SMS 303: Integrative Marine Sciences III

- Instructor: E. Boss, TA: C. Proctor

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- 5 weeks & topics: ~~Coriollis~~, ~~waves~~, ~~tides~~, diffusion and mixing.

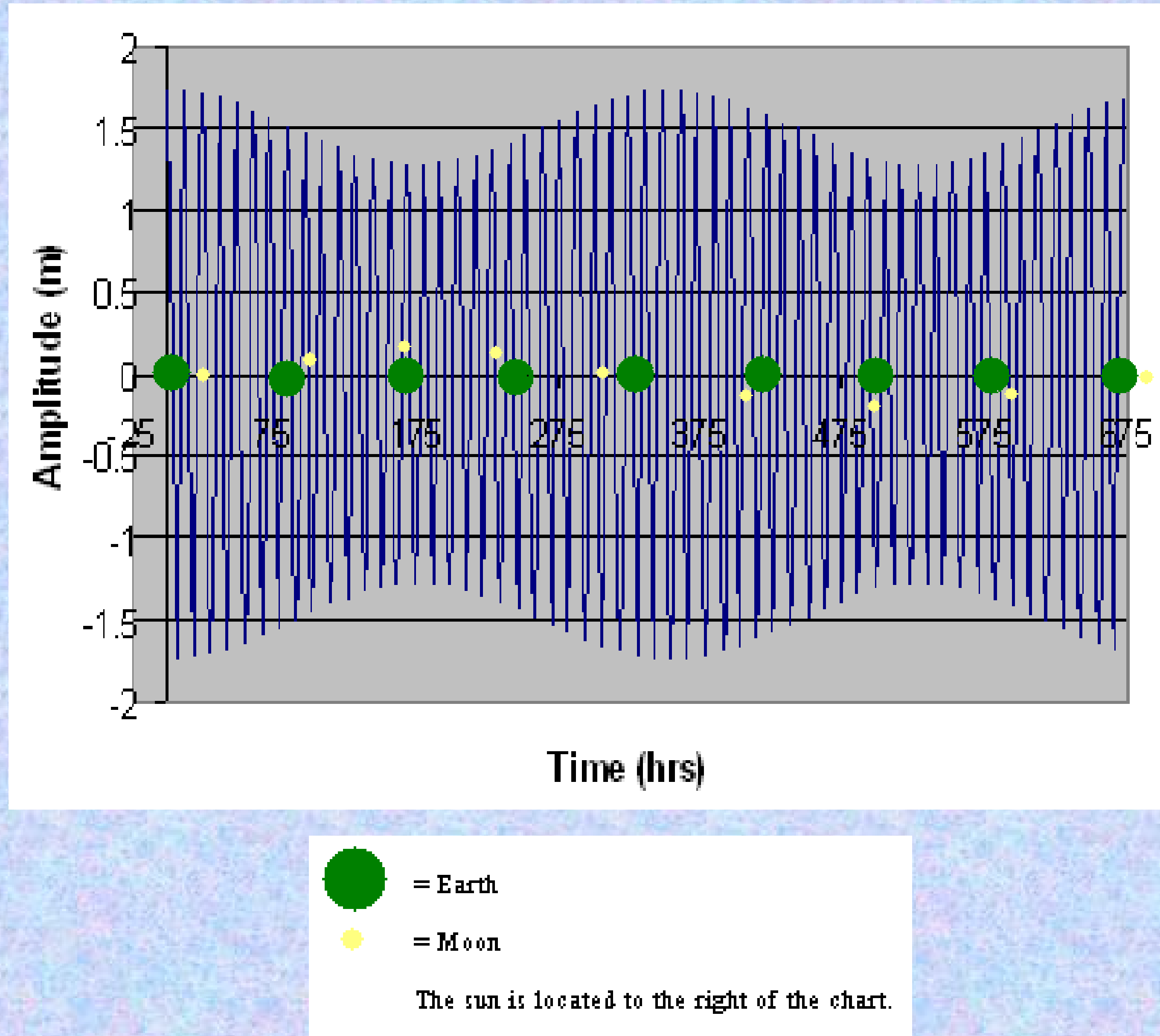
- Pre-class quiz:

  Moon and sun alignment in spring and neap tides?

  Most common method to predict tides?

- Homework: how many spring-neap cycles should we expect in a 28day period?

Spring-neap cycle:



- Diffusion
  - A collaborative exercise in class and corridors.

Short introduction:

•Diffusion in marine sciences.

•Fick/Fourier's laws.

•Differential equations are the needed math (what are these?).

•Macroscopic and microscopic descriptions.

•Diffusion and order are linked (Entropy).

•Irreversible without investment of energy.

- Diffusion in marine sciences

Examples of processes diffusion is important for:

•Nutrient uptake by phytoplankton and bacteria.

•Dissolution of particles.

•Exchange of solutes in sediment pore waters.

•Exchange of solutes within multi-cellular organisms.

•Gas diffusion is important for diving physiology.

•Release of waste products.

•Double diffusion.

Diffusion in a homogeneous fluid:

Fick's and Fourier's laws – down gradient flux of material and heat.

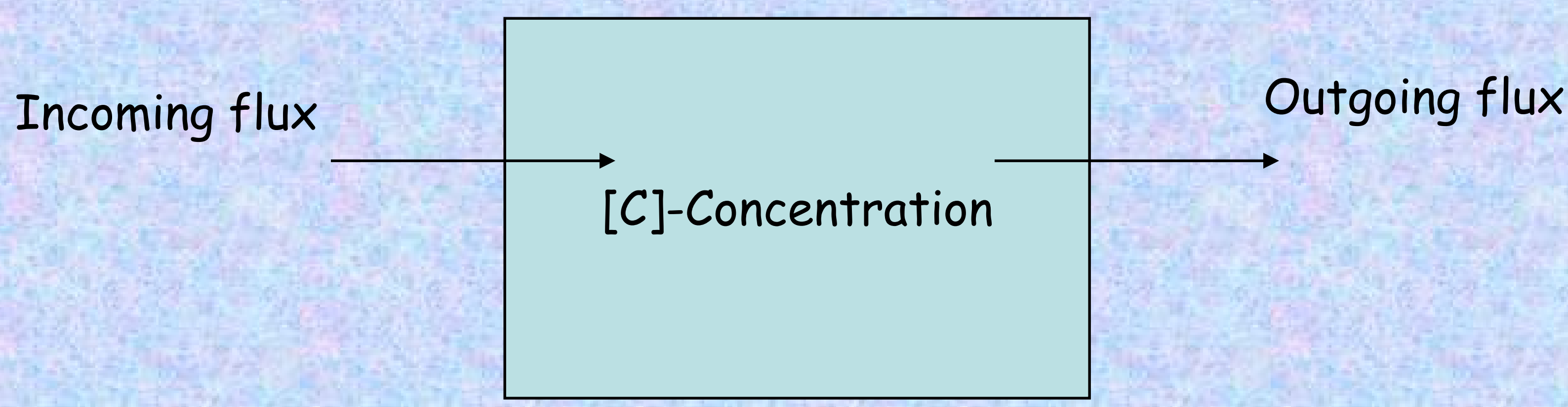
Friction – down gradient flux of momentum.

flux = -diffusion coefficient x gradient {e.g. [moles/s/m^2]}

In 1-dimension: $F=-D d[C]/dx$

What are the units of the diffusion coefficient?

Same units (not value) for momentum, temperature and scalars.



In 1-dimension: $\partial[C]/\partial t = -\partial F/\partial x = D\partial^2[C]/\partial x^2$

A **differential equation** is a mathematical equation for an unknown function (e.g. temperature) of one or several variables (e.g. time and space) that relates the values of the function itself and of its derivatives of various orders (Wikipedia).

An ordinary differential equation (ODE) is a differential equation in which the unknown function is a function of a *single* independent variable.

A partial differential equation (PDE) is a differential equation in which the unknown function is a function of *multiple* independent variables and their partial derivatives.

What kind of equation is:

In 1-dimension: $\partial[C]/\partial t = D\partial^2[C]/\partial x^2$

Answer: a 2nd order in space 1st order in time PDE.

Some analysis of the diffusion equation:

$$\partial[C]/\partial t = D\partial^2[C]/\partial x^2$$

What is the characteristic time scale in takes a molecule to diffuse a distance L?

Is this equation symmetric in time (invert t with –t)?

The concept of entropy (log of probability of state).

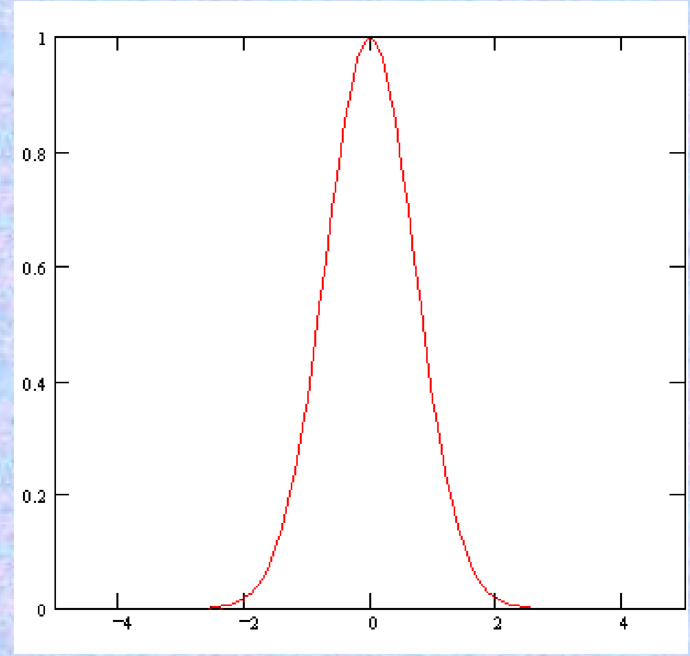
The 2nd law of thermodynamics.

Diffusion and entropy.

Boundary and initial conditions

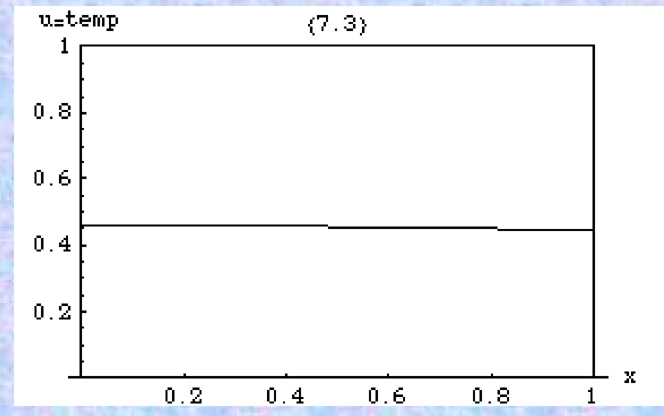
The macroscopic view of diffusion (continuum):

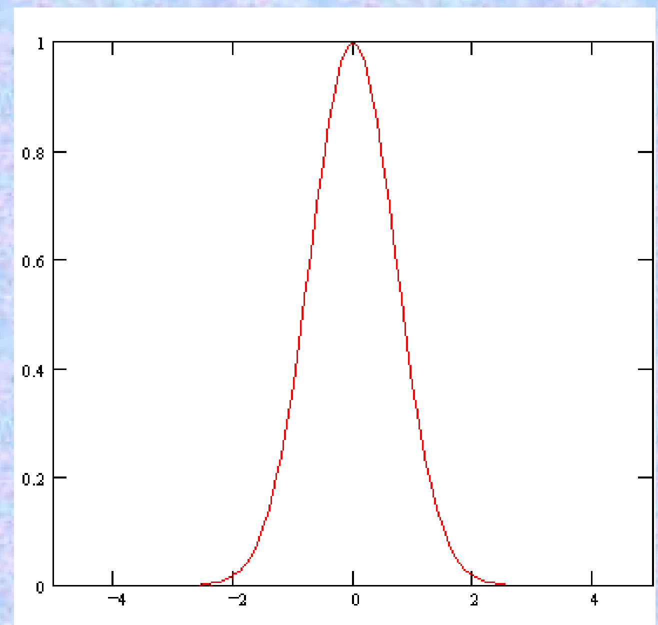
Initial condition:



http://galileo.phys.virginia.edu/classes/311/notes/dimension/node8.html

The macroscopic view of diffusion (continuum): Evolution:





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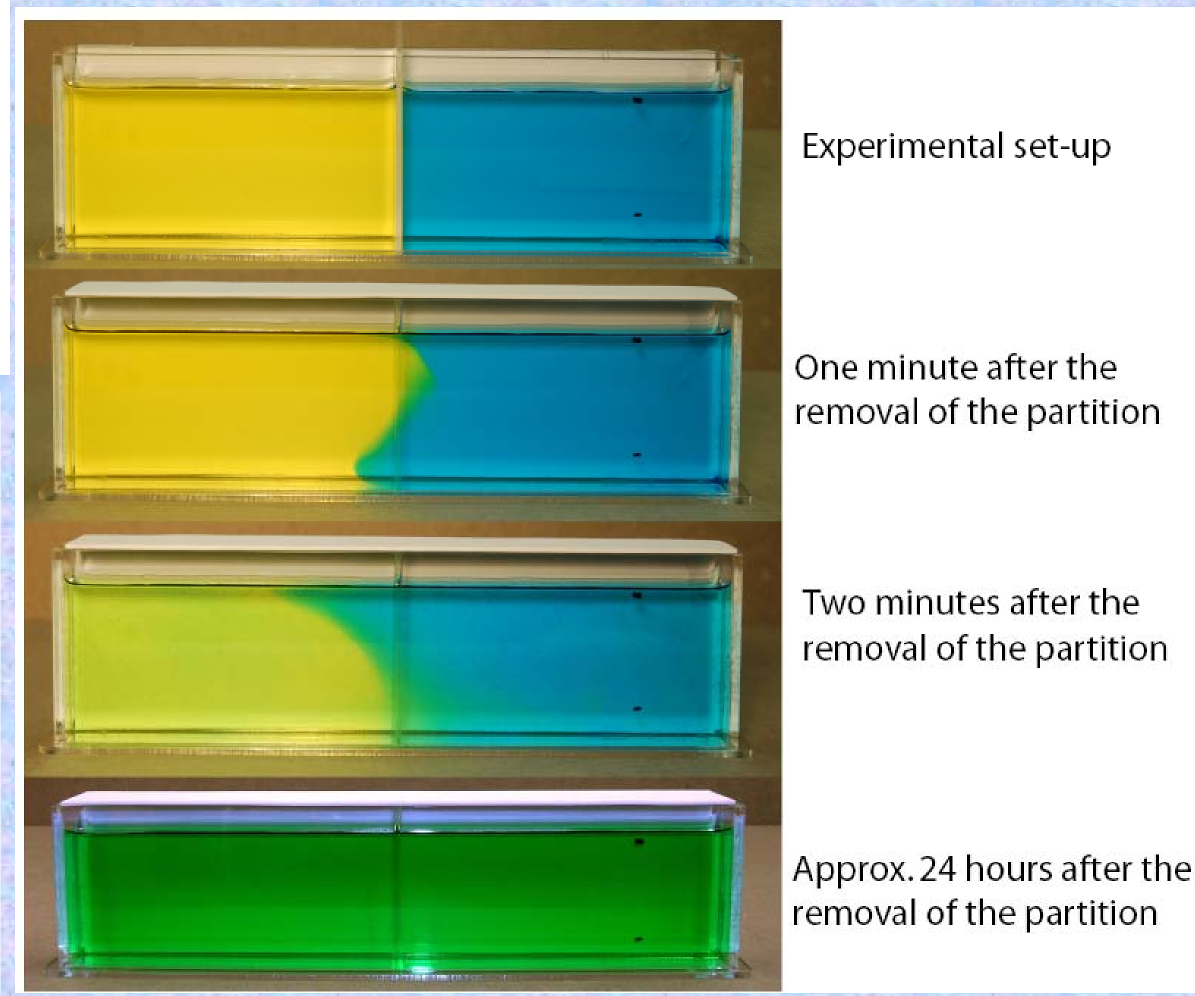


The microscopic view of diffusion:

Rather than a continuum the material is thought of as being comprised of discreet entities (molecules). The statistics of the molecular motion results in the macroscopically observed properties (concentration, temperature, momentum).

Molecules move randomly in the material. A dissolved substance introduced at one side of a tank eventually spread throughout it (diffusion of material).

Molecule bumping into one another transferring energy (mean kinetic energy) and momentum (resulting in diffusion of temperature and momentum).

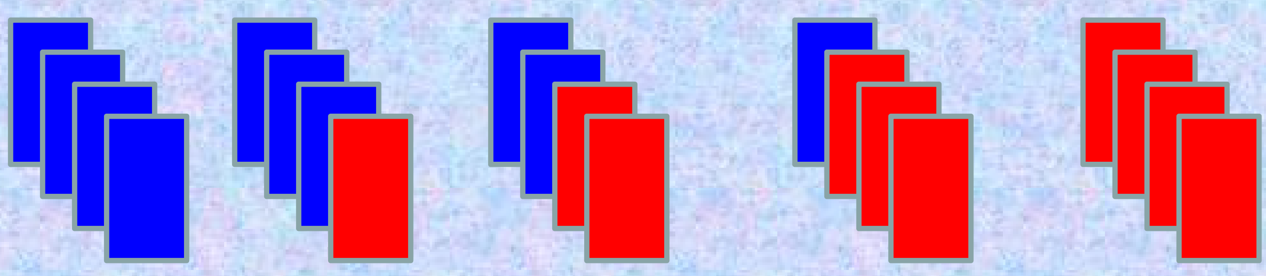
Some applets

http://www.geocities.com/piratord/browni/Difus.html

http://www.biosci.ohiou.edu/introbioslab/Bios170/diffusion/Diffusion.html

Class simulation 1:

time=0

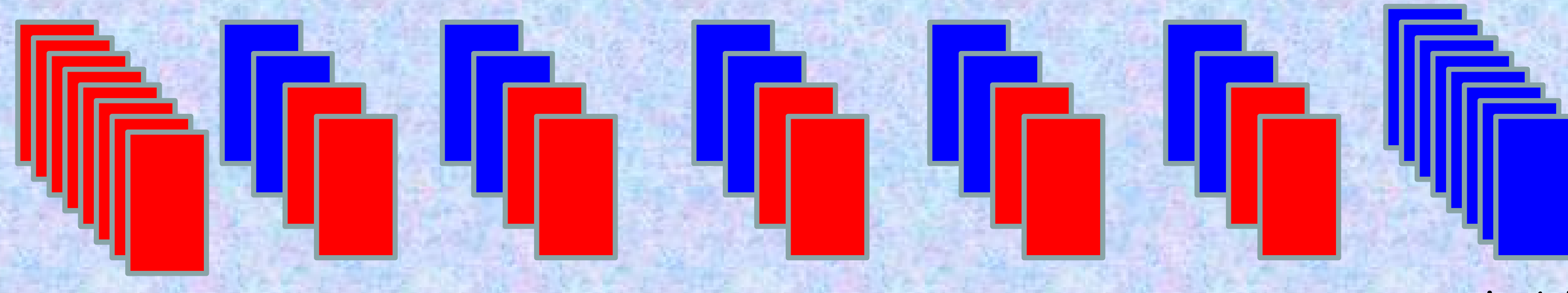


Rule: at each time step each student exchange one RANDOM cards with his/her neighbors.

Prediction: what will be the average concentration after 1 step? 2 steps? 10 steps?

Class simulation 2: changing the 'boundary conditions'

time=0



only red →

← only blue

Start with the same cards you finished the last simulation with.

Rule: at each time step each student exchange one RANDOM cards with his/her neighbor. The students at the ends simply put one card at the end and take the color associated with that end.

Prediction: what will be the average concentration after 1 step? 2 steps? 10 steps?

What is the difference between the two simulations?

How is the final state related to the solution of the diffusion equation?

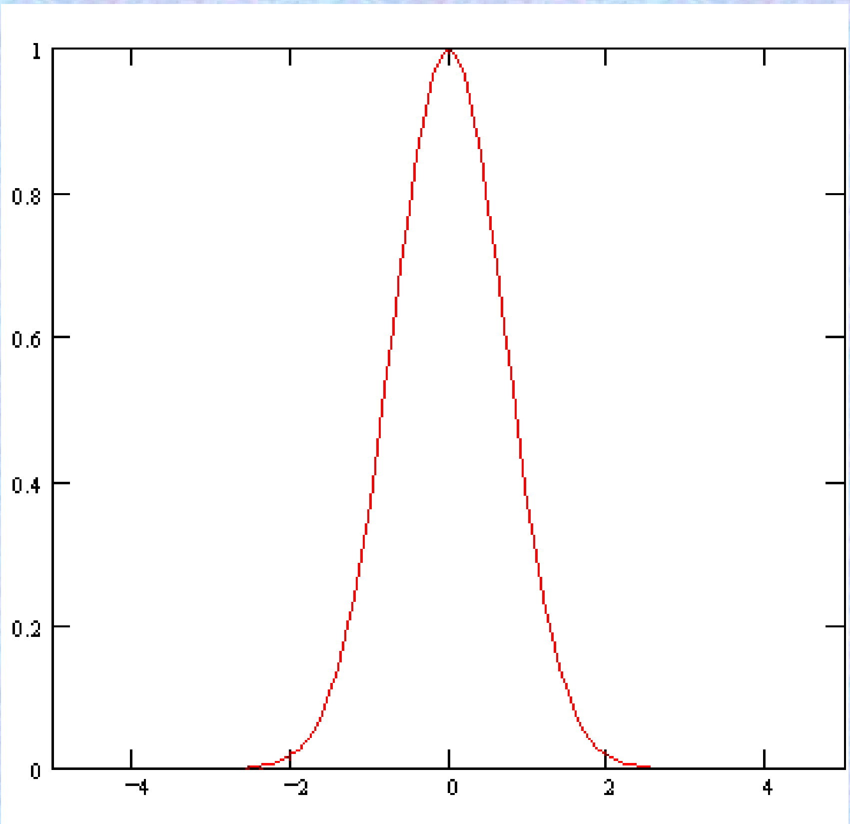
In 1-dimension: $\partial[C]/\partial t = D\partial^2[C]/\partial x^2$

Questions:

1. How does the final condition depend on the initial condition?

2. What is the steady state solution of $\partial[C]/\partial t = D\partial^2[C]/\partial x^2$?

Simulation 3: diffusion – students as molecules



http://galileo.phys.virginia.edu/classes
/311/notes/dimension/node8.html

How can we describe the changing concentration distribution as function of time?

Simulation 3: diffusion

Description of activity:

1. All student stand in line(s) of tile.
2. Every time step (1-16) toss a coin and move one tile left (head) or one tile right (tail).
3. Record your tile position (number) at each time step.

Homework part a (once you have the worksheet with all the data):

1. Plot the student concentration as function of time for t=0,4,8,12,16.
2. How did the mean student position change as function of time?
3. How did the standard deviation around the mean change as function of time? What units does it have?
4. Assuming a time-step of 20sec and a step-length of 25cm, estimate from dimensional analysis the diffusion coefficient of the students in the corridor ($[D]=m^2/s$).

Simulation 4: biased diffusion

Description of activity:

1. All student stand in line(s) of tile.
2. Every time step (1-16) toss a coin and move left one tile (head) or right TWO tiles (tail).
3. Record your tile position (number) at each time step.

Homework part b (once you have the worksheet with all the data):

1. Plot the student concentration as function of time for t=0,4,8,12,16.
2. How did the mean position changed as function of time?
3. How did the standard deviation around the mean change as function of time? What units does it have?
4. Assuming a time-step of 20sec and a step-length of 25cm, estimate from dimensional analysis the diffusion coefficient of the students in the corridor ($[D]=m^2/s$). Using part 1, what is the mean drift speed of the mean position (the rate by which the position of the mean drifts $[v]=m/s$)?

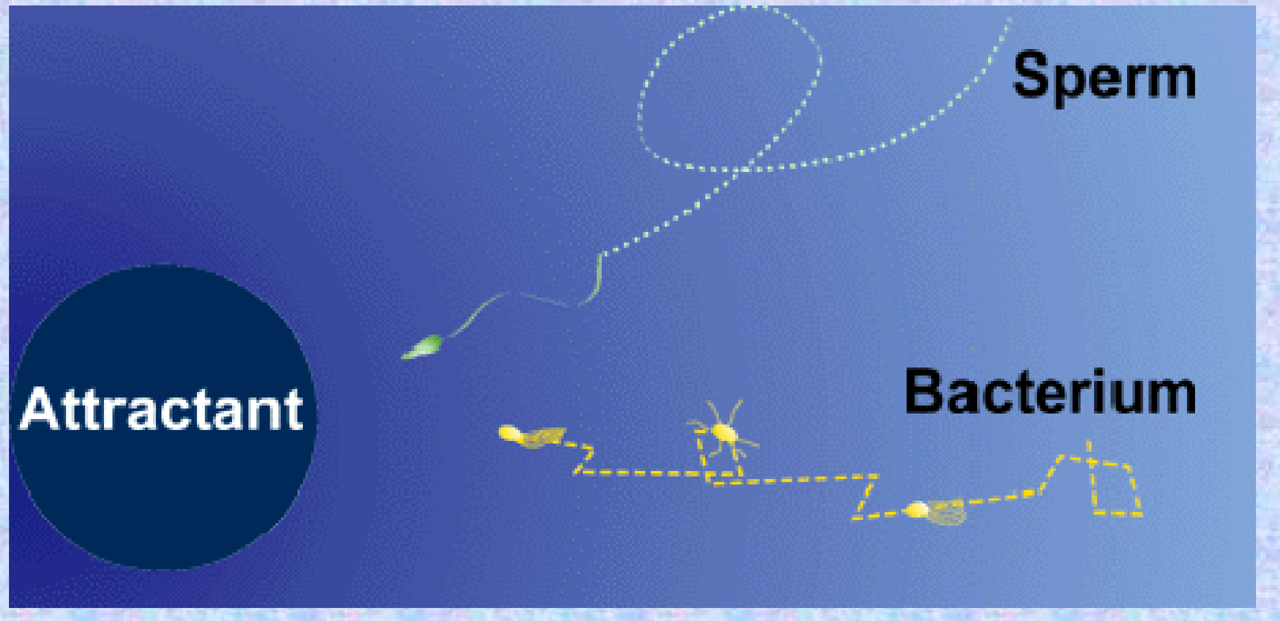
Other related processes:

Brownian motion:

Brown was studying pollen particles floating in water under the microscope. He then observed minute particles within the vacuoles of the pollen grains executing a jittery motion. By repeating the experiment with particles of dust, he was able to rule out that the motion was due to pollen particles being 'alive', although the origin of the motion was yet to be explained.

Brownian motion applet:
http://galileo.phys.virginia.edu/classes/109N/more_stuff/Applets/brownian/brownian.html

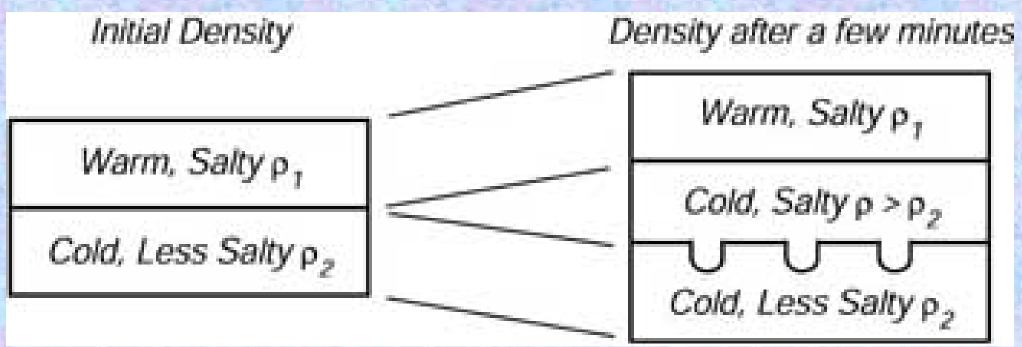
Biased random walk:



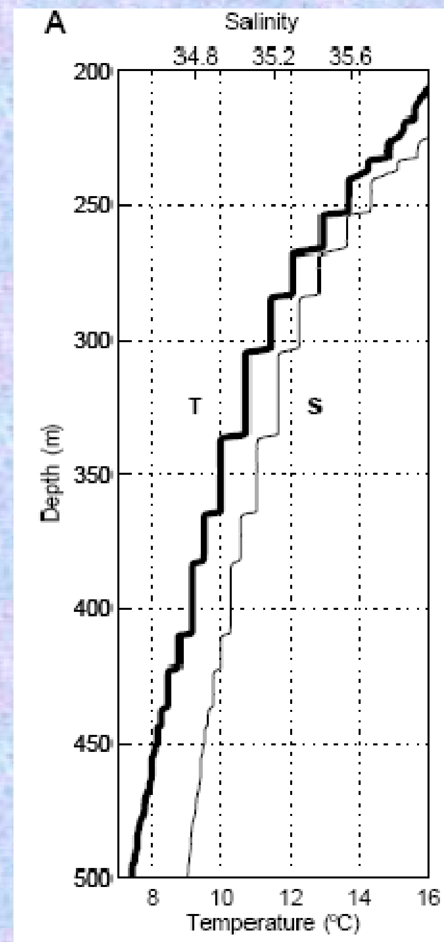
Osmosis:

Osmosis applet: http://www.eeb.uconn.edu/people/plewis/applets/Osmosis/osmosis.html

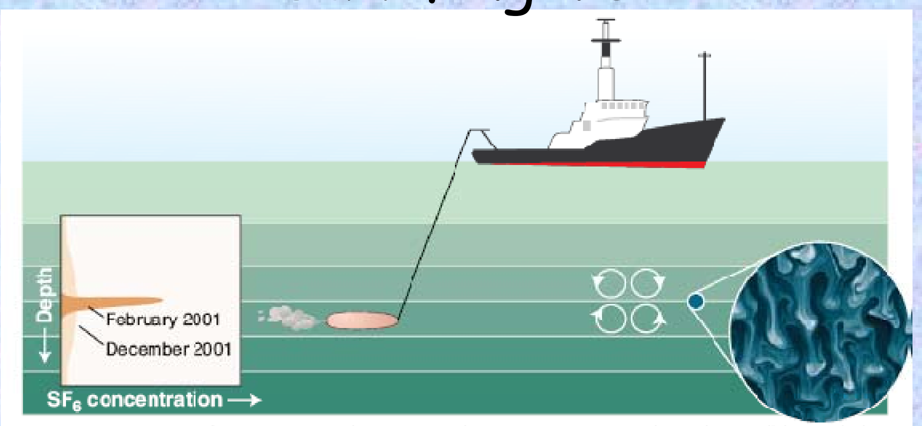
Double diffusion- $D_{temperature} = 100 \times D_{salt}$.



http://oceanworld.tamu.edu/resources/ocng_textbook/contents.html



Salt fingers



Staircase mixing in the ocean. In early 2001, 175 kg of inert SF_6 were released into a "thermohaline staircase" in the western tropical Atlantic. Subsequent vertical dispersion of this tracer (**inset, bottom left**), measured 10 months later, revealed the extent of mixing by salt fingers in the thin interfaces (**inset, bottom right**) and by convection within the thicker layers. The mixing rate, which applies to salinity, was approximately double that of heat.

Enhanced Diapycnal Mixing by Salt Fingers in the Thermocline of the Tropical Atlantic

R. W. Schmitt,* J. R. Ledwell, E. T. Montgomery, K. L. Polzin, J. M. Toole

SCIENCE VOL 308 29 APRIL 2005

Vertical eddy diffusion~ 0.9×10^{-4} m^2/s

A lab example of a double diffusion oscillator:

movie