SMS 303: Homework I. Coriolis, Inertial Oscillation and Baseball

From Newton's 2^{nd} law we know that:

Mass*acceleration=Force.

It is often convenient to divide both sides by mass:

Acceleration=Force/Mass.

Now, consider an object on a rotating platform that rotates at angular velocity Ω (= 2 π radians / period, where the period is the time it takes for one full rotation). In order to account for the rotation of the platform an apparent force known as the Coriolis force is added to the equations. In two dimensions those are:

$du/dt=F_x/Mass+2\,\Omega v$

$dv/dt=F_y\,/Mass-2\,\Omega u$

where (u,v)=(dx/dt, dy/dt), are the velocities in the x and y directions respectively. Here we assume x is eastward and y is northward.

Let's assume that we give a kick to the object in the direction y at time zero and observe how it moves without applying any extra force. Initial condition $v(t=0)=V_0$. Let us also denote $f=2\Omega$ (On the Earth and latitude ϕ, f is the Coriolis parameter and $f=4\pi\sin\phi / 24\ hr^{-1}$).

$du/dt=fv$

$dv/dt=-fu$

Solution (please check):

$u=-V_0\cos(ft)$, $v=V_0\sin(ft)$

This solution was actually wrong (they don't satisfy $v(t=0)=V_0$). It should have read:

$u=V_0\sin(ft)$, $v=V_0\cos(ft)$

Subsequently I will solve both cases

Homework (be careful regarding units).

1. Solve for the position (x, y) as function of time, assuming x(t=0)=y(t=0)=0 (remember, u=dx/dt, v=dy/dt).

Integrating the original equations we find:

$x = -V_0\sin(ft)/f + constant\_1$, $y = -V_0\cos(ft)/f + constant\_2$

substitute, $x(t=0)=0 \rightarrow constant\_1=0$

substitute, $y(t=0)=0 \rightarrow constant\_2 = V_0/f$.

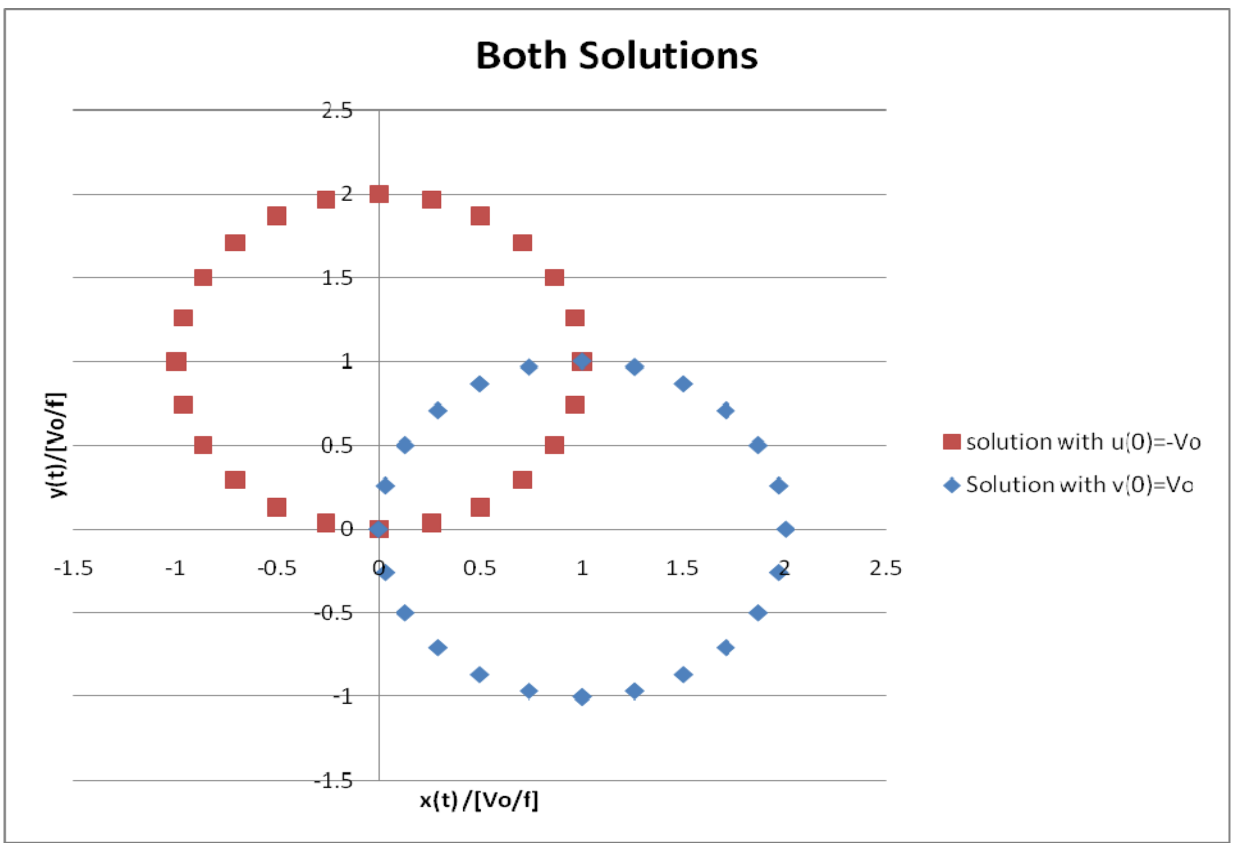
$\rightarrow (x(t),y(t)) = V_0/f(-\sin(ft), 1-\cos(ft))$.

If I similarly integrate the solution that satisfies ($v(t=0)=V_0$) I get:

$(x(t),y(t)) = V_0/f(1-\cos(ft), \sin(ft))$.

2. Plot the position of the object as function of time for 24 hours (every 1hr) assuming a latitude of 30°N. What is the shape of the trajectory? How long does it take for the object to come back to its initial position?

At 30°N $\sin(30°)=0.5$, $\rightarrow f = 2\pi / 24\ hr^{-1}$. It follows that every 24hrs, the position of the particle (x, y) will be back to the same position (when $ft=2\pi$).



Both solutions rotate clockwise.

3. Now, assume that we are dealing with baseball and Fenway park (~42°N). The speed of the ball leaving the bat is 40m/s. Neglecting friction, what would be the Coriolis deflection of the ball be after 2 seconds?

The deflection is the deviation of the ball from the direction we throw it to. We could use either solution and get the same deviation. Lets use the original one noting that the ball is thrown to the East (u(t=0)=-Vo) and see how much it will be deflected to the north due to coriolis within two seconds.

At 42°N sin(42°)~0.67, → $f = 4\pi * 0.67 / (24x3600) = 9.73x10^{-5}$.

Substituting to y(t)=(1-cos(ft))Vo/f with t=2sec and Vo=40m/s, *we get*

y(t)=0.0078 m, or 8mm, a very small deviation.

A simpler and approximate way to solve it is using a Taylor series. For small ft,

$Cos(ft)=1-(ft)^2/2$ → $y(t)=ft^2Vo/2$ =0.0078 m.

This page was last edited on 10/23/2007