

SMS-303: Integrative marine sciences III.

Answer to assignment #4

1. (Total points 36, 12pts for each subsection)

In the small tanks you sloshed water with water depths of 1.5cm and 6cm back and forth and found the number of sloshes as function of time.

- a) Translate your results into the wave speeds (how fast did crest move from one side to the other). How did they compare with theory?

The median over all the groups: 6 sloshes/5sec in the 1.5cm deep tank and 13sloshes/5sec in the 6cm deep tank.

*Tank length is 30.5cm. Each slosh represent a wave propagating the whole length of the tank. Therefore: speed=length * (# of sloshes/time) = 36.6cm/s (1.5cm deep) and 79.3cm/s (6cm deep).*

- b) Was the ratio of the waves speed with 6cm depth and 1.5cm depth consistent with theory?

The ratio of the velocities measured: $79.3/36.6=2.16$

Expected from theory: $\sqrt{gH_1}/\sqrt{gH_2}=\sqrt{H_1/H_2}=\sqrt{6/1.5}=2$

Good agreement (In particular given the crude way we did it in the class).

In all cases we are dealing with shallow water waves the wavelength (twice the tank length=61cm) divided by depth is $\gg 1$. In the 6cm case it is not strictly shallow (not bigger than $20 \times \text{depth}$) but not too far from it. Thus we expect the formula $c=\sqrt{gH}$ to work. $\sqrt{g \cdot 0.015}=0.38\text{m/s}=38\text{cm/s}$ compares well with 36.6cm/s from above and $\sqrt{g \cdot 0.06}=0.77\text{m/s}=77\text{cm/s}$ compares well with 79.3cm/s above.

- c) What is the natural (or intrinsic) period of the tank (1st mode) at each depth? (hint: you should be able to calculate it using only the length of the tank (30.5cm) and the wave speeds computed above). This is analogous to a Seich.

The natural period of the tank for the 1st mode is the time it takes the wave to propagate across the tank twice (so it gets back to the same position):

For the $H=1.5\text{cm}$ case: $2 \cdot 30.5\text{cm}/36.6\text{cm/s}=1.7\text{sec}$

For the $H=6\text{cm}$ case: $2 \cdot 30.5\text{cm}/77\text{cm/s}=0.8\text{sec}$

I did not take points off for those who calculated only half the period.

- d) (extra credit, 5pts) What is the intrinsic period of the North-South oriented Seich in Pushaw lake given an average depth of 20feet and a length of 4miles? Would the rotation of the Earth be an important factor for this Seich (how does the period of this seich compare with $1/f$, where f is the Coriolis frequency from last week)?

The natural period of the tank for the 1st mode is the time it takes the wave to propagate across the lake twice (so it gets back to the same position).

The wave speed is $\text{sqrt}(gH) = \text{sqrt}(9.81\text{m/s}^2 \times 20\text{ feet} \times 0.3048\text{m/feet}) = 7.7\text{m/s}$

The length of lake is: 4miles=6437.376m

Seich period = (twice the length of the lake)/speed of seich = 1672sec ~ 28min.

$f = 4\pi \sin(45^\circ)/24\text{hr} = 0.37\text{ rad/hr} \rightarrow 1/f = 2.7\text{hrs/rad} \gg 28\text{min}$ (one should compare with $T = 2\pi/f = 16.97\text{hr}$ which is the inertial period) \rightarrow rotation is not likely to be a factor.

2. (Total points 64. a. 14pts. b. 30pts and c. 20pts)

The effect of the sun on the tides in the Gulf of Maine (GOM).

The sun and the moon both contribute to the tides in the GOM, with the moon being the major contributor. For simplicity we will consider only two tidal components (M2-lunar and S2-solar) which often have the largest amplitude of all tidal components and which will be the major contributor were the moon, sun and earth all on the same plane perpendicular to the axis of rotation of the earth.

The elevation (in m) near Rockport, ME for the two tidal wave components is given by:

Lunar (M2): $1.2\cos(28.984t)$

Solar (S2): $0.3\cos(30t)$

Where t is in **hours** and the frequencies (28.984 & 30) are given in **degrees per hour** (not radians). At time $t=0$ the sun and the moon are at the same side relative to the earth (new moon).

- a) Which component dominates the tidal amplitude?

Obviously the lunar component since $1.2 \gg 0.3$.

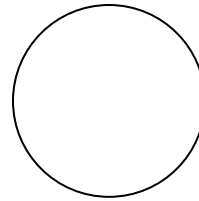
- b) Add the two components together on a spreadsheet and plot the resulting amplitude as function of time for a 28day lunar month (remember, t has to be in hours, 1 day = 24hours). Time step for the plot should be no longer than 3hours. Annotate on your plot the different arrangement of the planets every 3.5 days. For example:

t=0



Earth

Moon



Sun

Pay attention to units and to the fact that programs such as excel assume radians in the argument of trigonometric functions.

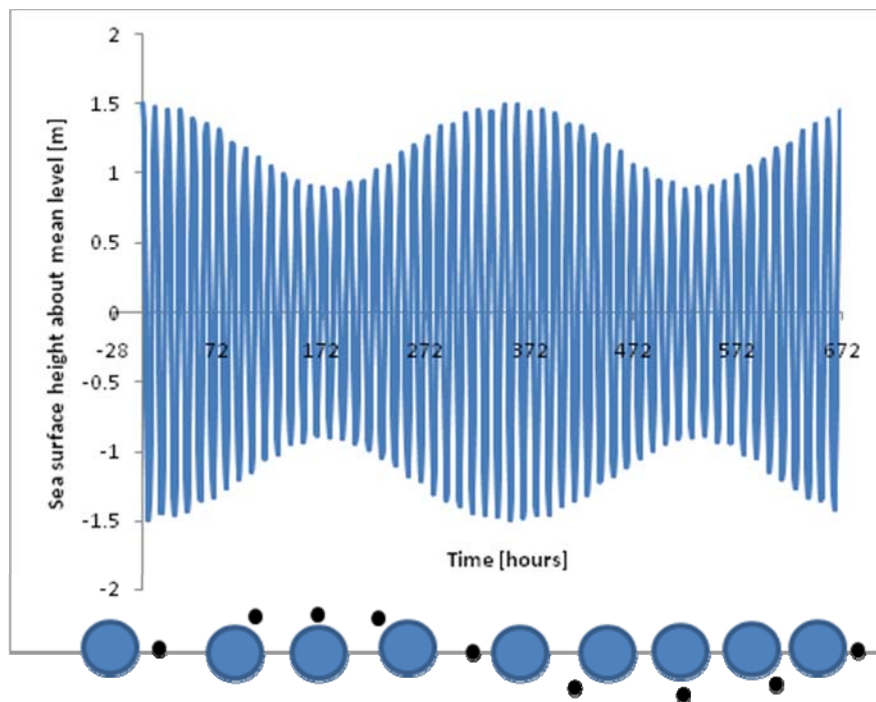


Figure 1. Height above mean sea level based on the two component give. Also drawn (not to scale) is the schematic of the respective alignment of the moon (small black circle), Earth (blue circle) and sun (yellow circle).

- c) Add the two tidal components using a trigonometric relationship for the addition of two sines ($\sin(A)+\sin(B)$), by first breaking the lunar component to two parts one of which has the same amplitude as the solar component (you can find the appropriate formula at: <http://www.sosmath.com/trig/Trig5/trig5/trig5.html>). The phenomena you are describing is called beating, the interaction of two waves with slightly different frequencies. Explain your results in section b in light of what you have found.

$$1.2\cos(28.984t)+0.3\cos(30t)=0.9\cos(28.984t)+0.3*(\cos(30t)+\cos(28.984t))=$$

$$0.9\cos(28.984t)+0.6*\cos(0.5*(30t+28.984t))\cos(0.5(30t-28.984t))=$$

$$0.9\cos(28.984t)+0.6*\cos(29.492t)\cos(0.508t).$$

Both fast terms have very similar frequencies; The second one is modulated by the 0.508degrees/hr frequency \rightarrow period=360degree/(0.508degrees/hr)=708.7hr=29.5days. . Because the modulating wave multiply a higher frequency wave (that of 12hr) the magnitude of the modulation is what is important. The magnitude has twice the frequency of the modulating wave and thus the 14.76period observed. This is the period of the spring/neap cycle of the tides!

- d) (Extra credit, 5pts): What is the name of this phenomenon (the wave beating of lunar and solar tides) when applied to ocean tides?

Neap and Spring tides.

Reference:

Applet on seich: <http://www.coastal.udel.edu/faculty/rad/seiche.html>

Applet on beating: <http://www.surendranath.org/Applets/Waves/Beats/BeatsApplet.html>

Tides: <http://en.wikipedia.org/wiki/Tide>

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