SMS-598, Introduction to Acoustical Oceanography. Fall 2005
Answers to assignment #2.

1. Compute the sound speed values for the following S,T, P values (to simplify your life you may want to use the matlab routine sndspd.m from ftp://acoustics.whoi.edu/pub/Matlab/oceans/programs/):
Using the Chen and Millero, "The Sound Speed in Seawater", J. Acoust. Soc. Am. 62 (1977), 1129-1135:

Conditions	**Sound speed**
Near surface arctic: 31, 2, 20	**1453.2 m/s**
Near surface Red Sea: 41, 24, 20	**1538.8 m/s**
Deep Atlantic Ocean: 36, 2, 5000	**1544.0 m/s**

2. A normal human can hear sound from 0db to 100db without experiencing too much pain. What are the values in term of pressure units (e.g. Pascal)? How do they compare to the atmospheric pressure on the Earth's surface?

P (in dB) = 20 log [P(in μPa)/20 μPa] → P(in μPa) = 10^{P (in dB)/20} x 20 μPa

P(0 dB)=20 μPa
P(100dB)=2 Pa

Atmospheric pressure is about 10^5Pa (The atmospheric pressure is similar to the pressure due to the mass of a 10m x $1m^2$ water column of water = 9.81m s^2 x 10m x 1000kg/m3 x $1m^2$ ~ 10^5 N/m2 = 10^5Pa).

Thus the pressures associated with audible sounds are less than 5 orders of magnitude relative to observed atmospheric pressures). Our ears are VERY sensitive organs that measure slight pressure changes, not absolute pressure background.

3. A stratified marginal sea has 4 layers with the following sound speed distribution:

	Layer 1	Layer 2	Layer 3	Layer 4
Sound speed	1530	1490	1500	1540
Depth range	0-100	100-400	400-800	800-1200

An explosion has occurred at 100m depth. Plot the ray trajectories starting in the zenith angles: θ=80° 90° and 100° until you get to 40km from the source. Assume the bottom and top boundaries to be reflective. How long did it take each ray to reach a 40km range from the source? At what depth did they arrive there (extra credit: automate the solution by programming it)?

For the solution I will be using Snell's law, and the fact that rays are symmetric as function of reflection when the medium does not vary in the horizontal (i.e the path

up in a layer is the mirror image of the path down within the same layer for the same ray).

The 90° ray: The first ray to get 40km from the source is that with θ=90°. Since sound speed is not varying along the ray it will stay on course at 100m depth. Assuming an average sound speed of 1510m/s, it takes 40000/1510=24.49s to arrive at depth of 100m.

The 80° ray: For this ray, 1530/ sin80°=1553.6 is constant along the path. The ray will reach the surface and reflect at horizontal distance $x_1=\Delta z$ tan80°=567.1282m covering a path of Δz/cos80°= 575.877m long. Time=t_1=ray length/speed= 0.3764s. Δz denotes the layers depth (100m for layer 1)
Upon reflection the mirror image ray propagate from the reflection point back to the 100m depth ($x=2x_1$, z=100, $t=2t_1$).
As it enters layer 2 the ray changes zenith angles from 80° in Layer 1 to 180°-73.55°=106.45° in layer 2. x2=Δz_2tan73.55°=1016m,
pathlength= Δz/cos73.55°=1059.4 and t_2=0.711s.
As it enters layer 3 the ray refracts and changes zenith angle to 180-74.9055=105.0945°. x3= 1483m, pathlength=1536 and t_3=1.024s.
As it enters layer 4 the ray refracts and changes zenith angle to 180-82.4125=97.5875°. x4= 3002.9m, pathlength=3029.4 and t_4=1.9671s.
Thus a trajectory between the top and bottom boundaries has a total horizontal length of: Σx_i=6069m, and takes Σt_i=4.0785s. Since the problem is symmetric, we can propagate the rays as close as we can to 40km-x_1 without additional work, i.e 6 times. At which time we are at the top boundary (z=0) after covering
x_1+ 6Σx_i=36,981m at t_1+ Σt_i=24.8474s.
It turns out that at the bottom of the third layer (z=800m) the total horizontal length covered by the ray will be: 40047m, at t=26.9588s. Correcting for a horizontal distance of 47m at layer 3 we backtrack to a depth of 800-12.68m=787.32m and a time of 26.9588-0.0325=26.9263s.

The 100° ray: For this ray, 1490/ sin100°=1513m/s is constant along the path. The ray will totally reflect from any layer where the sound speed is faster than 1513m/s and will thus only propagate in layers 2 and 3.
Within layer 2 the ray propagates a horizontal distance of $x_2=\Delta z_2$tan(180-100°) =1701.4m, covering a path of Δz_2/cos80°=1727.6m at t2=1.1595s.
As it enters layer 3 the ray refracts and changes zenith angle to 180-82.4879=97.5121°. x3= 3033.4m, pathlength=3059.6 and t_3=2.0397s.
One ray trajectory from top of layer 2 to the bottom of layer 1 cover a horizontal distance of Σx_i=4761m and takes Σt_i=3.1992s
Again, using symmetry we propagate rays and their mirror images as close as possible to the 40km mark, that will take 4 pairs of rays and their mirror images to bring us to the top (z=100) at a distance of: 38088m and time: 25.5936s.
Adding another travel in layer 2, z=400m, x=x_2+6Σx_i=39789m, and t=26.7531.
We need to go an extra 420.2m in layer 3 to get to x=40k. This will bring us to a depth of z=400+55.41=455.41m at a time t=26.7531+0.2826=27.0357