SMS-204: Integrative marine sciences.

Assignment #1

Knowledge base: Linking mass, volume and density. Skills: uncertainties, propagation of uncertainties, graphing data, regression, adding error bars to graphs. Computing basic statistical estimators with a spreadsheet.

The assignment can be done by a group. However, each student hands-in *individual*

homework that is NOT IDENTICAL to other group members. In the case of IDENTICAL homework the grade will be split among the students. A rubric addressinglevel of expertise in treatment of data is provided on the class's web site.

Submit the homework electronically to: sms204.umaine@gmail.com by next Monday at 12:10pm. Late homework will suffer an automatic 10pt penalty for the first week and afterwards will get a grade of zero. Name your homework: lastname_intial_HW#.doc or .pdf (e.g. Smith_N_HW1.doc for the first homework).

1. A. Compute the volume and densities and their uncertainties for all the rods your group measured. Divide the rods into ***two groups*** with similar densities (5pts).

1. The uncertainty in the scale varies with the scale we used. +/-0.0005gr is an upper bound based on our list accurate scale.
2. The uncertainty due to the ruler is +/-0.5mm=+/-0.05cm.

The uncertainties in volume are based on those for width (diameter, D) and length (H):

Below I will assume the uncertainty in the diameter to be based on the caliper (δD=0.01cm) and in the length to be based on the ruler (δH=0.05cm).

Volume (cylinder) = π width^2 $\times$ length / 4= $\pi D^2 \times H / 4$

Using the formula for propagation of errors provided to you in the lab, we can calculate of the uncertainty in the volume of a cube:

$$Volume = x \cdot y \cdot z \rightarrow \frac{dVolume}{Volume} = \sqrt{\left(\frac{dx}{x}\right)^2 + \left(\frac{dy}{y}\right)^2 + \left(\frac{dz}{z}\right)^2}$$

$$\rightarrow dVolume = Volume\sqrt{\left(\frac{dx}{x}\right)^2 + \left(\frac{dy}{y}\right)^2 + \left(\frac{dz}{z}\right)^2}$$

For a cylinder this will change to:

$$\frac{\delta Volume}{Volume} = \sqrt{\left(\frac{\delta H}{H}\right)^2 + 2\left(\frac{\delta D}{D}\right)^2}.$$

Uncertainties in density are computed from those of a ratio:

$$z = x \cdot y \rightarrow \frac{dz}{z} = \sqrt{\left(\frac{dx}{x}\right)^2 + \left(\frac{dy}{y}\right)^2}$$

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hence:

$$\frac{\delta density}{density} = \sqrt{\left(\frac{\delta Volume}{Volume}\right)^2 + \left(\frac{\delta mass}{mass}\right)^2}$$

Since the mass of *each* cylinder was different so will the uncertainty.
For example, for the 2.522cm long rod, with diameter of 1.277cm that weighed 3.63gr:

$$\frac{\delta density}{density} = \sqrt{\left(\frac{0.2}{3.1}\right)^2 + \left(\frac{0.0005}{3.652}\right)^2} = 0.06 \rightarrow \delta density =$$

$$0.06 \cdot \frac{3.652}{2.3 \cdot 1.3^2 \cdot \pi / 4} = 0.07 gr / cm^3$$

Rod #	1	2	3	4	5	6	7	8
Mass (g)	3.652	5.37	5.111	7.132	6.571	8.895	10.978	14.303
Length (cm)	2.3	2.9	3.3	4	4.4	5	7.5	7.8
Diameter (cm)	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.3
mass	Repeat of the above. No need to input anything here							
Δmass (g)	0.0005	0.0005	0.0005	0.0005	0.0005	0.0005	0.0005	0.0005
Volume (cm^3)	3.05	3.85	4.38	5.31	5.84	6.64	9.95	10.35
Δvolume (cm^3)	0.2	0.2	0.2	0.3	0.3	0.4	0.5	0.6
Density (g/cm^3)	1.20	1.40	1.17	1.34	1.13	1.34	1.10	1.38
Δdensity (g/cm^3)	0.07	0.08	0.07	0.07	0.06	0.07	0.06	0.08

Table 1. Rod measured and derived data. Highlighted data represents a group of rods with different density than those that are not highlighted

The rods you analyzed where of two densities, about 1.15g/cm^3 and about 1.37g/cm^3 (taking into account that the uncertainty in density is at least +/-0.06g/cm^3). Given the above uncertainties we should easily be able to differentiate the two groups of rods. Here I use the data that is on the class web site . I kept significant digits in the table based on the magnitude of the uncertainties (no point having 5 digits after the decimal for a value that has an uncertainty on the order of +/-0.01). More on rounding and reporting errors on a handout in the class's website

B. Plot the **mass** as a function of **volume** for these ***two groups***. Use the your experience from previous IMS classes to make a good plot (one that has: labels for each axis which includes units, tick marks on axis and a symbol to denote each datum with *no* line connecting them and no grid lines) (10pts).

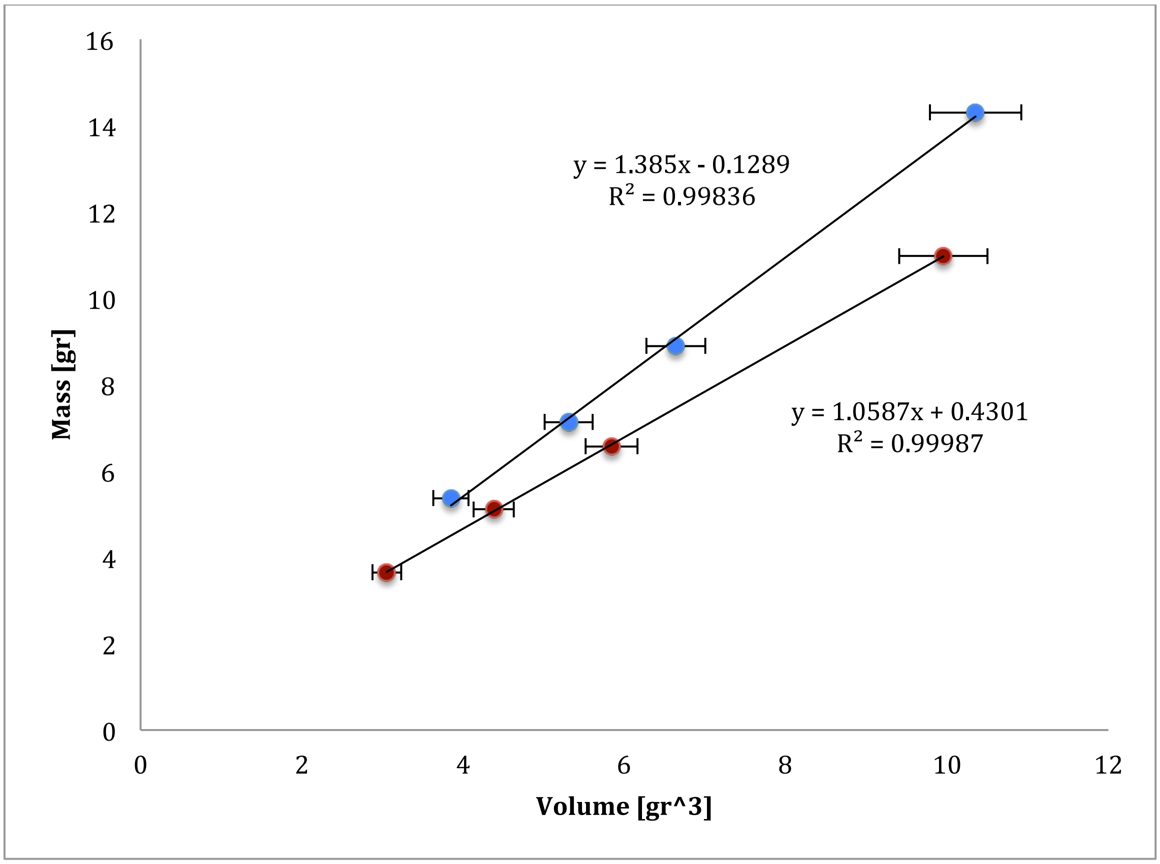

Figure 1. Rod mass as function of volume. Error bars indicate uncertainties in the measurements. The uncertainties in mass are too small to show. The lines are the linear regression fit to the two different groups of rod of similar density.

Note: it may be easier for you to separate the two groups of similar density rods into separate graphs. That is OK.

C. For both groups of rods, obtain the linear regression line between Mass and volume, a (this can be done by right clicking the data points in Excel, see links below). (5 pts)

See Figure 1.

D. Display the equation and the line on the graph. You should get an equation of the type: Mass=$A \times$ Volume + B. (5pts)

See Figure 1.

E. Does there seem to be a linear relationship between Volume and Mass? (5pts)

Yes. Mass is linearly correlated with volume and a straight line fits well within the data.

F. What does the slope of the regression line represents (A in the equation in question 1D) (5pts)? What are its units (5pts)?

The slope represents the density=mass/volume. Units are [g/ml] or[g/cm^3] .

G. What are the units of the intercept (B in the equation in question 1D) (5pts)?

Unit is [g] as the equation is one for mass.

I. Add uncertainties (in the form of error bars, see links below) to your plot (5pts).

See Figure 1.

J. Explain how the uncertainties were computed (5pts).

Uncertainties in mass were assumed constant based on the rounding of values provided in the table, e.g.+/-0.0005g. Uncertainties in volume are based on propagating error associated with measurements of length and width. I assumed we measured width and length with ruler had the same uncertainty (+/-0.05cm) and propagated using:

$$\frac{\delta Volume}{Volume} = \sqrt{\left(\frac{\delta H}{H}\right)^2 + 2\left(\frac{\delta D}{D}\right)^2}\ .$$

2. A. Using an Excel spreadsheet (or another program of your choice) and the **data your whole class collected** on sinking speeds of different beads, compute for each bead the mean, median, maximum, minimum, 16th percentile, 84th percentile, and the standard deviation of the sinking speed for that bead (10pts, those program have built-in function to compute these, to learn how to use them, for example google 'how to compute percentile with excel' + some tutorial on YouTube will walk you through it).

Note: Data were converted to mm. Data from two teams were clearly erroneous. Team 'Mola Molas' data for diameter should have been for radius (compared to all others) and were corrected. Team 'Mystery Inc''s diameters were clearly wrong (in value and with relations to the other measurements) and hence their data were not used. Problematic data is highlighted in table 2.

Group	Bead 1	Bead 2	Bead 3	Bead 4	Bead 5
1	3.18	4.77	6.36	10.52	12.71
2	3.16	5.76	6.36	10.54	12.71
3	3.12	5.72	6.32	10.51	12.7
4	3	5	6	11	13
5	3.6	4.6	7	10.8	12
6	3	4	6	11	13
7	3.18	5.77	6.35	11.55	13.7
8	3.17	4.76	6.34	9.52	12.73
9					

10	3.18	5.77	6.35	10.57	13.71
11	3.18	5.82	6.36	10.54	13.72
12	3	5	6.5	10	13
Average [mm]	3.16	5.18	6.36	10.60	13.00
Median [mm]	3.17	5.00	6.35	10.54	13.00
Standard deviation [mm]	0.17	0.62	0.26	0.53	0.53

Table 2. Bead size in [mm] and its statistics for all the groups.

Median bead size [cm]	0.317	0.5	0.635	1.054	1.3
Sinking speeds [cm/s]	3.52	7.73	13.2	27.4	37.3
	3.7	7.19	12.7	25.6	38.4
	3.57	7.5	13.2	24.5	37.3
	3.53	7.6	13.2	25.85	38
	3.6	7.5	12.79	25.33	36.89
	3.48	7.67	12.92	25.33	36.89
	3.63	7.59	12.362	24.49	38.2
	3.64	7.23	12.44	24.97	38.2
	3.37	7.69	12.73	23.43	37.09
	3.61	7.78	12.7	25.03	38.88
	3.55	7.57	12.58	25.03	38.3
	3.53	7.49	12.87	25.53	38.69
	3.29	7.3	12.53	32.84	38.1
	3.31	7.09	12.33	26.64	36.99
	3.3	7.13	11.41	23.52	35.61
	3.4	7.3	12.3	23.5	35.8
	3.4	7.1	11.7	23.5	38
	3.4	7.1	11.6	23.5	32.8
	3.44	7.40	12.21	23.46	35.94
	3.544	7.313	12.33	23.8125	36.99
	3.423	7.299	12.451	23.962	39.278
	3.38	6.13	12.45	22.54	36.99
	3.35	7.22	11.94	22.54	36.99
	3.37	7.1	12.21	23.37	36.63
	3.73	7.949	13.523	25.85	38
	3.766	8.05	13.33	27.14	38
	3.718	8.025	13.149	26.389	42.697
	3.8	8.1	13	28	45
	3.7	7.9	13	27	42
	3.7	7.9	13	26	43
	3.66	7.73	13.24	25.33	35.84
	3.61	7.85	13.24	25.33	40.43
	3.63	7.74	13.06	25.85	40.43
average speed [cm/s]	3.53	7.49	12.66	25.23	38.17
median speed [cm/s]	3.54	7.50	12.70	25.33	38.00
standard deviation [cm/s]	0.15	0.39	0.52	1.96	2.39
maximum [cm/s]	3.80	8.10	13.52	32.84	45.00
minimum [cm/s]	3.29	6.13	11.41	22.54	32.80
16 percentile	3.37	7.14	12.22	23.50	36.66
84 percentile	3.70	7.89	13.20	26.61	40.29
(84%-16%)/2	0.16	0.38	0.49	1.55	1.82

Table 3. Sinking speeds and their statistics.

B. How do the median and mean compare? How does the standard deviation compare to (84th percentile-16th percentile)/2? We expect them to be similar for a normal distribution (5pts).

They are similar. Hence the underlying statistical distributions of settling velocities are probably not very different from normal. A few outliers in settling speed cause the standard deviation to be larger for large beads.

C. Plot the **median** sinking speed of the beads (*y* or vertical-axis) as function of their cross-sectional area (=π × radius^2) (*x* or horizontal axis). (10pts)

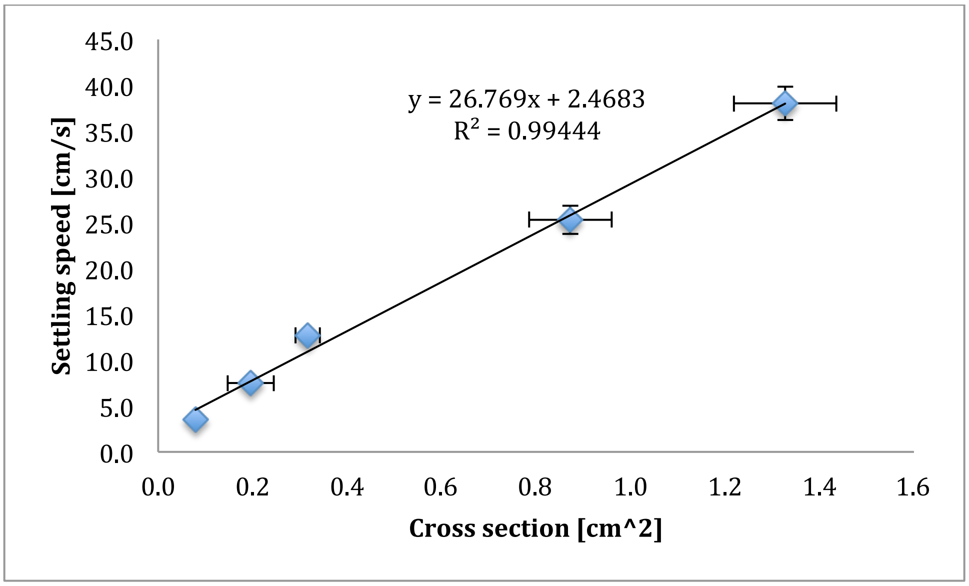

Figure 2. Settling velocity as function of cross-sectional area for beads. The line is the regression line.

D. Does there seem to be a relationship between cross-sectional area and median sinking speed (determine a relationship by plotting the regression line and displaying its equation on the plot)? Note: When the bead reaches terminal velocity, a balance exists between the downward pool of gravity and the drag force. We will get back to these data later in the semester when we will analyze the forces acting on bodies immersed in water. (5pts)

There is a monotonic relationship though a concave curve could match data better.

E. Add the uncertainties of the data points to the graph (these are called 'error bars', see links below). Use the statistics from the whole class dataset for uncertainties. What do such uncertainties represent? How do they compare to uncertainties in individual measurements of sinking speed? (5pts)

The uncertainties represent the measurement uncertainties associated with many different students measuring independently the same phenomenon. They are typically much larger than uncertainties based on an individual's measurements.

Uncertainties in cross-section:

$$\frac{\delta Cross-section}{Cross-section} = \sqrt{2\left(\frac{\delta D}{D}\right)^2} \text{ or } 2\left(\frac{\delta D}{D}\right).$$

The reason I put the 'or' is that the first one is the right one if you took two measures of diameter. If you simply measured once (as we did) then the larger error is strictly the correct one. I would accept both.

3. (15pts): What are ALL the curious phenomena related to drag that are demonstrated in the movie "Fluid dynamics of drag, part I" (find the link to on-line movie on class web site. If you have any problem viewing the movie we can set up a computer for you)?

The curious phenomena related to drag that were demonstrated in the movie are:

1. *Sometimes an increase in speed can cause a decrease in drag on an object.*
2. *Through one range of speeds a smooth ball can have more drag than a rough ball, while through another range of speeds the rough ball has less drag.*
3. *Under certain conditions, streamlining can reduce drag, while in other conditions, streamlining increases drag.*
