SMS-303: Integrative marine sciences III. Concepts in ocean physics.

Lecture 1: Waves

What is a wave:

A wave is a periodic disturbance in space and/or time. By periodic we mean that its manifestation is seen several times (in time and/or space). Waves transfer energy (information) through a medium without displacing much the medium itself. A wave packet is a finite disturbance in space and is encountered in many applications (earthquake, tsunamis, tidal bores etc').

There are many examples of waves in fluids; From gravity waves (where the restoring force is gravity), to capillary waves (where the restoring force is surface tension), to sound (pressure waves) and light (electromagnetic waves), just to name a few. Tsunami, tides, beach surf waves and ripples are all gravity waves.

Surface waves occur at the boundaries between two fluids (in our case, water and air).

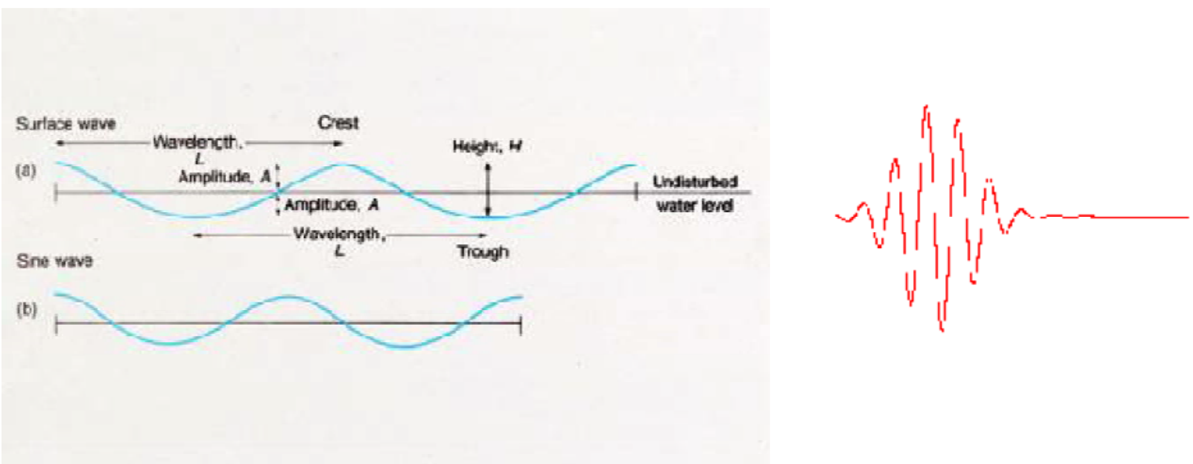

Figure 1: Schematics and nomenclature associated with surface waves (left) and an example of a wave packet (right).

The wavelength(λ) is defined as the distance between two troughs (or crests, Fig. 1). The wave's amplitude ($\mathcal{A}$) is half the wave height (H), the vertical distance between crest and trough. The period of the wave (T) is measured at a given location and is the time between two consecutives crests (troughs). The frequency (f=1/T) of a wave is the number of waves passing at a given point per second.

To first order, surface waves do not, in average, move water. Rather, it is the wave form that is propagating (thus energy is propagating). Each point on the surface traces a circle (Fig. 2).

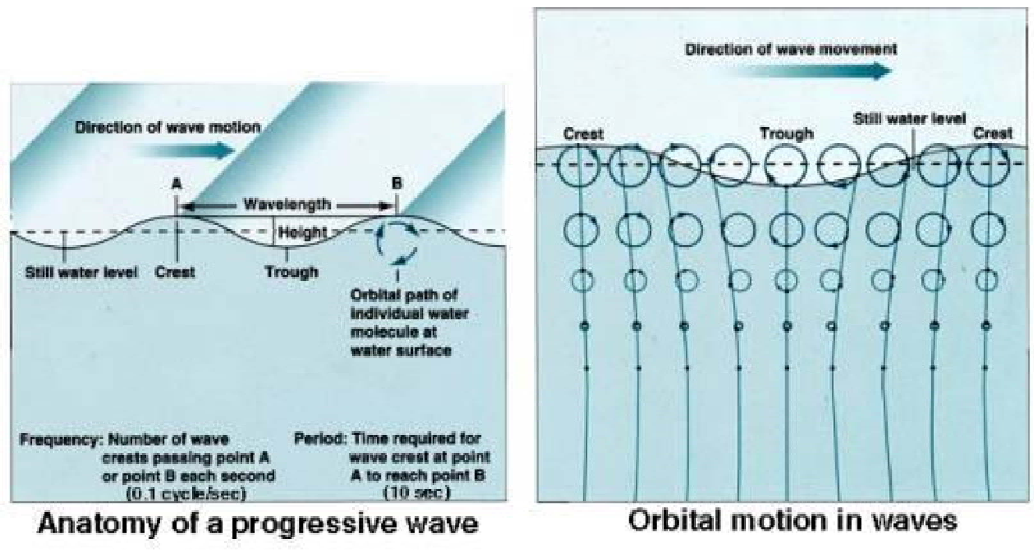

Figure 2: right panel: wave structure, left panel: trajectories of particles at different depth and different positions relative to a wave. (Figure 9.2-9.3 in Garrison, 2001)

The phase speed of waves is the speed by which a trough (crest) propagates ($c=\lambda/T$). The speed of a group of waves is called the group-speed (c_g) of the wave and is the speed by which energy (or information) propagates. The two are not necessarily the same and can even propagate in different direction (as is the case for internal wave in continuously stratified fluids) . For example, in a boat's wake one can observe how crests rise from the rear end of the edge formed behind the boat and disappear in its front edge, moving faster (in fact twice as fast) as the propagation of the wedge itself (the 'group').

The position of the surface of a wave traveling in the x direction is approximated as a sinusoidal: $\eta=A\sin\{2\pi(x/\lambda-t/T)$. The expression in bracket is the phase of the wave and goes from 0 to 2π from crest to crest. Is is a mathematical fact that any surface can be decomposed uniquely into a sum of sines and cosines of different wavelengths (or frequencies, this is a result of the work of Fourier).

The period of a wave may look shorter or longer to us depending on whether the wave is riding on top of a mean current. This phenomena is known as the Doppler shift. If the current propagates in the same direction as the wave, the period an observer standing at point will measure will be shorter (more wave will come by us per unit time) while when they are in opposite directions the period will be longer. Moving with the current, the wave will have the same period as in the absence of current.

Deep and shallow water waves:

Surface gravity waves are divided into two groups depending on whether they do or do not feel the presence of the bottom (Fig. 3).

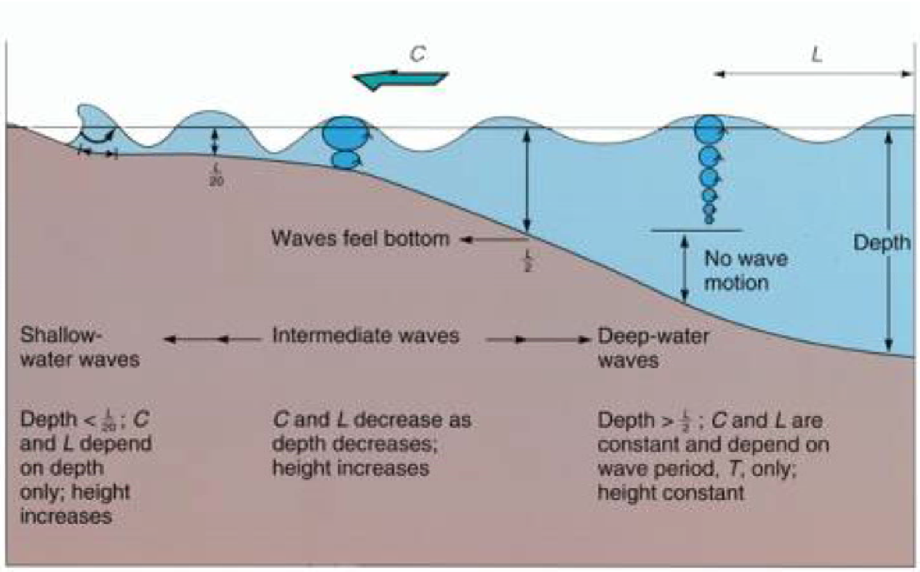

Figure 3: Changes in a wave as the depth shallows. Note change in particle trajectories, wavelength and steepness (H/λ). (Figure 9.11 in Duxbury, Duxbury and Sverdrup, 2000)

For gravity waves it can be shown that the phase speed is:

$c=\lambda/T=[g\lambda/2\pi \cdot \tanh(2\pi h/\lambda)]^{1/2}$ where h is the depth of the fluid.

When $\lambda<2h$ (deep water waves), $\tanh(2\pi h/\lambda)\sim 1$, and $c=[g\lambda/2\pi]^{1/2}$

When $\lambda>20h$ (shallow water waves), $\tanh(2\pi h/\lambda)\sim 2\pi h/\lambda$, and $c=[gh]^{1/2}$

While deep water waves of different length travel at different speeds (the long ones faster than the short ones), all shallow water waves travel at the same speed.

Group speeds (the speed of energy propagation) are equal to phase speed for shallow water waves and are half the phase speed of deep water waves.

The amplitude of surface gravity waves decays exponentially with depth (z), ($\exp(-kz)$) with a decay constant being the wavenumber (k), $k=2\pi/\lambda$.

Tsunamis are waves generated by seismic activity (earthquakes). A tsunami's has a typical wavelength $\lambda\sim$200km. Thus, even in deep ocean (h~5km) they behave like shallow water waves. The amplitude of Tsunamis in the middle of the ocean has O(1m). Their speed is similar to sound speed in air, ~300m/s.

Generation of Surface waves:

The primary generation process of surface waves is the wind (Figure 1.3 below). Differential stresses and pressure along the interface pushes the waves and steepens them.

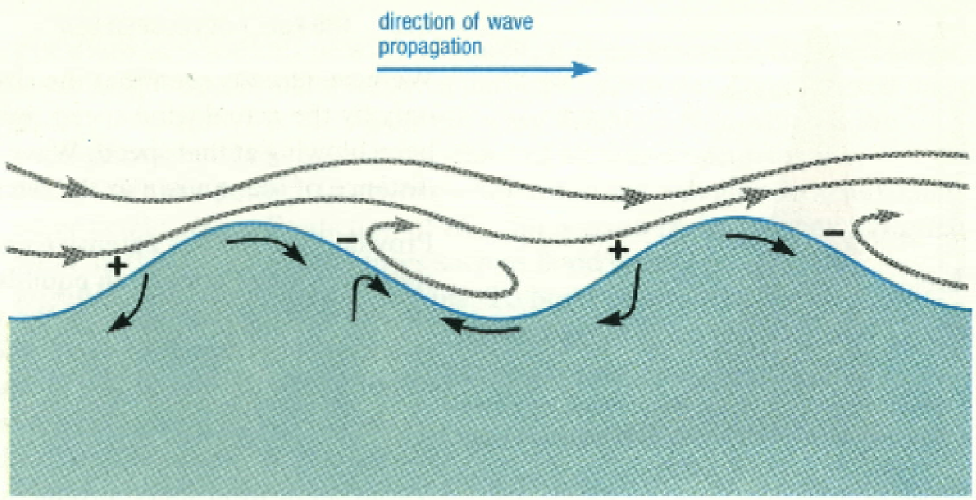

Figure 1.3 Jeffreys' 'sheltering' model of wave generation. Curved lines indicate air flow; short, straight arrows show water movement, which will be explained more fully in Section 1.2.1. The rear face of the wave against which the wind blows experiences a higher pressure than the front face, which is sheltered from the force of the wind. Air eddies are formed in front of each wave, leading to differences in air pressure. The excesses and deficiencies of pressure are shown by plus and minus signs respectively. The pressure difference pushes the wave along.

(Figure 1.3 from Waves, Tides and Shallow-water processes, The Open University)

Another source of surface waves are earthquake around the world. The devastating wave they produce is called a Tsunami (see above).

The surface of the ocean is made of a superposition (namely the sum) of many waves. Waves travel in different directions and their amplitude add up (Fig. 4)

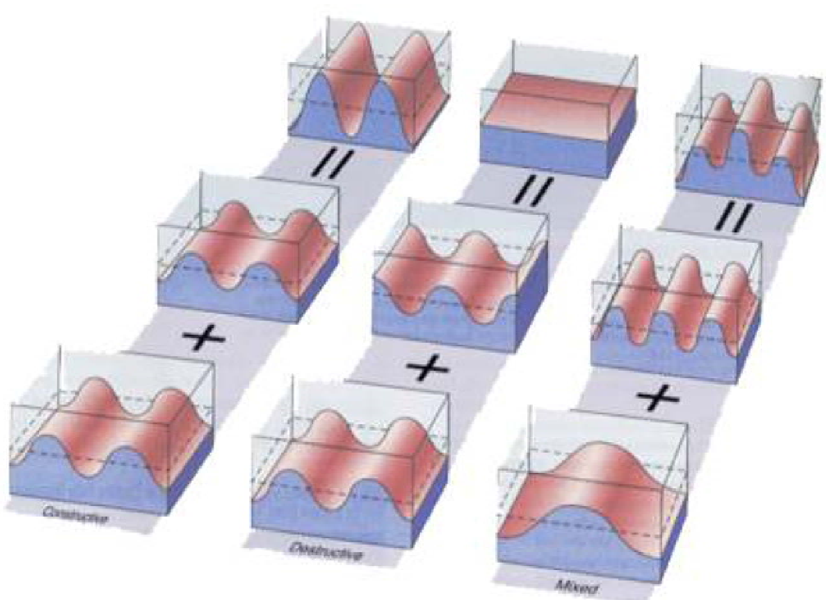

Figure 4. Superposition of waves (Figure 9-9 in Thurman, 1997).

Most of the energy in waves is due to wind waves with periods from 1 to 10 seconds (Fig. 5). Waves have both kinetic and potential energy. The total energy per unit area of a wave is given by:

$E=(\rho g H^2)/8.$

Where ρ is density, g-gravitational acceleration, and H, the wave's height.

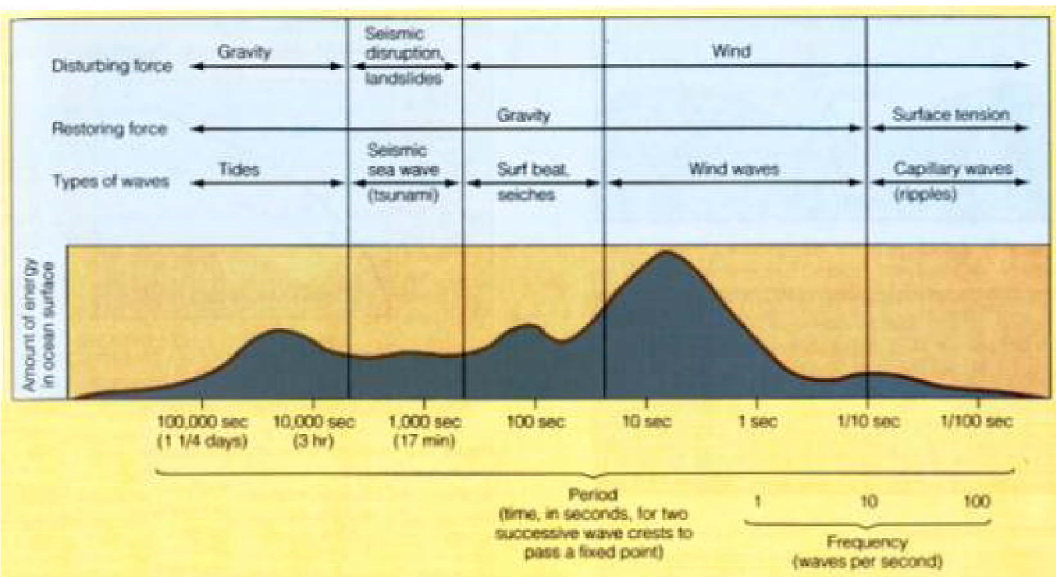

Figure 5. Distribution of energy as function of wave period and type. (Figure 9.4 in Garrison, 2001)

Breaking waves:

As deep water waves get to shallow areas their speed decrease and their amplitudes increase accordingly (to conserve mass). Once the wave achieve critical steepness ($H/\lambda \sim 1/7$) they break (Fig. 6). The energy the wave had is mostly dissipated into heat. Some of the energy may be channeled into creating strong 'rip' currents.

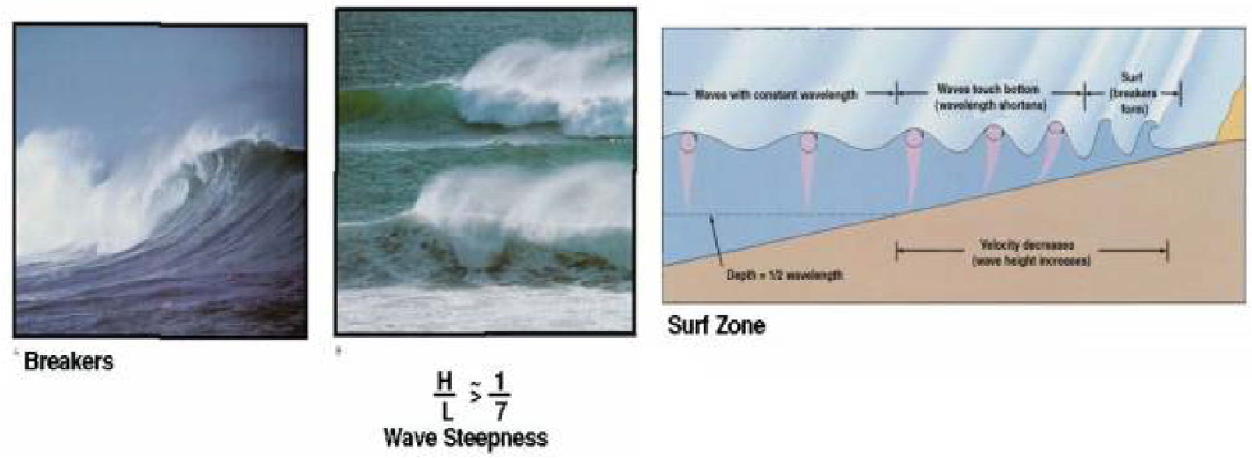

Figure 6: Waves approaching the shallow areas steepen and than break.

Seich:

Just as with musical instruments such as drums, lakes and embayment have their own natural frequency for surface gravity waves that depend on their depth .and their horizontal dimensions. When excited at the appropriate frequency large oscillations are produced having that frequency.

Hull speed:

An object moving along on the fluid surface (boat, duck), generates a surface wave with two crests at both ends of the object and the trough in between. The length is approximately equal to the water-line length of the hull (L). The natural phase speed of this wave is $c=(gL/(2\pi))^{1/2}$, and is called the hull speed. If the objects tries to swim faster than its hull speed it will need to move uphill over the wave it is forming, creating extra resistance in the process. The energetic cost of moving faster than the hull speed is such that ducks do not attempt it and so do not boats. Thus the length of a boat (swimmer) is very important in determining the maximal speed that can be sustained for a while.

Internal waves:

In oceans and lakes not only surface waves exist but possibly also internal waves. The simplest to understand are those that travel along the interface between dense deep fluid and the overlaying light fluid as this is similar to waves on the surface of the water (but below the light atmosphere). In continuously stratified fluid internal wave can propagate vertically. What is counter intuitive is that their phase and group velocities are perpedicular. These wave transmit energy from surface to depth and back and are beleived to play an important role in ocean mixing as they break over steep topography. Beautiful movies of such waves can be found at:

http://www.gfd-dennou.org/library/gfd_exp/exp_e/index.htm.

References and additional reading:

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