Topic 1: Buoyancy and the Archimedes's principle.

Archimedes's principle: The buoyancy force on an object is equal to the weight of the liquid displaced by the object.

\[ F_{\text{buoyancy}} = m_w g = \text{Volume}_{\text{displaced}} \cdot \rho_w g \]

Where \( m \) denotes mass, \( \rho \) denotes density and \( g \) the acceleration of gravity.

Another force acting at the same time is gravity:

\[ F_{\text{gravity}} = m_{\text{object}} g = \text{Volume}_{\text{object}} \cdot \rho_{\text{object}} g \]

If the object is sinking (it is 'negatively' buoyant), the resulting force on it:

\[ \Delta F = \text{Volume}_{\text{object}} (\rho_{\text{object}} - \rho_w) g > 0 \]

The apparent weight of the object (the net gravitational force pulling it down) has been reduced by an amount equal to the mass of water displaced.

Now suppose the object does not sink beyond a certain depth (i.e. it is neutrally buoyant). At that depth \( \Delta F = 0 \) \( \Rightarrow \rho_{\text{object}} = \rho_w \).

Positively buoyant objects float on the water. The buoyancy force on an object that floats equals the weight of the object. Thus,

\[ \text{Volume}_{\text{displaced}} \cdot \rho_w g = mg \] \( \Rightarrow \) The trick with floating steel (for example a tanker) is to displace a large volume (a bowl shaped object)!

Topic 2: Energy, work, power, heat and temperature

(see [http://hyperphysics.phy-astr.gsu.edu/hbase/enecon.html](http://hyperphysics.phy-astr.gsu.edu/hbase/enecon.html) for excellent discussion and definitions of concepts associated with energy).

Energy is the capacity to do work, and thus have the same units (force times distance, ability to push a load from here to there).

Energy comes in many flavors. The energy of a mass in motion is called kinetic energy (it equals the work needed to accelerate it or to stop it). The energy of photons is called radiative energy.

Potential energy, is, as its name convey, the stored energy that can be released in a certain sets of circumstances. For example, the gravitational potential energy is the energy of an object that can fall from height \( H \) to the ground (=mgH). If we make a hole below the
object, we could obtain more energy at its point of impact. Other forms of potential energies are elastic (stretched spring), pressure-volume (Champagne bottle), chemical (the energy released when two substances interact), Nuclear energy (energy released in nuclear chain reaction) and electrical due to the attraction of oppositely charged particles.

**Heat** is another form of energy. Heat is related to temperature though it is not exactly proportional to it (we need to invest heat to change water from ice to liquid without raising the temperature). Temperature is proportional to the mean kinetic energy of the molecules in our system. The faster the movement of molecules the higher is the temperature.

**Friction** is the process through which mechanical energy is converted to heat. The breaks in our car heat when we use them and so do our hands when we rub them against each other. Friction can often be an undesirable conversion of energy to heat. In that process a loss of some energy (to heat, another form of energy) occurs.

Energy is another conserved physical quantity (in addition to mass and momentum we explored previously). It wears many forms and can change from one form to another (e.g. a steam engine converts chemical potential energy stored in coal to heat to pressure-volume energy that is, in turn, converted to kinetic energy).

The amount of work done per unit time and similarly the change of energy as function of time is the **power**.

**Topic 3: Energy conservation, Bernoulli's principle and its application.**

In fluids we have several forms of energy:

e-internal energy per unit volume, comprised of kinetic and potential molecular energy.

PV-pressure-volume energy (P-pressure, V-volume).

Mgz-mechanical potential energy associated with the fluid’s mass (M-mass, g-gravitational constant, z-height).

Mv²/2-mechanical kinetic energy (v-velocity).

When work is being done by/on the fluid (W) or when heat is exchanged with the surrounding, some energy can be gained/lost (note, within the whole system energy is conserved).

When (a) the flow is steady, (b) no work is performed, (c) the flow is frictionless, and (d) the fluid is incompressible (ρ is constant), then energy is conserved along the flow (from point ‘1’ to point ‘2’) and can be written (per unit mass of the fluid) as:

\[
E_1/m = v_1^2/2 + gz_1 + p_1/\rho = v_2^2/2 + gz_2 + p_2/\rho = E_2/m
\]
This is known as Bernoulli's principle.

Application of energy conservation: water squirting through a hole in a tube (Fig. 1).

\[ mgz_f = mv^2 \frac{1}{2} \rightarrow v = (2gz_f)^{1/2} \]

![Diagram of a squirting fountain, where potential energy is converted to kinetic energy.](image)

*Figure 1. A squirting fountain, where potential energy is converted to kinetic energy.*

Note that increasing the size of the hole or the diameter of the tank will not change the speed of the fluid squirting out of the tube (except when the hole is too small such that friction by its side affect the flow considerably).

One of the interesting and unintuitive result of Bernoulli’s principle is that when a fluid approaches a constriction and accelerate (to satisfy continuity) the pressure at the constriction drops (the pressure must increase ahead of the constriction as we show below).

This principle can also be derived directly from Newton's 2nd law (\( ma = F \)). Neglecting for simplicity the gravitational term:

\[ \frac{mdv}{dt} = -VdP/dx \]

where \( v \) is velocity and \( V \) volume.
multiplied by vdt=dx:

\[ \Rightarrow \rho \, dv = -dP. \]

Integrating both sides:

\[ \Rightarrow \frac{v_1^2}{2} + \frac{p_1}{\rho} = \frac{v_2^2}{2} + \frac{p_2}{\rho} \]

Application, flow over a hill: Flow above a bottom intrusion changes its velocity. This change in momentum is done by the pressure force. Fluid accelerates down a pressure gradient and decelerates when the pressure gradient reverses (Fig. 2).

**Figure 2.** Fluid above a hill (clam, boulder, etc') is accelerated and decelerated (lines represent streamlines). The pressure field supplies the force to accommodate that adjustment.

**Topic 4: A fluid zoo: streaklines, streamlines, pathlines, and flow tubes.**

It is often helpful to be able to visualize the flow using dyes and particles. One way of visualizing flows is to continuously release dye from a series of points in the fluid. If the fluid is not turbulent, the dye will form streaks, which are termed streaklines. The paths of individual particles released in a flow that are reconstructed using long exposure photography are called pathlines (the trajectory of single particles). Finally, if we use particles in the fluid to map the velocity field (by high-frequency photography) and connect all the instantaneous velocity vectors are the streamlines (everywhere tangent to the velocity).

In steady flows all three lines coincide and do not change, while in an unsteady flow (where velocity at a point varies in time) they change in time and need not coincide. In steady flows, therefore, these lines are all tangent to the velocity field. If we define an area \((A_1)\) at a point we can map it to area \((A_2)\) where the fluid will be after a time interval \(\Delta t\). To do so we simply map the points on the boundary of \(A_1\) along the streamlines for an
interval of $\Delta t$. In this process we define a flow tube. The tube may contract or expand but no fluid can leave the tube from its sides (since by definition its sides are parallel to the flow). By continuity $v_1A_1=v_2A_2$. Thus if the flow tube contracts the fluid must accelerate. Thus if we observe streamlines group together around a body we know that the flow there must be faster than upstream where they were further apart. From Bernoulli's principle (assuming no change in the average vertical position) we know that the pressure must change as well between the streamlines.

![Flow tube diagram](image)

Figure 3. A flow tube, constructed mapping the points on the boundary of $A_1$ along the streamlines for an interval of $\Delta t$

References and additional reading:
Faber, T. E., 1995, Fluid dynamics for physicists, Chapter 1.

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