SMS-204: Integrative marine sciences II.

Lecture 2:

**Topic 1: Mass, mass conservation, volume and density.**

*Mass* is a fundamental physical property of matter. It is conserved, which makes it a very useful quantity. *Conservation* does not mean that the amount of water in a reservoir is always constant. It means that the total mass changes only if we remove some of it (for example by evaporation, by pumping it out or by using some for a chemical reaction such as photosynthesis) or add some (for example by rain). Mass is related to volume and density:

\[
\text{density} = \frac{\text{Mass}}{\text{volume}}.
\]

When the fluid’s density changes (for example, due to changes in temperature), its volume changes (since mass stays constant). In solids the changes in density with temperature are smaller than in fluids. When comparing fluids (for example to determine which will float over which) mass itself is not useful; we may have lots of one fluid and little of the other, yet it is the one that is denser that will sink below the less dense fluid. We care about the mass per unit volume, the density.

**Topic 2: Continuity, flux.**

*Continuity* is a principle which incorporates motion (dynamics) into the conservation of mass. Assume a flow in a pipe with diameter \( D_1 \). Assume downstream the pipe is attached to a narrower pipe of diameter \( D_2 \) (Figure 1). Conservation of *mass* (for a *non-compressible* fluid) implies that the same *volume* of fluid passes through any position across the pipe. If the velocity at \( B_1 \) is \( v_1 \) and at \( B_2 \) is, in average, \( v_2 \) then:

\[
\frac{\pi D_1^2}{4v_1} = \frac{\pi D_2^2}{4v_2} \Rightarrow v_2 = v_1 \frac{D_1^2}{D_2^2}.
\]

More generally,

\[
v_2 = v_1 \frac{A_1}{A_2},
\]

where \( A_1 \) and \( A_2 \) are the cross sectional areas at \( B_1 \) and \( B_2 \).
Figure 1. Conservation of mass implies that the same volumetric amount of fluid passes across the pipe at any given time at both $B_1$ and $B_2$. Since $D_2 < D_1$ the fluid must accelerate. Fluid cannot 'concentrate' within the system (assuming it is not compressible).

The amount of fluid passing through a pipe (or river, or a vein) per unit of time is called the mass flux. Mass flux has dimensions of $M/T$. It is related to other quantities we encountered by:

$$\Phi_M = v \rho A,$$

where $\Phi_M$ denotes the mass flux, $A$ - the cross sectional area, $\rho$ - the fluid's density, and $v$ - the velocity perpendicular to the cross sectional area.

What makes water flow through a pipe in the first place? There must be force acting on it. It is the pressure difference between the two ends of a pipe that causes water to go through (more about it later).

**Topic 3: Momentum, forces, and Newton's law of dynamics.**

Momentum is the product of mass and velocity. Newton's 1st law states that in the absence of any force the momentum of a body stays constant. This is another conservation principle. The only way momentum can change is by application of a force (Newton's 2nd law):

$$\frac{d(mv)}{dt} = F.$$

This statement is more general than, $mdv/dt = F$, because it allows for changes in mass. It illustrate well why even a slow moving SUV has more force than a fast driving Honda civic when it comes to a sudden stop during a collision (the change of momentum is greater).

With fluids we experience many forces; some forces act on the boundaries of fluid (e.g. friction by the bottom, wind stress at the top) and propagate into the fluid through viscosity and pressure. Some forces act everywhere within the fluid, such as gravity (a 'body' force).
The amount of momentum passing through a pipe (or river, or a vein) per unit of time is called the \textit{momentum flux}. Momentum flux has dimensions of $M\,L/T^2$. It is related to other quantities we encountered by:

$$\Phi_{\text{Mom}} = v^2 \rho A.$$

\textbf{Topic 4: Pressure and stress.}

It is often the force per unit of area, rather than the force itself, that matters in many problems. Case in point, a blunt object cannot penetrate the skin at a given force, while the same object, when sharpened, can. The skin can withstand a certain pressure (force per unit area perpendicular to it) which is exceeded when the same force is applied to a smaller area.

Force is a vector and we can always decompose a vector into its three spatial components. So is the force per unit area a vector. The component of the force per unit area perpendicular to the surface is the \textit{pressure} while the two components parallel to the surface are called \textit{stresses}. Thus the wind applies a stress on the ocean surface (and some pressure when waves are present) while the weight of the atmosphere applies a pressure.

Within a fluid \textit{at rest} excess pressure is the same in all direction (isotropic) at any given point within the fluid and is perpendicular to the boundaries encompassing the fluid. Excess pressure is the pressure when the hydrostatic pressure is removed from the total pressure. \textit{Hydrostatic pressure} refers to the pressure due to the weight of the fluid above. We feel the pressure of the atmosphere above us. Organisms in the ocean fill in addition to the weight of the atmosphere the pressure due to the weight of the water. If the pressure were not isotropic at each point, fluid would move from high to low pressure in response. Pressure does change in the vertical; however that pressure change is balanced exactly by the weight of the fluid and creates no motion. It does cause the value of the pressure to increase with depth.

Let's assume a cylinder filled with fluid and compute the pressure due to the fluid on the bottom. This pressure is simply:

$$P = \text{Force/Area} = (m_{\text{air}} + m_{\text{water}})g/A = P_{\text{air}} + Ah\rho_w g/A = P_{\text{air}} + h\rho_w g,$$

Where we assumed a uniform density for water ($\rho_w$) and a height of the fluid equal to $h$ (thus the volume is $Ah$). The atmospheric pressure was assumed constant ($P_{\text{air}} \approx 1.01 \times 10^5 \text{Pa} \pm 10\%$). The pressure varies with depth and is given at depth ($h$) by the same formulae. If the fluid is in a container, then at each level the pressure applied by the fluid on the walls is applied back by the walls on the fluid (a consequence of Newton’s 3\textsuperscript{rd} law of motion).

\textbf{Problem:} What is the side force applied on a vertical Dam of width W and height H by a fluid?
Solution: At any depth h the net sideway pressure felt by the dam is: \( h \rho_w g \). The average pressure is \( H \rho_w g / 2 \). The pressure is applied over the whole area of the dam (\( A=W \cdot H \)). Thus the force applied by the fluid on the dam is: \( F=PA=WH^2 \rho_w g / 2 \).

Now, what if instead of a vertical container we had a container shaped like an inverted cone? It turns out that the pressure is exactly the same as for a cylinder. Can you explain why?

Divers (and animals who have air cavities), need to equalize pressure with their surrounding as they dive (why?). Similarly a diver needs to exhale into the mask to prevent it from collapsing inward.

Problem (from http://www.uvi.edu/Physics/SCI3xxWeb/Plumbing/FluidStatics.html): What will happen to a diver if he/she dives to 10m without equalizing his/her mask?

Solution: At the surface the pressure inside the mask is similar to that outside and equals \( P_a \). At depth h the pressure outside is \( h \rho_w g + P_a \) while that inside stayed the same, \( P_a \). The net difference at 10m is \( 10 \cdot 1030 \text{kg/m}^3 \cdot 9.81 \text{m/sec}^2 \approx 10^5 \text{Pa} \) (about the same as the atmospheric pressure on its own). A typical mask has a surface area \( 0.1 \text{m} \cdot 0.15 \text{m} = 0.015 \text{m}^2 \). Thus the net force on the mask is \( F=PA=1500 \text{N} \) equals to the weight of 150Kg (~330 pounds) person sitting on ones face!

Pressure exerts another (dynamic) force on fluids: fluids flow from high to low pressure. Presence of a pressure gradient forces fluid motion (Remember, \( \text{mdv/dt}=F \) and pressure is a force per unit area. Gradients in pressure indicate that there is a net force in a given direction). Similarly, water will not flow through a pipe unless there is a pressure change along it, it will flow from high to low pressure. This is what allows us to drink through a straw (dropping the pressure in our mouth relative to the atmospheric pressure)!

The hydraulic press (Fig. 2) illustrates how we can use fluids to raise a heavy object. Pressure is transmitted to all part of the fluid. The work required (force*distance) to raise a heavy object is equal to the work performed in pushing the fluid. Pushing a small area requires less force \( (F_1=PA_1) \), than a large area. The pressure is transmitted in the fluid and raises the car \( (F_2=PA_2) \). The excess pressure (above the hydrostatic) in the fluid is the same everywhere. So is the work (force*distance).

\[
\text{Work}=F_1d_1=F_2d_2 \rightarrow d_2=d_1(F_1/F_2)=A_1/A_2
\]

Note that continuity implies that the volume of fluid displaced downward (\( d_1A_1 \)) equals the volume of fluid displaced upward (\( d_2A_2 \)). This is the same result as above! Note that the car is not lifted as high as the fluid we pushed, yet with less force applied a greater distance we were able to move an object we couldn’t elevate directly.
Could this mechanism work with air?

**Topic 5: Compressibility and the equation of state.**

Compressibility of a fluid is its tendency to change its density under pressure. Water has very little compressibility and for our purposes can be treated as incompressible (sound, however, as we will see later, depends on compressibility to propagate). Air is compressible. The equation of state of an ideal gas, which is applicable to air in normal conditions, is:

\[ P \cdot V = n \cdot R \cdot T, \]

where \( T \) is the absolute temperature (°K), \( n \) the numbers of mole of the gas (1 mole=6.022·10^{23} \text{ molecules}) and \( R \) (8.314 J/g-mole K) a constant. Note that unlike incompressible fluid, for an ideal gas increase in Pressure result in decrease in volume. Thus, if we sink a balloon (or our ears) in water its volume will shrink. Similarly, releasing a helium balloon to the atmosphere will cause it to expand. How much expansion or contraction occurs depends on the elastic forces associated with the balloon's (or ear drum’s) skin. This is the reason divers need to equalize their ears and mask. If ears were filled with an incompressible liquid (as is our body), no equalization was needed.

By definition:

\[ \rho = \frac{\text{Mass}}{\text{Volume}} = \frac{n \cdot M_w}{\text{Volume}} \]

where \( M_w \) is the molecular weight. It follows that for an ideal gas:

\[ \rho = \frac{M_w P}{(RT)}. \]
Thus the density of an ideal gas increases with pressure at a constant temperature. In the Earth’s atmosphere density decreases with distance from the Earth (z) in approximately exponentially:

$$\frac{\rho(z)}{\rho(z=0)} = \frac{p(z)}{p(z=0)} = \exp(-z/8400 \text{m}).$$

For water the equation of state is a complicated function of temperature, salinity, and pressure (Fig. 3 and 4).

![Salinity vs. Freezing Point of Water](http://geoserv.geology.wmich.edu/dave/otln7.htm)

*Figure 3: density as function of salinity and temperature at sea level. from: http://geoserv.geology.wmich.edu/dave/otln7.htm*

When water freezes it is less dense than when liquid (that is often referred to as the anomaly of water).
For many purposes (for example modeling winds and currents near the coast) we can assume that air and water are incompressible. However, compressibility of these substances is essential to the transfer of sound waves within them, as well as explaining why the local temperature in the water increases at deeper depth.

References and additional reading:

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