

# SMS 303: Integrative Marine Sciences III

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- 4 weeks & topics: ~~waves~~, ~~tides~~, ~~mixing~~ and Coriolis.

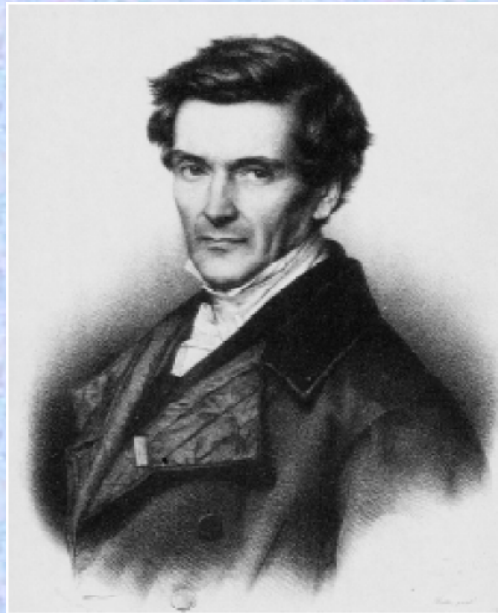


FIG. 5. Gaspard Gustave Coriolis (1792–1843).

From: Persson, 1998, BAMS

# Real and fictitious forces on a rotating planet

Newton's 1<sup>st</sup> law of motion:

Every object in a state of uniform motion tends to remain in that state of motion unless an external force is applied to it.

Corollary: if you want to change an object's trajectory you need to apply force.

Q: how can one throw a curved ball?

This law is for motion with respect to a non-moving reference frame - called *inertial* frame of reference.

When we measure motion on the Earth we do it relative to a *rotating* reference frame. If we are to predict where a ball will go we need to take this rotation into account.

## Real and fictitious forces on a rotating planet

What happens when you sit on a merry-go-round that begins to spin?

Assume you are held in place to the merry-go-round. What happens if you throw a ball to somebody?



[http://ww2010.atmos.uiuc.edu/\(Gh\)/guides/mtr/fw/crls.rxml](http://ww2010.atmos.uiuc.edu/(Gh)/guides/mtr/fw/crls.rxml)

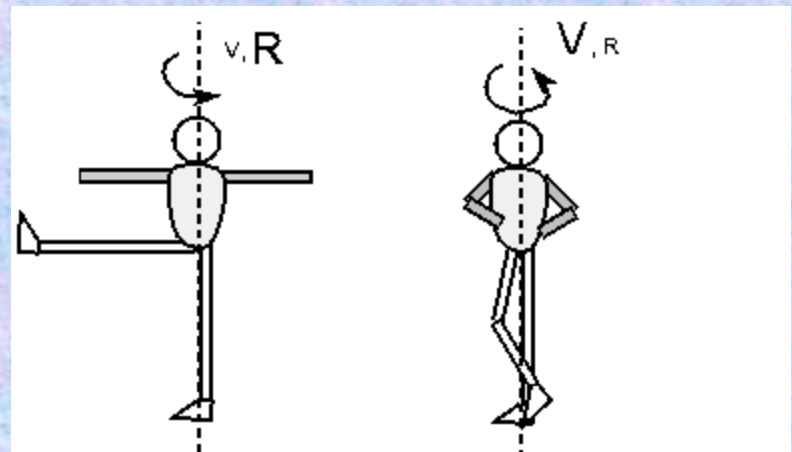
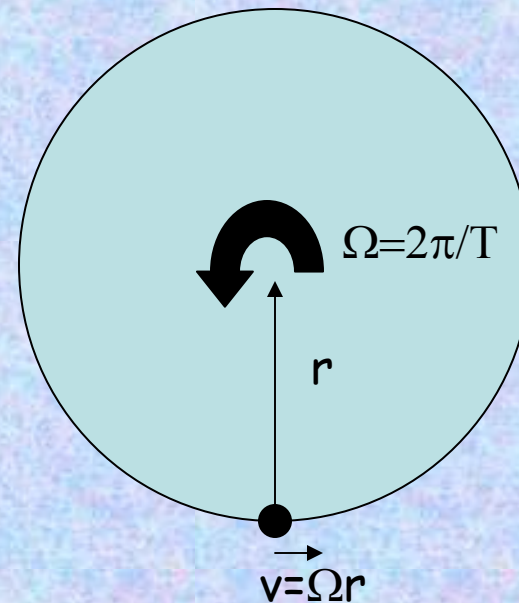
How different does it seem for an observer that is not rotating?

## Analysis of the merry-go-round:

Angular momentum:  $mv \times r = m\omega r^2$

When the ball is pushed in  $r$  changes.

If  $r$  is shortened  $v$  needs increase (to conserve angular momentum)  $\rightarrow$   
 $\rightarrow$  added rotation relative to the merry-go-round (think about an ice skater).



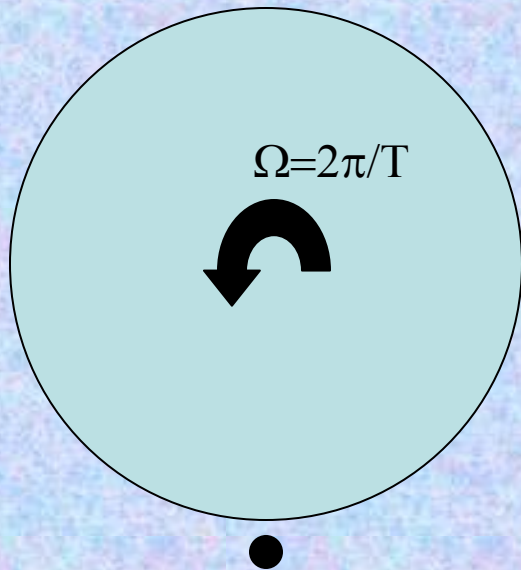
In addition the position of the observer is changed:

[www.astronomynotes.com/evolutn/s12.htm](http://www.astronomynotes.com/evolutn/s12.htm)

**Turn table** Simulation

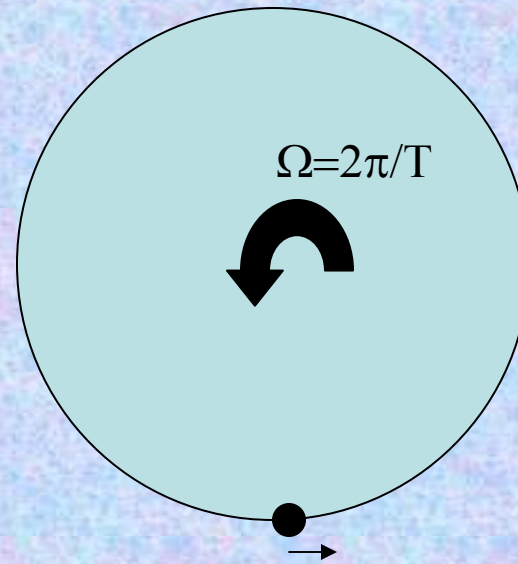
([http://des.memphis.edu/lurbano/vpython/coriolis/Coriolis\\_model.html](http://des.memphis.edu/lurbano/vpython/coriolis/Coriolis_model.html))

Analysis of turn table simulation:



The ball has no angular momentum at the start

Analysis of merry-go-round:

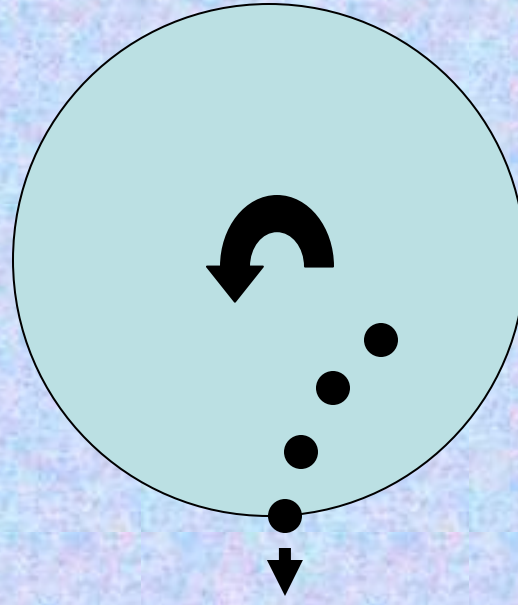
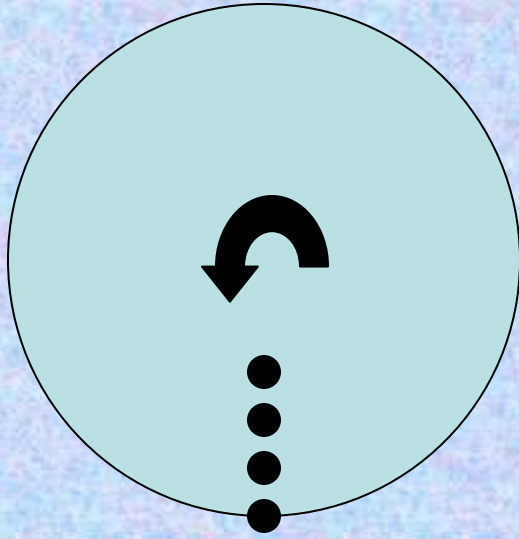


At the rim the angular velocity is that of the merry-go-round is  $\Omega R$  in the direction of the tangent.

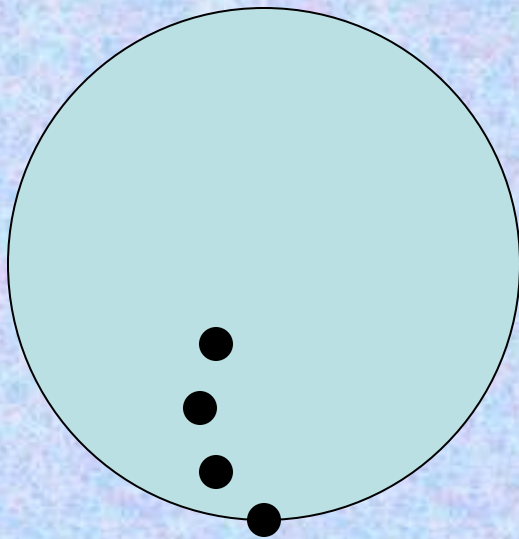
What will happen if you push the ball towards the center?

Analysis of turn table simulation:

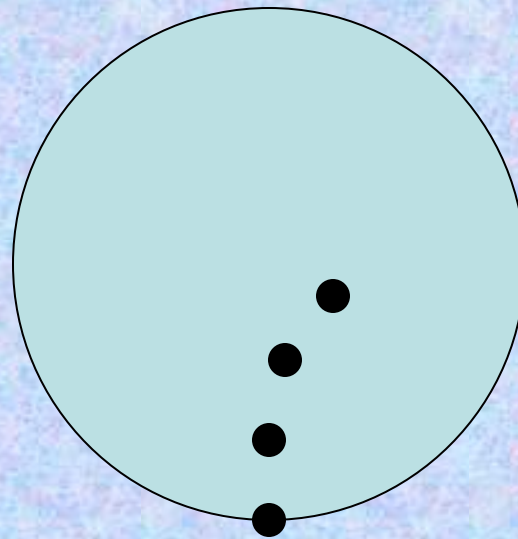
Analysis of merry-go-round:



Inertial frame

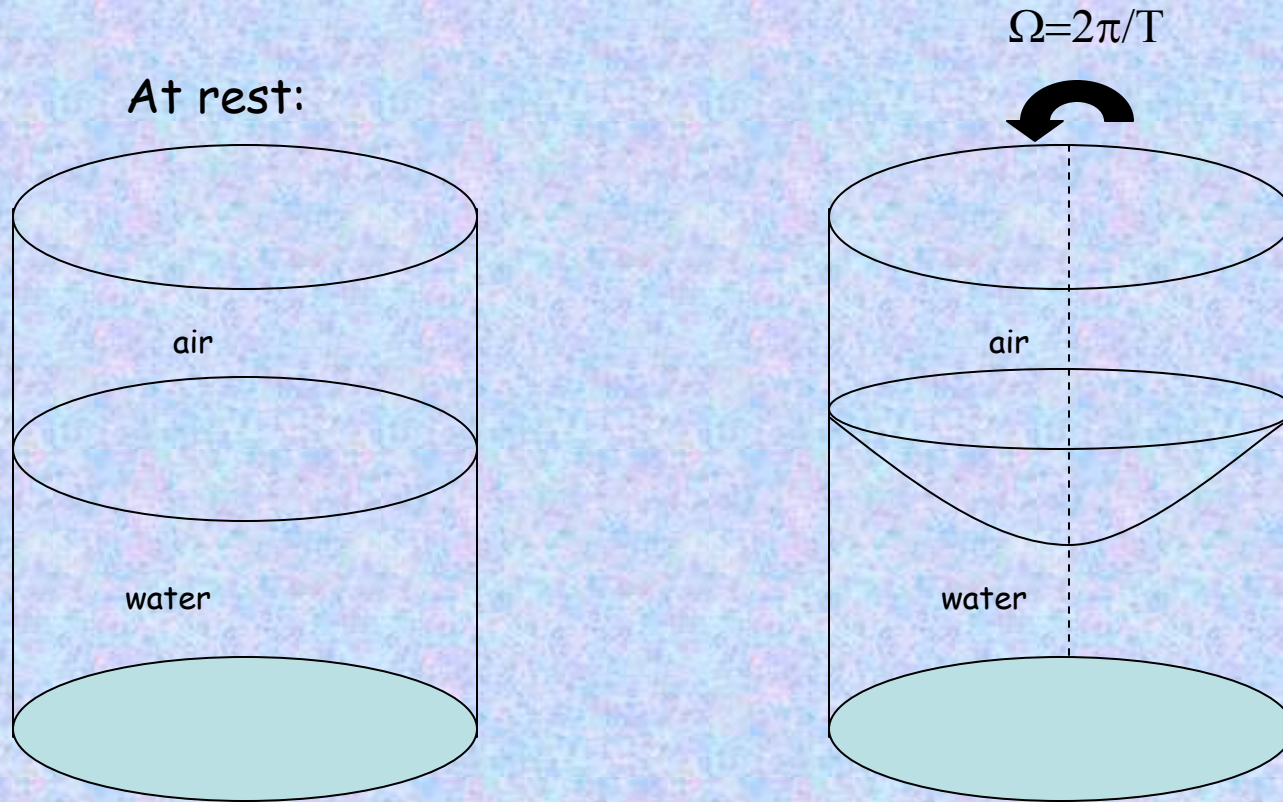


Rotating frame  
→



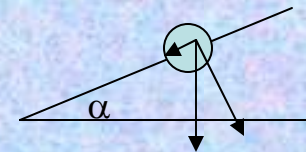
An apparent force (apparent to a rotating observer) is present (Coriolis).

# The *geopotential* surface: fluid at rest in a rotating world.



gravity balances the centrifugal force for fluid in solid-body rotation:

$$g \sin \alpha = \Omega^2 r \cos \alpha \rightarrow \tan \alpha = dh/dr = \Omega^2 r / g \rightarrow h(r) = h_0 + \Omega^2 r^2 / g$$



What happens if we push a parcel of water towards the center?

# Real and fictitious forces on a rotating planet

Forces affecting motion of fluids as viewed from an inertial frame:

1. Gravity (directed towards the center of gravity).
2. Centripetal force (directed towards to center of curvature).
3. Pressure gradient (directed from high to low pressure).
4. Friction (slows velocity without affecting direction).

Forces affecting motion of fluids as viewed from a rotating frame:

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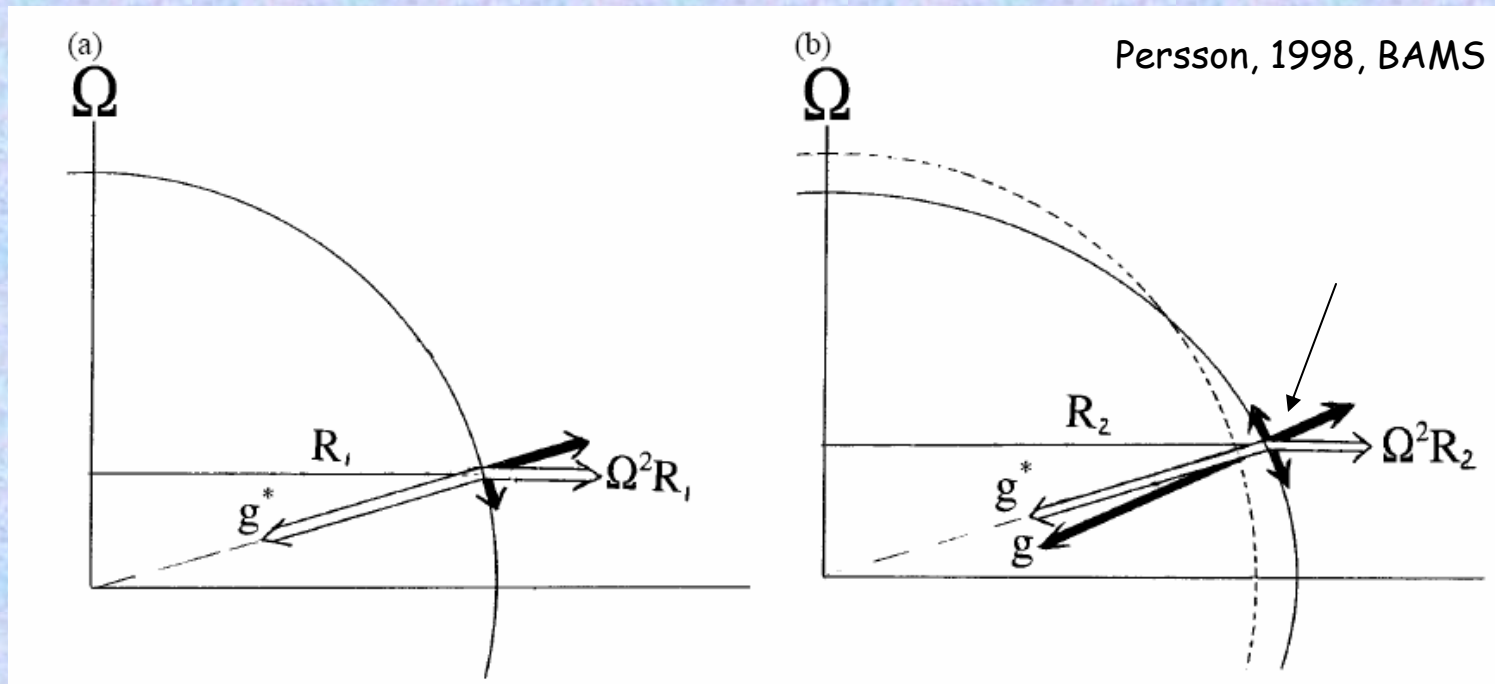
and:

5. Coriolis force (directed opposite the direction of rotation).



## Moving to a 3-D planet:

The balance of forces for a particle at rest on the Earth (e.g. rotating thousands of miles per day):



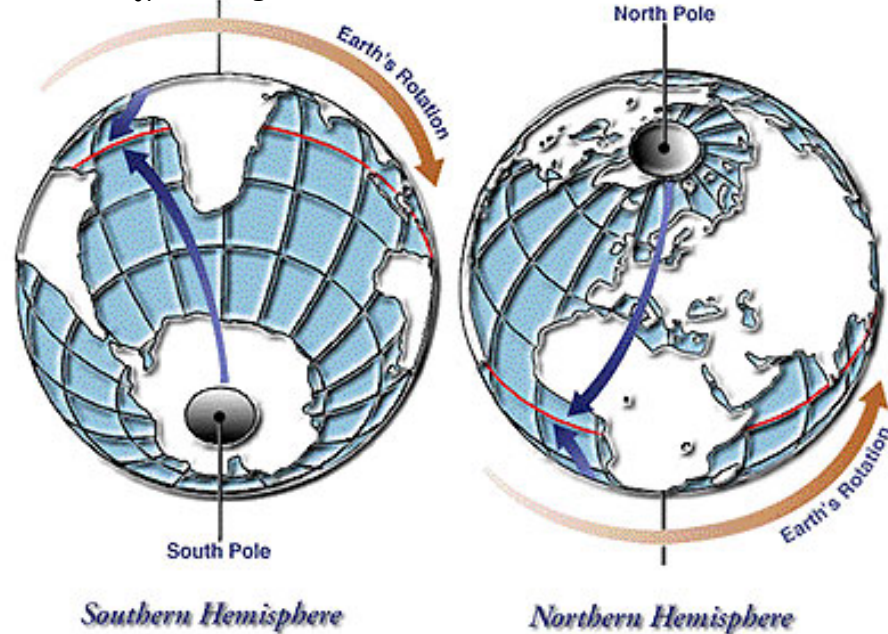
Spherical Earth: tangent components are not balanced.

Ellipsoidal Earth: tangent components are balanced for mass at rest relative to the Earth.

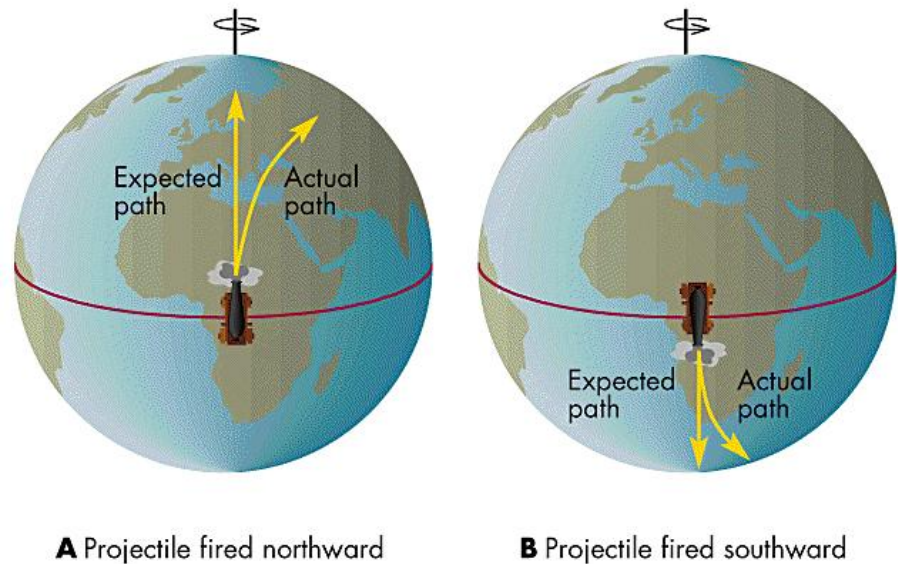
Geopotential: a surface on which particles are at rest.

It seems easy to explain the deflection of the trajectory in the N-S direction:

Sealevel.jpl.nasa.gov



www.nhhe.com

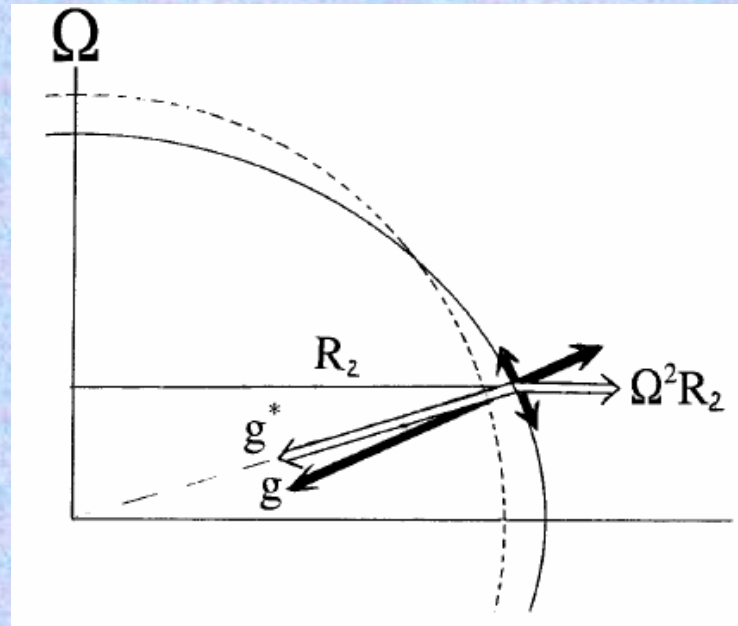


The velocity in the E-W direction of the particle is that of its initial latitude. That velocity is different from that of the new latitude where it is found causing the observed E-W motion in the new latitude.

In reality, this can only add up to half the deflection. It cannot explain the E-W case.

Because  $r$ , the distance to the rotation axis, is changing (one gets closer as one goes to the poles) so will the radial (E-W) velocity to conserve angular momentum (Similar to a skater with arms stretched out or in).

How can we explain the deflection of the trajectory in the E-W direction:



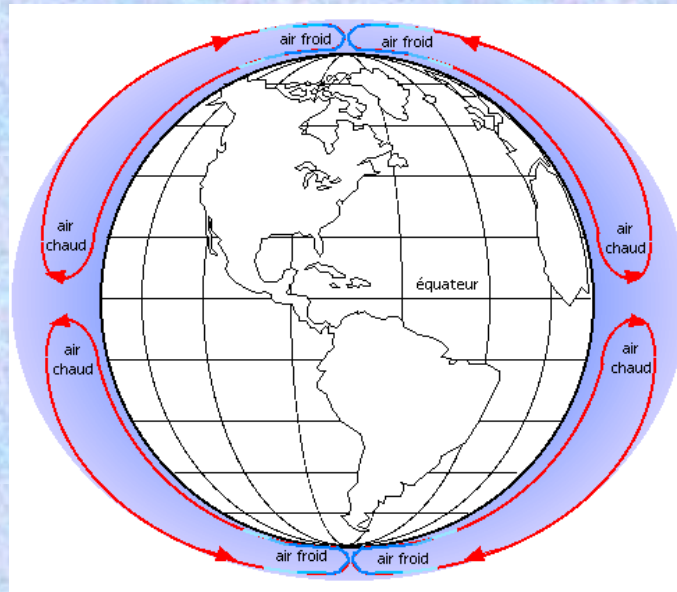
The balance between gravity and the centrifugal force holds for a particle at rest.

If the particle has velocity at the same direction as the Earth rotation (e.g. increasing the centrifugal component) a net equatorial force will act on it.

If the particle has velocity opposite the direction of the Earth rotation (e.g. decreasing the centrifugal component) a net poleward force will act on it.

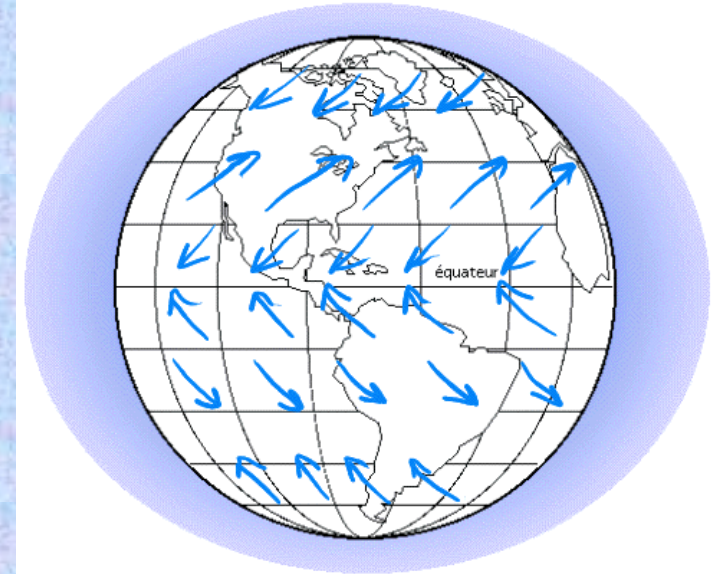
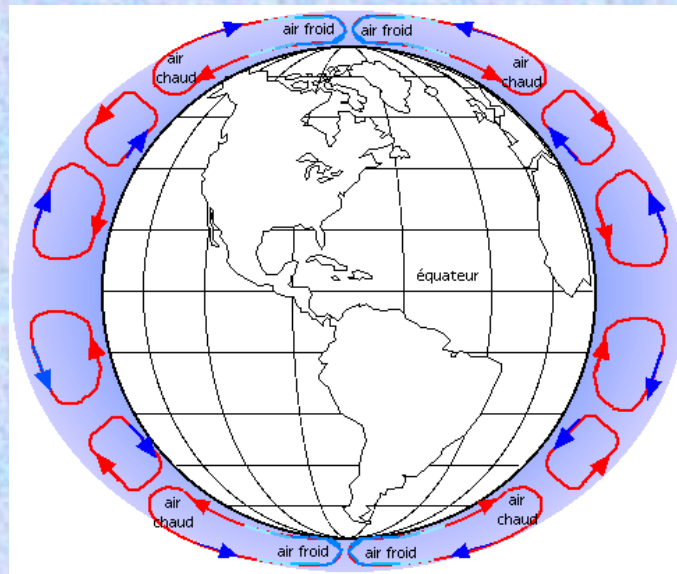
# The Earth rotation has profound effects on currents and wind:

No rotation:



Near surface winds:

With rotation:



A common misconception wrt Coriolis:

It controls the direction water drains in the bath or toilet.

When is Coriolis important?

I. Small spatial scales: motions with time scales on the order of the Earth's rotation period.

II. Small spatial scales: motions with large speeds (e.g. fast velocity)- e.g. a demonstrable effect ( $O(1\text{cm})$ ) can be shown for a baseball.

III. Large spatial scales + short time scales - e.g. missiles.