SMS 303: Diffusion, answer to homework # 1

1. Plot the student concentration (# of student at each position) as function of time for t=0,4,8,12,16 turns. x-axis could be the position.

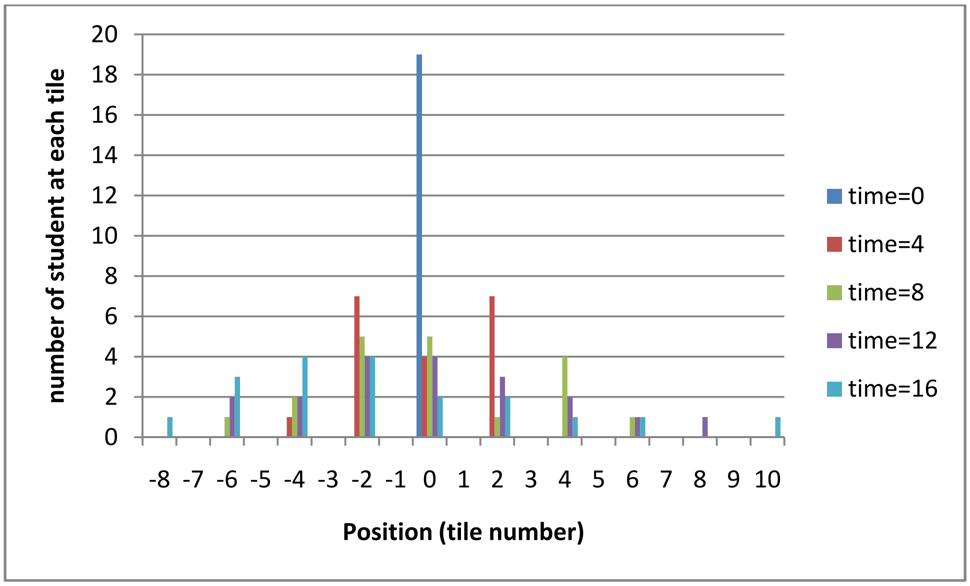

Figure 1. Number of students at each position for times t=0,4,8,12,16 turns.

2. How did the mean student position change as function of time (plot it)?

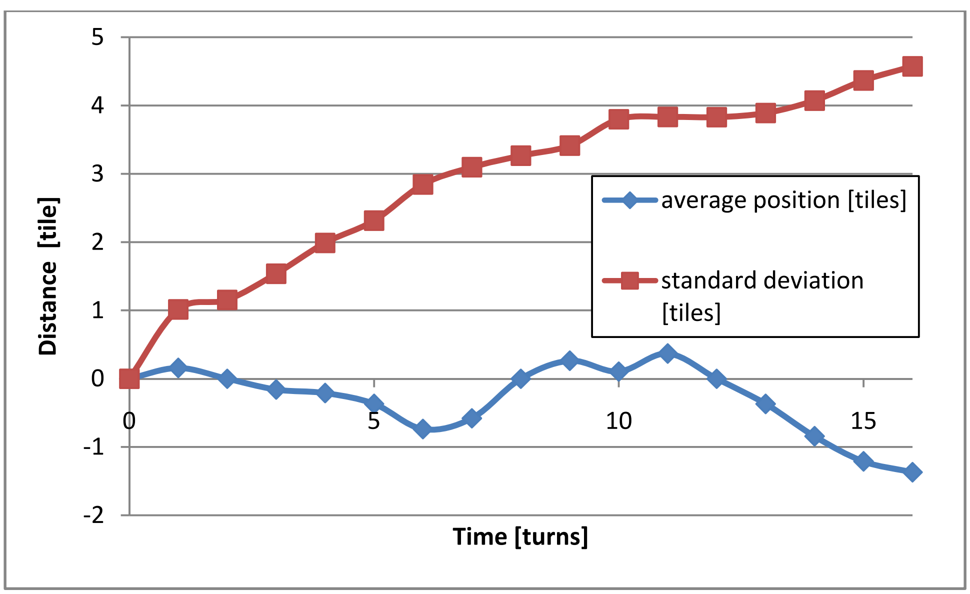

Figure 2. Average position and its standard deviation in units of tiles as function of time (in units of turns).

The mean position is wondering around zero with no clear trend (Fig. 2).

3. How did the standard deviation around the mean change as function of time (plot it)? What units does it have?

The standard deviation initially increased rapidly and then increased more slowly (Fig. 2). Its units are the same as those of position, tiles.

4. Assuming a time-step of 30sec and a step-length of 25cm, estimate from dimensional analysis the diffusion coefficient of the students in the corridor ($[D]=L^2/T$, where [] means dimensions off, D- the diffusion coefficient, L is distance and T-time).

By simple substitution we get an estimate for the diffusion coefficient: $D \sim (0.25m)^2/30sec \sim 0.002m^2s^{-1}$

Homework **part b** (40pts) – biased one dimensional random walk:

1. Plot the student concentration as function of time for t=0,4,8,12,16 turns.

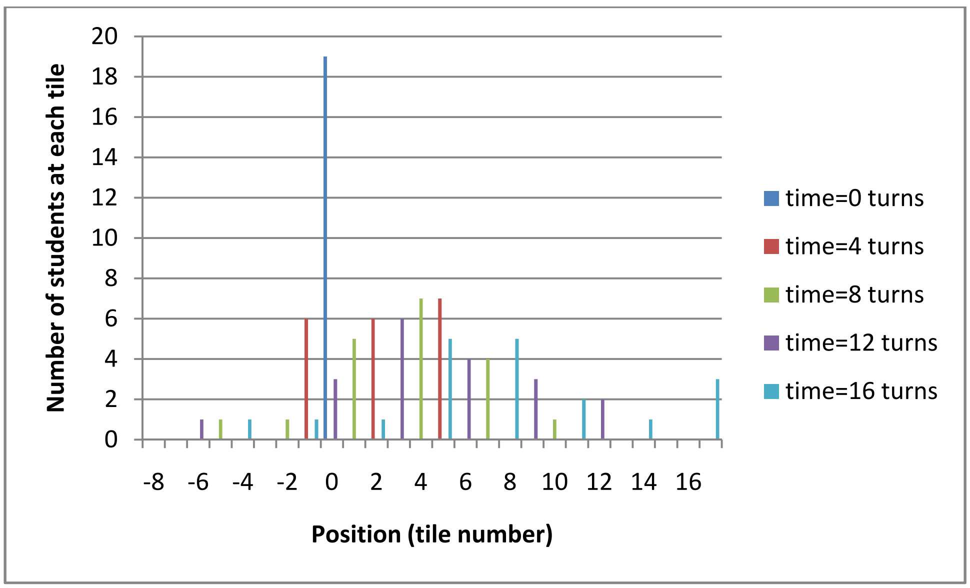

Figure 3. Number of students at each position for times t=0,4,8,12,16 turns.

2. How did the mean position changed as function of time? Plots it as function of time.

The mean position is increasing as function of time around zero with no clear trend (Fig. 4). The rate of increase is about 0.45 tiles per turn (based on fitting a line). I would have expected 0.5tiles per turn (in one turn you moved two tiles to the right (+2) or one to the left (-1); After 2N turns you would do about 2N steps to the right and N steps to the left hence your average would be N, or half the number of turns).

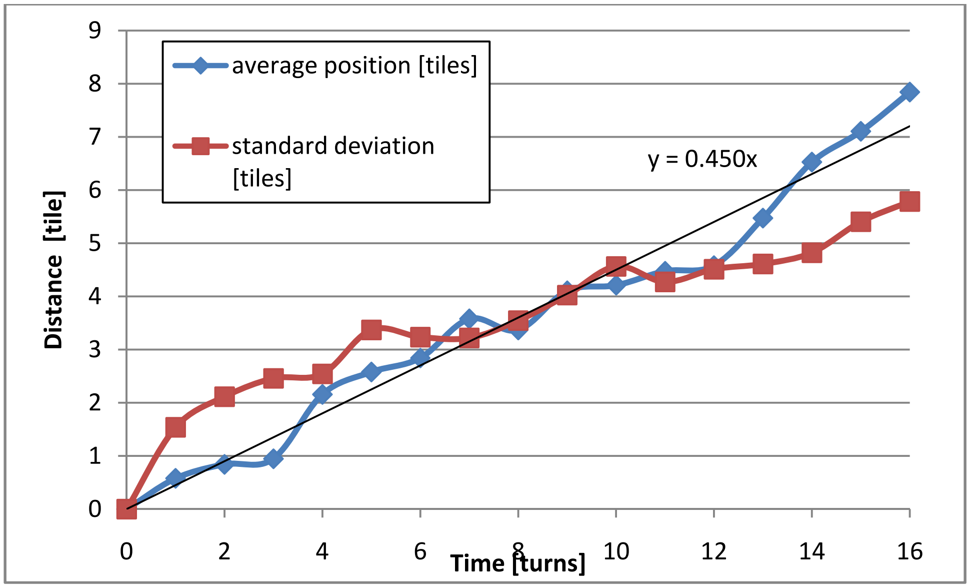

Figure 2. Average position and its standard deviation in units of tiles as function of time (in units of turns). The black line represents the fit to the mean position.

3. How did the standard deviation around the mean change as function of time (plot it)? What units does it have?

The standard deviation initially increased rapidly and then increased more slowly (Fig. 2). Its units are the same as those of position, tiles.

4. Assuming a time-step of 20sec and a step-length of 30cm, estimate from dimensional analysis the diffusion coefficient of the students in the corridor ($[D]=L^2/T$). How does the mean drift ($[U]=L/T$, U-speed) compare to what you may derive from dimensions alone?

The length scale, L, is the average step per turn which is (60cm+30cm)/2=0.45cm (1.5tiles).

$[D]\sim(0.45m)^2/20s\sim0.01m^2s^{-1}$

From dimensions: $[L]\sim1.5$tiles/turn=0.45m/20s$\sim0.02ms^{-1}$. This is larger than the actual drift (~0.5tile/turn, as it does not take into account the fact that the drift is due to the DIFFERENCE between right and left step.

Part c : Using dimensional analysis answer the following questions:

1. Assuming molecular diffusion alone, and a thermal diffusion coefficient of $D=(8.43 - 0.101\times T)\times10^{-3}$ cm^2/s with T=20°C, approximately how deep will a large atmospheric temperature change be felt after one month? (10pts)

$L\sim(D \text{ x time})^{1/2}\sim(0.00641cm^2/s \times 30day/mo \times 24hours/day \times 3600sec/hours)^{1/2}\sim129cm\sim1.3m$

2. A bacterium (1μm in size) swims at a speed of about 25 body-lengths a second. Every two seconds, on average, it tumbles and starts off swimming at another random direction (hence it mean-free-path is 50μm). Compute the time scale it will take a tiny bacterium drop released within a flat Petri dish to diffuse 2mm away from its release position (10pts).

By dimension alone $D \sim (\text{mean free path})^2/t \sim (50\mu m)^2/2s \sim 1.25 \times 10^{-9} m^2 s^{-1}$.

From dimensions alone: time to drift out $\sim L^2/D \sim (0.002m)^2/(1.25 \times 10^{-9} m^2 s^{-1}) = 3200s$ or about 1hour.