1. Compute the sound speed values for the following S,T, P values (to simplify your life you may want to use the matlab routine sndspd.m from ftp://acoustics.whoi.edu/pub/Matlab/oceans/programs/):


<table>
<thead>
<tr>
<th>Conditions</th>
<th>Sound speed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Near surface arctic: 31, 2, 20</td>
<td>1453.2 m/s</td>
</tr>
<tr>
<td>Near surface Red Sea: 41, 24, 20</td>
<td>1538.8 m/s</td>
</tr>
<tr>
<td>Deep Atlantic Ocean: 36, 2, 5000</td>
<td>1544.0 m/s</td>
</tr>
</tbody>
</table>

2. A normal human can hear sound from 0db to 100db without experiencing too much pain. What are the values in term of pressure units (e.g. Pascal)? How do they compare to the atmospheric pressure on the Earth’s surface?

\[ P \text{ (in dB)} = 20 \log \left[ \frac{P \text{ (in } \mu \text{Pa})}{20 \mu \text{Pa}} \right] \Rightarrow P \text{ (in } \mu \text{Pa}) = 10^{\frac{P \text{ (in dB)}}{20}} \times 20 \mu \text{Pa} \]

\[ P(0 \text{ dB}) = 20 \mu \text{Pa} \]
\[ P(100 \text{ dB}) = 2 \text{ Pa} \]

Atmospheric pressure is about \(10^5\)Pa (The atmospheric pressure is similar to the pressure due to the mass of a 10m x 1m\(^2\) water column of water = \(9.81 \text{ m s}^2 \times 10\text{m} \times 1000\text{kg/m}^3 \times 1\text{m}^2 \approx 10^5 \text{ N/m}^2 = 10^5\text{Pa}\)).

Thus the pressures associated with audible sounds are less than 5 orders of magnitude relative to observed atmospheric pressures. Our ears are VERY sensitive organs that measure slight pressure changes, not absolute pressure background.

3. A stratified marginal sea has 4 layers with the following sound speed distribution:

<table>
<thead>
<tr>
<th>Layer</th>
<th>Layer 1</th>
<th>Layer 2</th>
<th>Layer 3</th>
<th>Layer 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sound speed</td>
<td>1530</td>
<td>1490</td>
<td>1500</td>
<td>1540</td>
</tr>
<tr>
<td>Depth range</td>
<td>0-100</td>
<td>100-400</td>
<td>400-800</td>
<td>800-1200</td>
</tr>
</tbody>
</table>

An explosion has occurred at 100m depth. Plot the ray trajectories starting in the zenith angles: \(\theta=80^\circ\), \(90^\circ\), and \(100^\circ\) until you get to 40km from the source. Assume the bottom and top boundaries to be reflective. How long did it take each ray to reach a 40km range from the source? At what depth did they arrive there (extra credit: automate the solution by programming it)?

For the solution I will be using Snell’s law, and the fact that rays are symmetric as function of reflection when the medium does not vary in the horizontal (i.e the path
up in a layer is the mirror image of the path down within the same layer for the same ray).

The 90° ray: The first ray to get 40km from the source is that with $\theta=90°$. Since sound speed is not varying along the ray it will stay on course at 100m depth. Assuming an average sound speed of 1510m/s, it takes $40000/1510=24.49s$ to arrive at depth of 100m.

The 80° ray: For this ray, $1530/ \sin 80°=1553.6$ is constant along the path. The ray will reach the surface and reflect at horizontal distance $x_1=\Delta z \tan 80°=567.1282m$ covering a path of $\Delta z/\cos 80°=575.877m$ long. Time=$t_1=\text{ray length/speed}=0.3764s$. $\Delta z$ denotes the layers depth (100m for layer 1)
Upon reflection the mirror image ray propagate from the reflection point back to the 100m depth ($x=2x_1, z=100, t=2t_1$).

As it enters layer 2 the ray changes zenith angles from 80° in Layer 1 to 180°-73.55°=106.45° in layer 2. $x_2=\Delta z_2 \tan 73.55°=1016m$, pathlength=$\Delta z/\cos 73.55°=1059.4$ and $t_2=0.711s$.
As it enters layer 3 the ray refracts and changes zenith angle to 180°-74.9055°=105.0945°. $x_3=1483m$, pathlength=1536 and $t_3=1.024s$.
As it enters layer 4 the ray refracts and changes zenith angle to 180°-82.4125°=97.5875°. $x_4=3002.9m$, pathlength=3029.4 and $t_4=1.9671s$.
Thus a trajectory between the top and bottom boundaries has a total horizontal length of: $\Sigma x_i=6069m$, and takes $\Sigma t_i=4.0785s$. Since the problem is symmetric, we can propagate the rays as close as we can to 40km-$x_1$ without additional work, i.e 6 times. At which time we are at the top boundary ($z=0$) after covering $x_1+6\Sigma x_i=36,981m$ at $t_1+\Sigma t_i=24.8474s$.

It turns out that at the bottom of the third layer ($z=800m$) the total horizontal length covered by the ray will be: 40047m, at $t=26.9588s$. Correcting for a horizontal distance of 47m at layer 3 we backtrack to a depth of 800-12.68m=787.32m and a time of 26.9588-0.0325=26.9263s.

The 100° ray: For this ray, $1490/ \sin 100°=1513m/s$ is constant along the path. The ray will totally reflect from any layer where the sound speed is faster than 1513m/s and will thus only propagate in layers 2 and 3.
Within layer 2 the ray propagates a horizontal distance of $x_2=\Delta z_2 \tan (180-100°) =1701.4m$, covering a path of $\Delta z_2/\cos 80°=1727.6m$ at $t_2=1.1595s$.
As it enters layer 3 the ray refracts and changes zenith angle to 180°-82.4879°=97.5121°. $x_3=3033.4m$, pathlength=3059.6 and $t_3=2.0397s$.
One ray trajectory from top of layer 2 to the bottom of layer 1 cover a horizontal distance of $\Sigma x_i=4761m$ and takes $\Sigma t_i=3.1992s$
Again, using symmetry we propagate rays and their mirror images as close as possible to the 40km mark, that will take 4 pairs of rays and their mirror images to bring us to the top ($z=100$) at a distance of: 38088m and time: 25.5936s.
Adding another travel in layer 2, $z=400m, x=x_2+6\Sigma x_i=39789m$, and $t=26.7531$.
We need to go an extra 420.2m in layer 3 to get to $x=40k$. This will bring us to a depth of $z=400+55.41=455.41m$ at a time $t=26.7531+0.2826=27.0357$