SMS-303: Integrative marine sciences III.

Answers for assignment #2 (answers in bold italics)

1.(Total points 36, 12pts for each subsection)
In the small tanks you sloshed water with water depths of 1.5cm and 6cm back and forth and found the number of sloshes as function of time.

For many groups: 6 sloshes/5sec in the 1.5cm deep tank and 12sloshes/5sec in the 6cm deep tank.

a. Translate your results into the wave speeds (how fast did crest move).

Tank length is 30.5cm. each slosh represent a wave propagating the whole length of the tank. Therfore: speed=length * (# of sloshes/time) = 36.6cm/s (1.5cm deep) and 73.2cm/s (6cm deep).

How did they compare with theory?

In all cases we are dealing with shallow water waves the wavelength (twice the tank length=61cm) divided by depth is >>1. In the 6cm case it is not strictly shallow (not bigger than 20*depth) but not too far from it. Thus we expect the formula c=sqrt(gH) to work. sqrt(g*0.015)=0.38m/s=38cm/s compares well with 36.6cm/s from above and sqrt(g*0.015)=0.77m/s=77cm/s compares well with 73.2cm/s above.

b. Was the ratio of the waves speed with 6cm depth and 1.5cm depth consistent with theory?

The ratio of the velocities measured: 73.2/36.6=2
Expected from theory: sqrt(gH_1)/sqrt(gH_2)=sqrt(H_1/H_2)=sqrt(6/1.5)=2
Very good agreement (In particular given the crude way we did it in the lab).

c. What is the natural (or intrinsic) period of the tank (1^{st} mode) at each depth? (hint: you should be able to calculate it using only the length of the tank (30.5cm) and the wave speeds computed above). This is analogous to a Seich.

The natural period of the tank for the 1st mode is the time it takes the wave to propagate across the tank twice (so it gets back to the same position):
For the H=1.5cm case: 2*30.5cm/36.6cm/s=1.7sec
For the H=6cm case: 2*30.5cm/77cm/s=0.8sec
I did not take points off for those who calculated only half the period.

d. (extra credit, 5pts) What is the intrinsic period of the North-South oriented Seich in Pushaw lake given an average depth of 20feet and a length of 4 miles? Would the rotation of the Earth be an important factor for this seich (how does the period of this seich compare with 1/f, where f is the Coriolis frequency from last week)?

The natural period of the tank for the 1st mode is the time it takes the wave to propagate across the lake twice (so it gets back to the same position).
The wave speed is sqrt(gH)=sqrt(9.81m/s^2 x 20 feet x 0.3048m/feet)=7.7m/s
The length of lake is: 4miles=6437.376m
Seich period = (twice the length of the lake)/speed of seich = 1672sec ~ 28min.

$f=4\pi\sin(45°)/24hr=0.37$ rad/hr → $1/f=2.7$hrs/rad >> 28min (one should compare with $T=2\pi/f=16.97$hr which is the inertial period) → rotation is not likely to be a factor.

2. (Total points 64. a. 14pts. b. 30pts and c. 20pts)
The effect of the sun on the tides in the Gulf of Maine (GOM).

The sun and the moon both contribute to the tides in the GOM, with the moon being the major contributor. For simplicity we will consider only two tidal components (M2-lunar and S2-solar) which often have the largest amplitude of all tidal components and which will be the major contributor were the moon, sun and earth all on the same plane perpendicular to the axis of rotation of the earth.

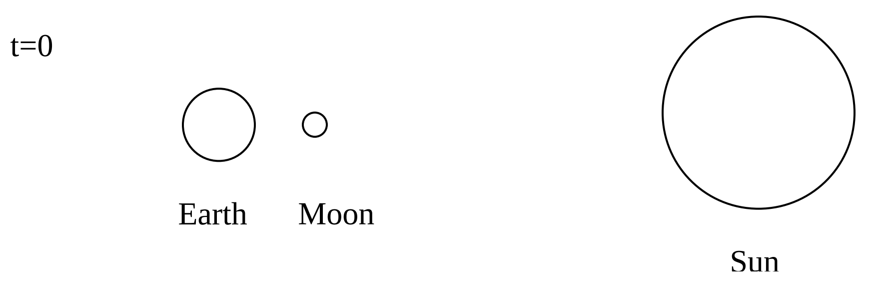
The elevation (in m) near Rockport, ME for the two tidal wave components is given by:
Lunar (M2): 1.5cos(28.984t)
Solar (S2): 0.23cos(30t)

Where t is in hours and the frequency is given in degrees per hour. At time t=0 the sun and the moon are at the same side relative to the earth (new moon).

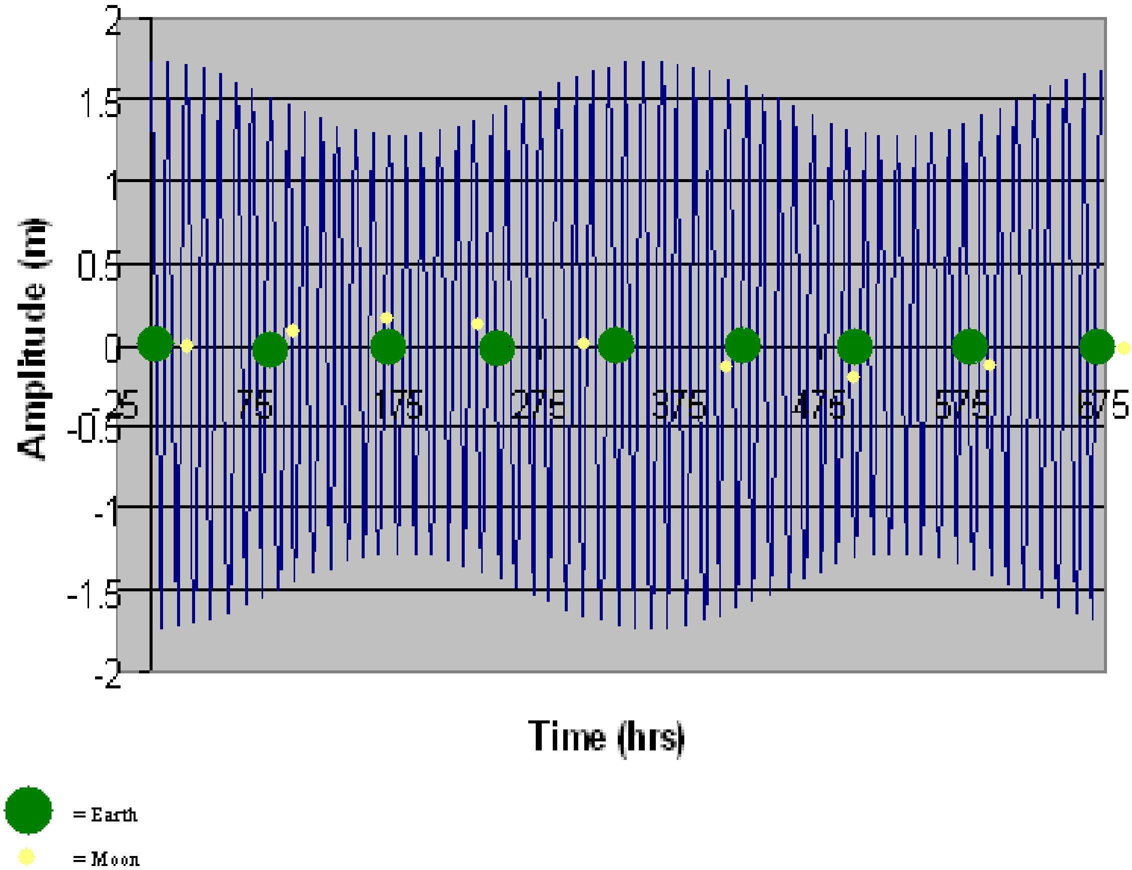
a) Which component dominates the tide?

Obviously the lunar component since 1.4>>0.28.

b) Add the two components together on a spreadsheet and plot the resulting amplitude as function of time for a 28day lunar month (remember, t has to be in hours, 1 day = 24hours). Annotate on your plot the different arrangement of the planets every 3.5 days. For example:



Below is the solution based on a student from a previous year. One detail that is not taken into account in her graph is the fact that the Earth-Sun angles varies within a month by a few degrees. The students who took it into account got extra credit!



The sun is located to the right of the chart.

c) Add the two tidal components using a trigonometric relationship for the addition of two sines (sin(A)+Sin(B)), by first breaking the lunar component to two parts one of which as the same amplitude as the solar component. The phenomena you are describing is called beating, the interaction of two waves with slightly different frequencies. Explain your results in section b in light of what you have found.

$$1.4\cos(28.984t)+0.28\cos(30t)=1.12\cos(28.984t)+0.28*(\cos(30t)+\cos(28.984t))=$$
$$1.12\cos(28.984t)+0.56*\cos(0.5*(30t+28.984t))\cos(0.5(30t-28.984t))=$$
$$1.12\cos(28.984t)+0.56*\cos(29.492t)\cos(0.508t).$$

Both fast terms have very similar frequencies; The second one is modulated by the 0.508degrees/hr frequency $\rightarrow$ period=360degree/(0.508degrees/hr)=708.7hr=29.5days. . Because the modulating wave multiply a higher frequency wave (that of 12hr) the magnitude of the modulation is what is important. The magnitude has twice the frequency of the modulating wave and thus the 14.76period observed. This is the period of the spring/neap cycle of the tides!

d) (Extra credit, 10pts): How is this phenomena resulting from the beating of the solar and lunar tides called when applied to the ocean tides?

Neap and Spring tides.
