SMS-204: Integrative marine sciences.

For homework- remember to provide uncertainties in graphs and display only significant figures for all your results.

HMWK 3 (please pay attention to provide uncertainties and round off numbers to the appropriate significant digits):

1. (25pts) Using results from lab station 1 (Archimedes' ball):

Work out mathematically how much air needs to be evacuated in order for the ball to barely start sinking in the surrounding water. How does it compare to your observations (are they within the observation uncertainties)?

Let's assume the dry ball is of mass m (about 125g± 0.5g), volume V (= $4\pi r^3/3$, with r the radius) and density $\rho = m/V$. The ball diameter is 7± 0.1 cm → V_{ball} = $\pi D^3/6$~180ml=1.8×10^{-4}m^3. ρ<1000Kg/m^3, as the ball is observed to float.

Let's assume you filled Xml (~52 ± 2.5ml) water in the ball (replacing the same volume of air). The ball is now barely floating, so we can assume all of its volume has displaced water. For that to happen:

Weight of ball = weight of water in the volume it displaced. From Archimedes principles we then find: $mg + X\text{ml} \times (\rho_{water} - \rho_{air}) \times g = V_{ball} \times \rho_{water} \times g$

Since $\rho_{water} >> \rho_{air}$, we can simplify to (after dividing by g):

$m + X\text{ml} * \rho_{water} = V_{ball} \times \rho_{water}$ → $X\text{ml} = V_{ball} - m/\rho_{water}$.

If everything was done right, Xml calculated in this way should be similar to the volume of air evacuated into the syringe.

This solution is identical to that which recognize that when the ball barely floats its density=density of water. $\rho_{water} = m/(V_{ball} - X\text{ml})$ → $X\text{ml} = V_{ball} - m/\rho_{water}$~ 180ml-125g/(1 g ml^{-1}) = 55ml.

The amount calculated is consistent with what was evacuated (52ml) when we take into account the uncertainty of the reading on the syringe (+/-2.5ml) and weight (+/-0.1g), and the fact that the ball was not exactly a ball in shape.

2. (30pts) Using results from lab station 4:

a. Measure/compute the volume and weight of the empty box.

b. For a series of 4 different internal amounts of weights (making sure in all cases that the box floats), measure:

- The weight outside the water using the spring.
- The weight of the box in water
- The depth to which the bottom of the box is immersed in water (each mark on the box is 1cm).

Note: weights are labeled 5 or 10 corresponding to 5g and 10g, respectively.

# and type of weights	Weight in air I accepted mass)	Weight in water (I accepted mass)	Immersion depth (cm)
0	25g	0g (floats)	1+/-0.2cm
2 × 10g + 1 × 5g	50g	0g (floats)	2+/-0.2cm
4 × 10g + 2 × 5g	75g	0g (floats)	3+/-0.2cm
6 × 10g + 3 × 5g	100g	0g (bearly floats)	4+/-0.2cm
# and type of weights	Weight in air	Weight in water	Immersion depth

c. Plot the depth to which the box is immersed in water as function of the weight of the box + the added weights (5pts). Obtain the slope of the best-fit line. (5pts)

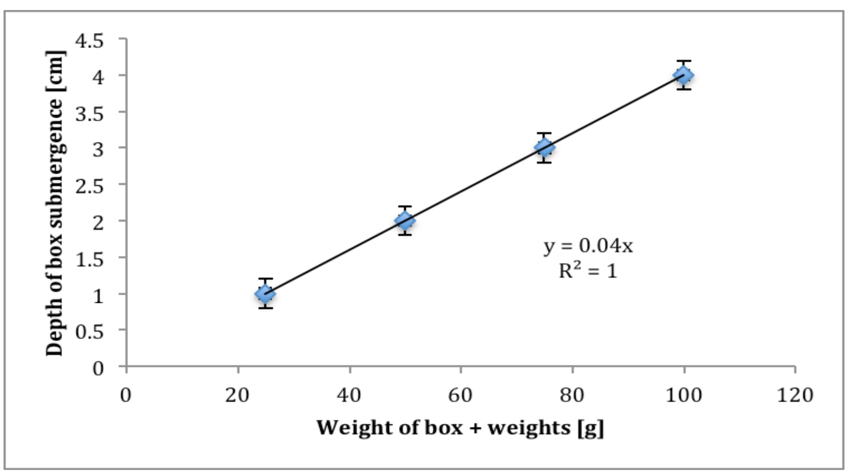

Figure 1. Immersion depth as function of weight of box + weights. The slope is ~0.04cm g^{-1} and the intercept ~0gr as expected (see below).

d. What should the slope be based on Archimedes's principle (5pts)? How does it compare to the slope of your plot? (5pts)

Answer:

For the floating box Archimedes tells us that: $Ah\rho_{water} g = mg$

→ $Ah = m/\rho_{water}$, **where** h **is the depth of submergence,** m **the mass of the box (with the inside weights) and** A **the area of the bottom of the box (25cm^2).**

→ the slope of the graph of h **as function of** m **should be** $1/A\rho_{water} \sim 1/(25\ \mathrm{cm}^2 \times 1\mathrm{gr/cm}^3) = 0.04\mathrm{cm\ g}^{-1} = 0.4\mathrm{m\ Kg}^{-1}$**.**

e. You have added enough weights so that the box sinks in water.

f. In that case, what is the weight of the box when immersed in water and when outside water? What is the difference between them? (5pts)

g. Is this difference reasonable given what you know about buoyancy? (5pts)

Answer: Let the weight in Air be m_1 **and the weight in water be** m_2**.**

By Archimedes principle, the buoyancy force equals to the force due to the weight of the water displaced (Volume*ρ_{water}*g), and equals the difference between weight in air (m_1*g) and in water (m_2*g). Dividing all sides by the gravitational acceleration g **we get:**

→ $m_1 - m_2 = \mathrm{Volume} * \rho_{water}$**.**

The Volume of the box=100cm^3 → $m_1 - m_2 \sim 100 * 1\mathrm{gr/cm}^3 = 100\mathrm{gr} = 0.1\mathrm{Kg}$**.**

If we measured this difference in weight than theory and measurements are consistent!

3. (25pts) In 2nd station of the 2nd lab you were studying water squirting out from a hole in a cylinder filled with water into a tub. Assume you have a 30cm head of water above the hole and that the hole is 10cm above ground.

1. How fast will the water leave the hole (think about converting potential energy per unit volume to kinetic energy per unit volume)?

Potential energy per unit volue: $\rho g h_1 \rightarrow \rho v^2/2$ **: kinetic energy per unit volume.**

→ $v^2=2gh_1$ → $v=2.43ms^{-1}$

h_1 denotes the distance from top of water to hole.

2. How long will it take it to reach the ground (think mechanics)?

You learned in mechanics that an object starting from rest (there is no vertical velocity to the water) obeys: $gt^2/2=h_2$ → $t^2=2h_2/g$ → $t=0.14s$

h_2 denotes the distance from hole to ground.

3. How far will the water reach by the time it hits the ground?

$L=vt$~0.35m or 35cm

4. How does the place where the water reaches change with the height of the water above the hole (provide an equation or a relationship)?

5. How does the place where the water reach change with the height of the hole above ground (again, provide an equation or a relationship)?

Answer for 4 & 5:

$L=vt=sqrt(2gh_1)$ x $sqrt(2h_2/g) = 2sqrt(h_1h_2)$ - where sqrt denotes square root. Hence the place where the water splash is proportional to the square-root of the height of the water above the hole times the square-root of the height of the hole above ground (it is their geometric mean and it increases with increase in any of them).

4. (20pts) based on Fluid dynamics of drag, part III:

a. What is the Reynolds number?

b. For low Reynolds number flows, what are the physical characteristics of the fluid, the object and the flow around an object that the drag is proportional to?

c. For high Reynolds number flows, to first approximation, what are the physical characteristics of the fluid, the object and the flow around an object that the drag is proportional to?

d. How do laminar and turbulent flows differ? How are they related to Reynolds number?

e. What is a boundary layer (BL)? How is the BL affected by viscosity for a given bottom type and a given fluid velocity in the interior of the fluid?

Answer:

a. The Reynolds number (Re) is a nondimensional number used to characterize flows in terms of the ratio of inertial forces to viscous forces. It is equal to the product of the length scale of the flow (L) its velocity scale (U) and the density of the fluid (ρ) divided by the fluid's viscosity (μ), Re=ULρ/μ.

b. At low Reynolds number flows viscous forces dominate, boundary layers around objects are wide and the flow is very regular. The drag force is proportional to the relative velocity of the object to the fluid, the object's size and the fluid's viscosity.

c. At High Reynolds number flows inertia dominates resulting in trubulent flows with thin boundary layers around objects. The drag force is proportional to the square of the relative velocity of the object to the fluid, the cross sectional area of the object (the square of its length scale) and the fluid's density.

d. Low Reynolds number flows are laminar, that is the fluid flows in an organized way (laminar means layered). Viscous forces are important in laminar flow. Turbulent flows have high Reynolds numbers and are highly disorganized.

e. A boundary layer is a region of fast transition in fluid flow, as is found near objects immersed in the fluid or walls. It arises as a consequence of the no-slip condition. The higher the viscosity the larger is the boundary layer (lower Reynolds number). The higher the velocity in the fluid interior the smaller is the boundary layer (higher Reynolds number). When the bottom is rough, the transition to turbulence can occur at lower velocity (or higher viscosities) resulting in smaller boundary layers in those cases.

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