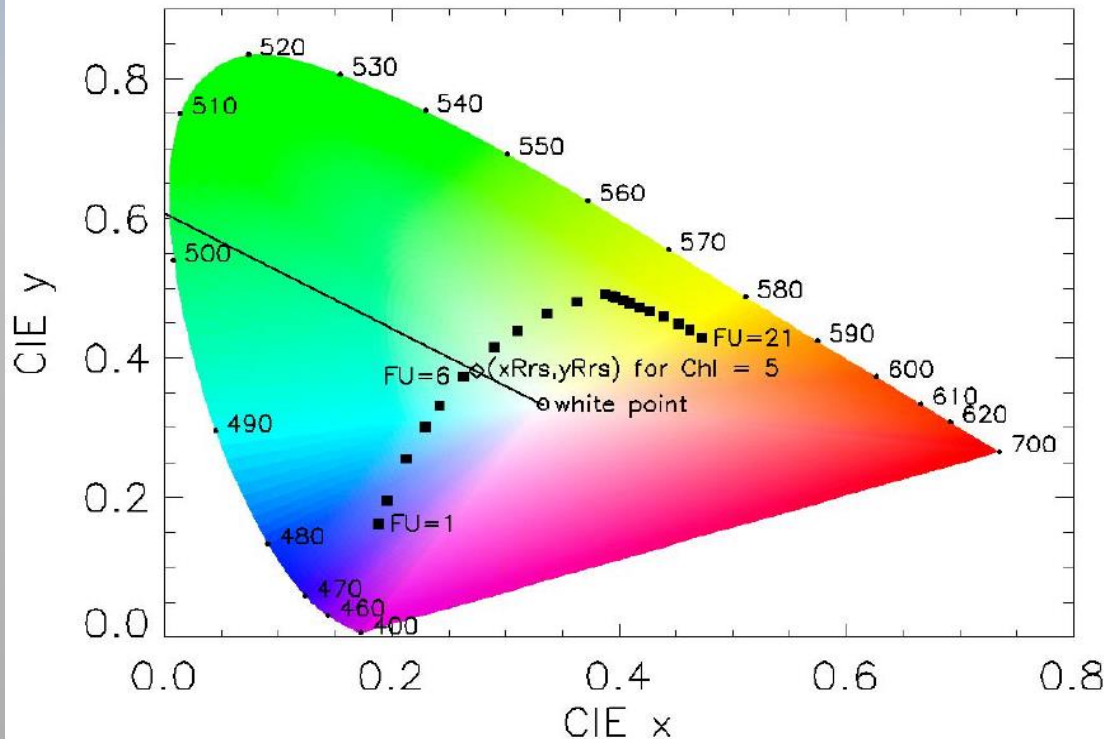


<https://www.nioz.nl/en/news/app-measures-colour-palette-of-lakes-seas-and-oceans>

The Forel-Ule Color Indices



Fig. 1. Approximate colors of the Forel-Ule indices FU = 1 to FU = 21 (copied from Wernand and van der Woerd, 2010a).



simulation	$Chl = 0.05$	$Chl = 0.5$	$Chl = 5$	$Chl = 50$	$Chl = 50 + a_{CDOM}(440) = 1 \text{ m}^{-1}$	$Chl = 50 + a_{CDOM}(440) = 2 \text{ m}^{-1}$	$Chl = 50 + a_{CDOM}(440) = 10 \text{ m}^{-1}$
FU for R_{rs}	1	3	6	11	17	19	21
FU for L_u	1	3	6	10	15	18	21

Sea Kayaking in Antarctica







