SMS 303: Homework 3. Coriolis, Inertial Oscillations calculus and Baseball

From Newton's 2nd law we know that:

Mass×acceleration=Force.

It is often convenient when dealing with fluids to divide both sides by mass:

Acceleration=Force/Mass.

Now, consider an object on a rotating platform that rotates at angular velocity Ω (= 2 π radians / period, where the period is the time it takes for one full rotation). In order to account for the rotation of the platform an apparent force known as the Coriolis force is added to the equations. In two dimensions those are:

$$du/dt=F_x/\text{Mass}+2\Omega\times v$$

$$dv/dt=F_y/\text{Mass}-2\Omega\times u$$

where (u,v)=(dx/dt, dy/dt), are the velocities in the x and y directions respectively. Here we assume x is eastward and y is northward.

Let's assume that we give a kick to the object in the direction y at time zero and observe how it moves without applying any extra force. Initial condition $v(t=0)=V_0$. Let us also denote $f=2\Omega$ (On the Earth and latitude ϕ, f is the Coriolis parameter = $4\pi \sin\phi / 24\ hr^{-1}$).

$$du/dt= f\, v$$

$$dv/dt=- f\, u$$

Homework (be careful regarding units).

1. Check that the following is a solution the equations above (5 pts):

$$u= V_0\cos(f\, t), \quad v= -V_0\sin(f\, t)$$

2. Solve for the position (x, y) as function of time, assuming x(t=0)=y(t=0)=0 (remember, u=dx/dt, v=dy/dt, so $x(t)=\int u(t')dt'$ and $y(t)=\int v(t')dt'$, with the boundaries of the integral being from 0 to t). (15 pt)

3. Plot the position of the object as function of time for 24 hours (every 1hr, using $f = 4\pi \sin\phi / 24\ hr^{-1}$) assuming a starting latitude of 30°N. a. What is the shape of the trajectory (15 pts)? B. How does the trajectory depends on V_0 and f (15 pts)? C. How long does it take for the object to come back to its initial position in terms of f and/or V_0 (15 pts)?

4. Now, assume that we are dealing with baseball and Fenway park (~42°N). The speed of the ball leaving the bat is 40m/s. Neglecting friction, what would be the position of the ball after 2 seconds (15pts)? Plot the trajectory of the ball in the x-y plane (10pts).

Further reading:

http://en.wikipedia.org/wiki/Coriolis_effect

Durran, D. R., 1993: *Is the Coriolis force really responsible for the inertial oscillation?*, Bull. Amer. Meteor. Soc., 74, 2179–2184; Corrigenda. Bulletin of the American Meteorological Society, 75, 261 (http://www.atmos.washington.edu/~durrand/pdfs/Coriolis_BAMS.pdf)

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