SMS-204: Integrative marine sciences

Homework 3

For homework- remember to provide uncertainties in graphs and display only significant figures for all your results.

1. (25pts) Using results from lab station 1(Archimedes' ball):
Work out mathematically how much air needs to be pulled out of the ball by the syringe in order for the ball to barely start sinking in the surrounding water. How does it compare to your observations (are they within the observation uncertainties)?

Let's assume the dry ball is of mass m (about 125g± 0.5g), volume V (= $4\pi r^3/3$*, with r the radius) and density* $\rho = m/V$*. The ball diameter is 7± 0.1 cm →* $V_{ball} = \pi D^3/6 \sim 180ml = 1.8\times10^{-4} m^3$*.* $\rho < 1000 Kg/m^3$*, as the ball is observed to float.*

Let's assume you filled Xml (~52 ± 2.5ml) water in the ball (replacing the same volume of air). The ball is now barely floating, so we can assume all of its volume has displaced water. For that to happen:

Weight of ball = weight of water in the volume it displaced. From Archimedes principles we then find: $mg + Xml \times (\rho_{water} - \rho_{air}) \times g = V_{ball} \times \rho_{water} \times g$

Since $\rho_{water} >> \rho_{air}$*, we can simplify to (after dividing by g):*

$m + Xml * \rho_{water} = V_{ball} \times \rho_{water} \rightarrow Xml = V_{ball} - m/\rho_{water}$.

If everything was done right, Xml calculated in this way should be similar to the volume of air evacuated into the syringe.

This solution is identical to that which recognize that when the ball barely floats its density=density of water. $\rho_{water} = m/(V_{ball} - Xml) \rightarrow Xml = V_{ball} - m/\rho_{water} \sim 180ml - 125g/(1\ g\ ml^{-1}) = 55ml$.

The amount calculated is consistent with what was evacuated (52ml) when we take into account the uncertainty of the reading on the syringe (+/-2.5ml) and weight (+/-0.1g), and the fact that the ball was not exactly a ball in shape.

2. (30pts) Using results from lab station 4:

a. Report the cross-section area, volume and weight of the empty box.

$25cm^2$, $100cm^3$ and 25g.

b. For the four box weights for which the box floated create a table showing the weights added, the box weight in air, the box weight in water, and the depth to which the box was immersed in water.

Mass of added weights	Weight in air (I accepted mass)	Weight in water (I accepted mass)	Immersion depth (cm)
0g	25g	0g (floats)	1+/-0.2cm
25g	50g	0g (floats)	2+/-0.2cm
50g	75g	0g (floats)	3+/-0.2cm
75g	100g	0g (barely floats)	4+/-0.2cm

c. Plot the depth to which the box is immersed in water as function of the weight of the box in air (5pts).

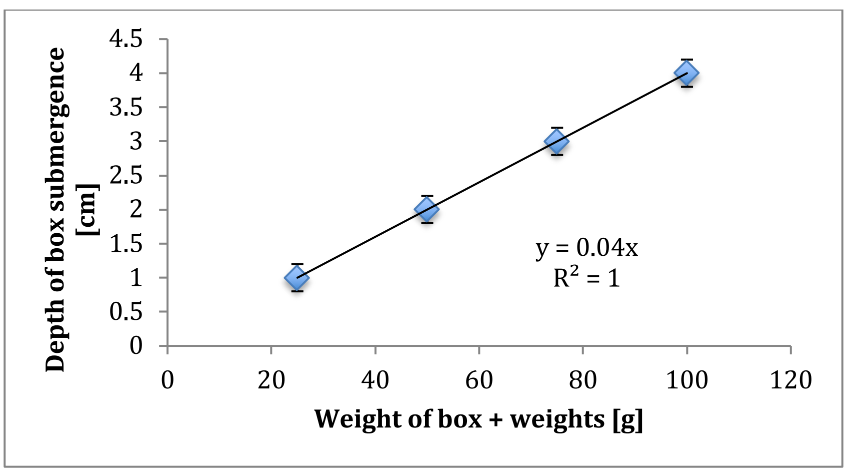

Figure 1. Immersion depth as function of weight of box + weights. The slope is ~0.04cm g^{-1} and the intercept ~0gr as expected (see below).

d. Obtain the slope of the best-fit line. (5pts)

See Fig. 1.

e. What should the slope be based on Archimedes's principle (5pts)?

For the floating box Archimedes tells us that: $Ah\rho_{water}g = mg$

→ $Ah = m/(A\rho_{water})$, where h is the depth of submergence, m the mass of the box (with the inside weights) and A the area of the bottom of the box ($25cm^2$).

→ the slope of the graph of h as function of m should be $1/(A\rho_{water}) \sim 1/(25\ cm^2 \times 1gr/cm^3) = 0.04cm\ g^{-1} = 0.4m\ Kg^{-1}$.

f. How does it compare to the slope of your plot? (5pts)

Very well, see Fig. 1.

g. Consider the final trial when you added enough weight so that the box *sinks* in water. In that case, what is the weight of the box when immersed in water and when outside water? What is the difference between them? (5pts)

Weight in air: 100+x g. Weight in air: x g. Difference: 100g.

Where x is whatever weight was added to make the box sink (25g for most).

h. Is this difference reasonable given what you know about buoyancy? Explain. (5pts)

Let the weight in Air be m_1 and the weight in water be m_2.

By Archimedes principle, the buoyancy force equals to the force due to the weight of the water displaced ($Volume*\rho_{water}*g$), and equals the difference between weight in air (m_1*g) and in water (m_2*g). Dividing all sides by the gravitational acceleration g we get:

→ $m_1 - m_2 = Volume*\rho_{water}$.

The Volume of the box=$100cm^3$ → $m_1 - m_2 \sim 100cm^3 * 1gr/cm^3 = 100gr = 0.1Kg$.

If we measured this difference in weight than theory and measurements are consistent!

3. (25pts) In the 2nd station of the 2nd lab you were studying water squirting out from a hole in a cylinder filled with water into a tub. Just as a falling ball converts potential energy to kinetic energy, water pressure pushed water out of the hole by converting potential energy per unit volume (ρgh) to kinetic energy per unit volume ($\rho v^2/2$). Assume you have a 40 cm head of water above the hole and that the hole is 20 cm above ground.

1. (5pts) What is the horizontal speed at which the water leaves the hole?

Potential energy per unit volume: ρgh_1 -> $\rho v^2/2$: kinetic energy per unit volume (h_1 height of water above hole).

→ $v^2=2gh_1$ → $v=(2 \times 9.81 \times 0.4)^{0.5}=2.8 ms^{-1}$

2. (5pts) How long will it take it to reach the ground (think mechanics)?

You learned in mechanics that an object starting from rest (there is no vertical velocity to the water) obeys: $gt^2/2=h_2$ → $t^2=2h_2/g$ → $t=(2 \times 0.2/9.81)^{0.5}=0.2s$

h_2 *denotes the distance from hole to ground.*

3. (5pts) How far will the water reach by the time it hits the ground?

L=vt~0.57m or 57cm, (depending when you round the answers, you may get 56 cm which is ok).

Answer for 4 & 5:

$L=vt=sqrt(2gh_1) \times sqrt(2h_2/g) = 2sqrt(h_1h_2)$ *- where sqrt denotes square root.*

Hence the place where the water splash is proportional to the square-root of the height of the water above the hole times the square-root of the height of the hole above ground (it is their geometric mean and it increases with increase in any of them).

4. (20pts) Working with data from profiling float:

a. Go to: http://www.mbari.org/science/upper-ocean-systems/chemical-sensor-group/floatviz/ Select a float in the 'Select Float' window. Choose density anomaly (=density-1000kg/m^3 called Sigma_theta) in the 'select one X Variable' window. Choose depth in the 'Select Y variable'. On the left most part, click on 'plot' to generate a plot of all the density anomalies and click on 'Send'. Copy the image to your homework.

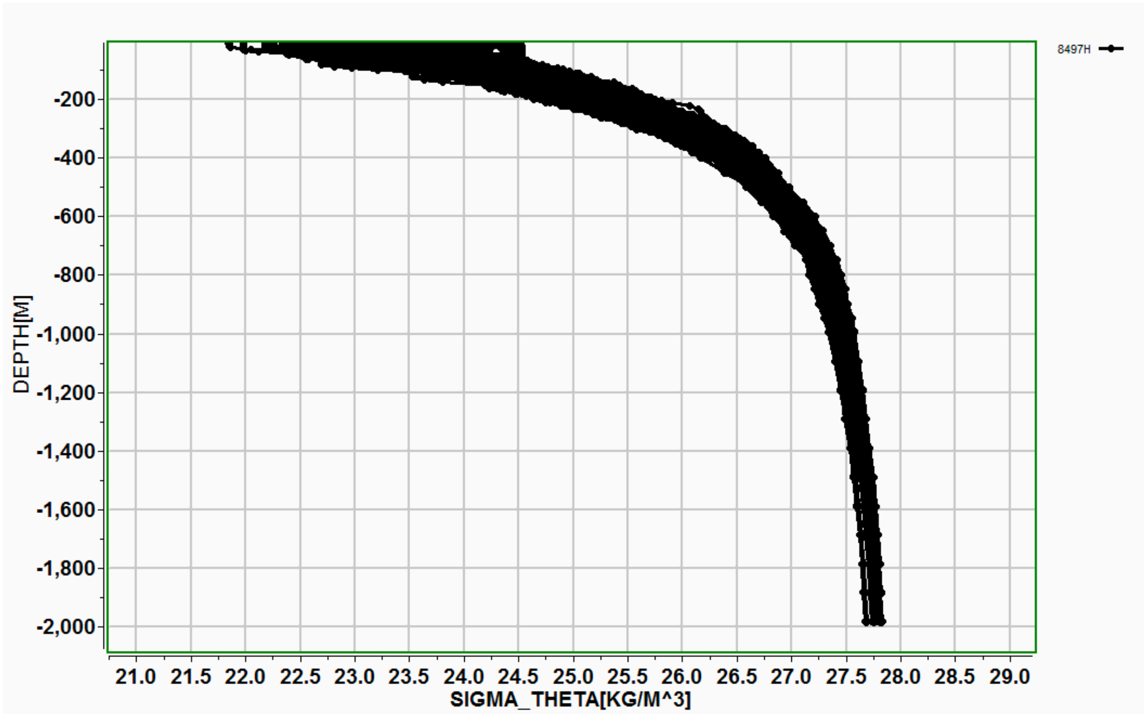

Figure 2. density anomaly profiles of float 8497Hawaii.

b. (5pts) On average, what is the difference in water density between the surface and depth? In what part of the water column is density most variable?

Maximal difference is about 6 kg m^{-3}. The largest variability in density is near the surface (top 300m).

c. (5pts) Given that a float profiles typically from 2000m (~10 days between profiles) to the surface, in the least, what is the range of densities it should be able to accommodate (in terms of its own density so it can float and sink)?

Float density should span the range from 1021.5 to 1028 kg m^{-3}

d. (5pts) The float is a perfect cylinder with a 35cm diameter and 1.5m length. What should be its mass, such that its own density without inflating a bladder, matches the density at depth?

*Volume = π 0.3^2m^2/4 * 1.4m = 0.09896m^3.*

If float density matches that of the water at depth, it's mass is:

Mass=Volume x density = 0.09896m^3 x1027.8 Kg/m^3 = 101.71Kg;

e. (5pts) Given the above mass and volume, how much should the bladder be inflated (in cm^3), to allow the float to reach the surface for all the conditions it encountered?

To reach the surface in all condition: density = 1021.7 kg m^{-3}*=101.71Kg /Volume → Volume = 0.09955*m^3 *→ an inflation of 0.0006* m^3 *= 600*cm^3 *of the bladder is necessary to change the float buoyancy to make it reach the surface (note that I rounded to +/-10*cm^3*).*
