The wave equation –

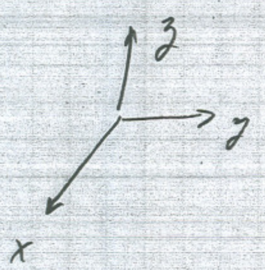
LANGUAGE – ACOUSTICS IMPLIES INVICID FLUID –

NO SHEAR –

USE SLINKY & DIAGRAM

A SOLID OR VERY VISCOUS FLUID HAS $G \neq 0$ –

INTERESTING NOTE AS $\dot{\varepsilon} \to \infty$
$G \to 0$




$$\nabla^2 \phi = \frac{1}{c^2} \phi$$

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

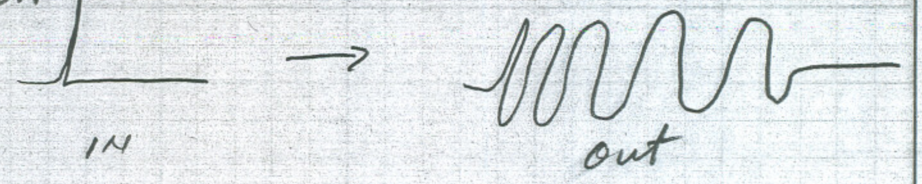
PROPAGATION – UNBOUNDED BODIES

$\left.\begin{array}{l}\text{REFLECTION} \\ \text{REFRACTION}\end{array}\right\}$ BOUNDARY OR INTERFACE

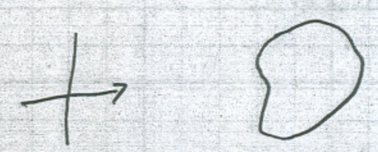
GUIDED WAVES ← LIKE 2 BOUNDARIES

1 UNBOUNDED DIMENSION

DISPERSION
GROUP VELOCITY



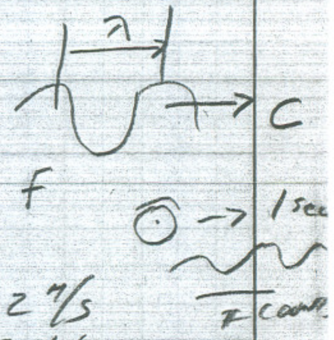

SCATTERING PROBLEMS → FINITE DIMENSIONAL BODY



LOOK AT IDEA OF DIMENSIONS!

CHARACTERISTIC DIMENSION INTRINSIC –

$\lambda = c/f$ HOW BIG IS A CAR




JAPAN MICRO CAR 3 METERS

SPEED OF SOUND IN WATER 1482 m/s

" " " " AIR 332 m/s

WORLD REFERED TO CHARACTERISTIC LENGTH

5 kHz – $\lambda = .296$ M WATER

5000 1/s – $\lambda = .066$ M AIR

MICROCAR IS BIG > $5\lambda$

LINEARIZED THEORY – $L(\alpha x + \beta y) = \alpha Lx + \beta Ly$

PULSES

FOURIER ANALYSIS –

SUPERPOSITION OF HARMONIC WAVES (TIME HARMONIC WAVES)

HARMONIC $\sin \frac{n\pi t}{T}$ $\cos \frac{n\pi t}{T}$

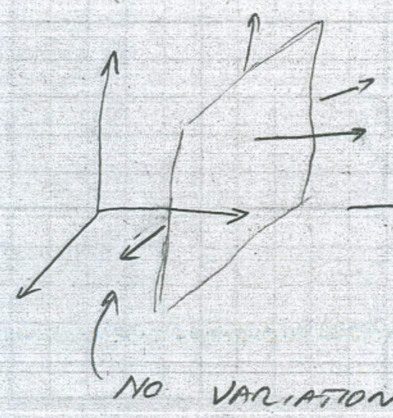
OR EULER'S IDENTITY

$$e^{i\theta} = \cos\theta + i\sin\theta$$

START IN 1-DIMENSION –

WAVE ON A STRING –

OR WAVE IN AN UNBOUNDED MEDIA - PLANE WAVE –



DISTURBANCE TRAVELING IN S DIRECTION

→ S

NO VARIATION IN PLANE –

PARTIAL DIFFERENTIAL EQUATION -

$$\frac{\partial^2 p}{\partial s^2} - \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} = 0$$

OR $\nabla^2 p + \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} = 0$

$$\left(\frac{\partial}{\partial s} - \frac{1}{c}\frac{\partial}{\partial t}\right)\left(\frac{\partial}{\partial s} + \frac{1}{c}\frac{\partial}{\partial t}\right)p = 0$$

P = PRESSURE - VARIES

$p = p_0 + p'$ — DISTURBANCE OF PRESSURE WITH SPATIAL VARIATION

$p_0$: PRESSURE IN FLUID

$\rho = \rho_0 + \rho'$ DENSITY IS PERTURBED BY PRESSURE VARIATION

MOST GENERAL SOLUTION IS A PAIR OF DISTURBANCES MOVING IN THE +S & -S DIRECTION OF ARBITRARY SHAPE

d'Alembert Solution $p = f(t - \frac{1}{c}s) + g(t + \frac{1}{c}s)$

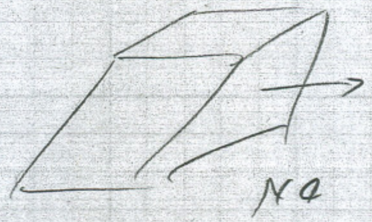
$f$ ↳ MOVES IN +S

$g$ ↳ MOVES IN -S

VELOCITY OF DISTURBANCES IS C

NO CHANGE IN SHAPE -

TALK ABOUT LINEAR THEORY -



→ NET ENERGY FLUX -

NO CHANGE IN MASS -

WE CAN ALWAYS DESCRIBE AS A VELOCITY INSTEAD OF A PRESSURE

$$\rho_0 \frac{\partial v}{\partial t} = \frac{\partial p}{\partial t}$$

$$v = \frac{1}{\rho c}\left[f(t - \tfrac{1}{c}s) - g(t + \tfrac{1}{c}s)\right)$$

SINGLE FREQUENCY

$$p = P_{max} \cos(\omega t - \phi)$$
$$= P_{max} \sin(\omega t - \phi')$$
$$= Re\, \hat{P} e^{-i\omega t}$$

NOT ANY DIFFERENT

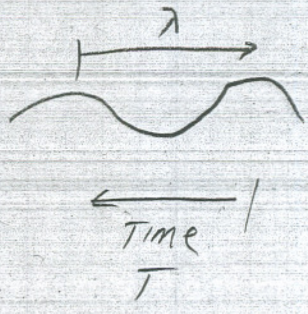
$f(t - \frac{1}{c}s)$

$= \sin[k(s - ct)]$

$= \sin[ks - \omega t]$

Same Notation

Remember

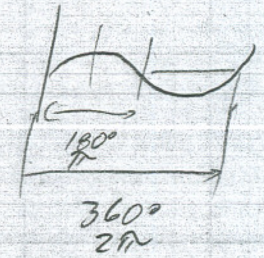



$T = 1/f$

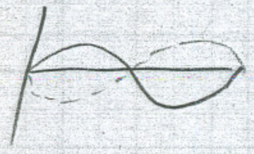
$\omega = kc$ $\quad k$ = WAVE NUMBER

$c$ = VELOCITY

$\phi$ PHASE




2 WAVES 180° OUT OF PHASE $\phi = \pi$ RADIANS



NOW THE BIG DOG –

SINGLE POINT SOURCE IN AN UNBOUNDED DOMAIN

$$r^2 = x_1^2 + x_2^2 + x_3^2$$
$$2r \frac{\partial r}{\partial x_i} = 2x_i$$

$$\underbrace{\frac{\partial}{\partial x_i} \overbrace{p(r,t)}^{\text{SPHERICAL}}}_{\text{OUR WAVE EQUATION}} = \frac{\partial p}{\partial r}\frac{\partial r}{\partial x_i} = \frac{x_i}{r}\frac{\partial p}{\partial r}$$

! SPREAD OUTWARD FROM A POINT NO $\phi$ $\theta$ DEPENDENCE

$$\frac{1}{r}\frac{\partial^2}{\partial r^2} rp - \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} = 0$$

PLANE WAVE EXCEPT FOR $\frac{1}{r}$

MEANING ATTENUATION WITH $r$

AS $r \to \infty$ PLANE WAVE!

AS $r \to 0$ BLOWS UP

$$P(r,t) = \frac{1}{r} f\left(t - \frac{r}{c}\right) + \frac{1}{r} g\left(t + \frac{r}{c}\right)$$

$f$ & $g$ ARE CONTINUOUS FOR AT LEAST A SECOND DERIVATIVE

MEANING

$t - \frac{r}{c} \to$ MOVES AWAY

$t + \frac{r}{c} \to$ MOVES INWARD

CAUSAL - HIT IT BEFORE IT RESPONDS

$g \Rightarrow 0$

$\frac{\Delta r}{c}$ TIME FROM DISTURBANCE TO LISTENER

$\frac{1}{r}$ ATTENUATION WITH TIME

MORE ADVANCED CONCEPT - FUNDAMENTAL SOLUTION

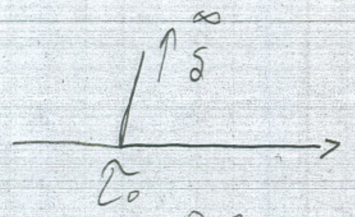
$$\iint_S P(r,t)\, ds \text{ TO USE}$$

SOLUTION FROM POINT SOURCE

OR WITH AMPLITUDE

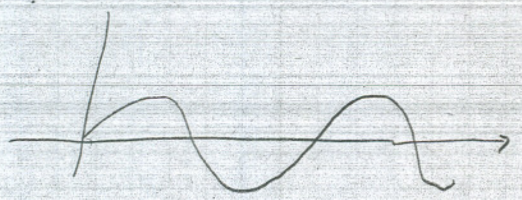

RADIATED FIELD FROM A SOURCE

WE WILL FOCUS ON HARMONIC WAVES — WILL RETURN BUT 2 FUNDAMENTAL SOLUTIONS —




$$\Rightarrow \int_{-\infty}^{\infty} \delta(t-\tau_0)\, dt = 1$$

DIRAC DELTA FUNCTION




$$\Rightarrow 1 \sum_{w} \cos wt \Rightarrow \delta(t)$$

HARMONIC SOLUTION

ADD UP ALL HARMONIC SOLUTIONS GET A $\delta(t)$ AT 0

FOR OUR SPHERICAL CASE

$$\frac{\partial^2 p}{\partial r^2} + \frac{2}{r}\frac{\partial p}{\partial r} = \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2}$$

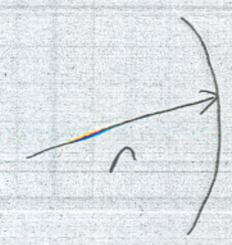
OR $$\frac{\partial^2 (rp)}{\partial r^2} = \frac{1}{c^2}\frac{\partial^2 (rp)}{\partial t^2}$$

$$\tilde{p} = \frac{\tilde{A}}{r} e^{i(wt-kr)}$$

RECALL EULER'S IDENTITY

$$e^{i\theta} = \cos\theta + i \sin\theta$$

AT A LARGE DISTANCE $r$




$r \to \infty$

PLANE WAVE SOLUTION

THUS WE SEE THAT A SPHERICAL SPREADING WAVE BECOMES THE PLANE WAVE AS $r \to \infty$

USING TEXT NOTATION NOTE

$u$ IS VELOCITY NOT DISPLACEMENT

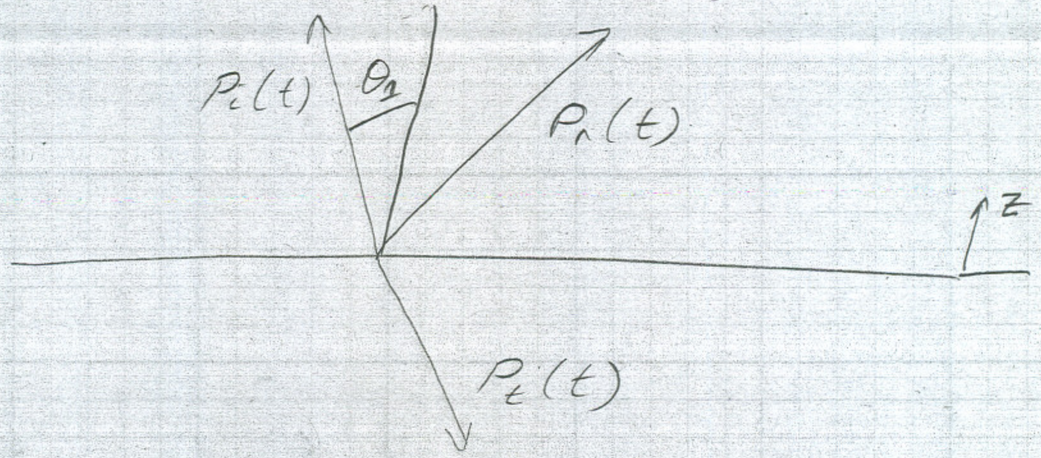
WE REITERATE

$$\frac{\partial u}{\partial t} = -c\frac{\partial u}{\partial x}$$

$$\overset{F}{\frac{\partial p}{\partial x}} = \rho \overset{a}{\frac{\partial u}{\partial t}} \qquad F = ma$$

$$\frac{\partial p}{\partial x} = \underbrace{\boxed{\rho_A c}}_{\text{ACOUSTIC IMPEDANCE}} \frac{\partial u}{\partial t}$$

ACOUSTIC IMPEDANCE IS THE CHARACTERISTIC ABILITY TO TRANSFER ENERGY BETWEEN MEDIA —




B.C. PRESSURE IS CONTINUOUS

NORMAL COMPONENTS OF VELOCITY ARE EQUAL —

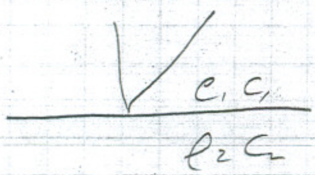
$$p_i(t) + p_r(t) = p_t(t)$$

$$u_{zi}(t) + u_{zr}(t) = u_{zt}(t)$$

$$p = \rho c u$$

$$\frac{\sin\theta_2}{c_2} = \frac{\sin\theta_1}{c_1} \quad \text{SNELL'S LAW}$$

THE VERTICAL (z) VELOCITIES




$$u_{zi}(t) = u_i(t)\cos\theta_1$$
$$u_{zr}(t) = u_r(t)\cos\theta_1$$
$$u_{zt}(t) = u_t(t)\cos\theta_2$$

PRESSURE $\quad p = \mp(\rho c)u$

THEN $$u_{zi}(t) = \frac{p_i(t)}{\rho_1 c_1}\cos\theta_1$$
$$u_{zr}(t) = \frac{p_r(t)}{\rho_1 c_1}\cos\theta_1$$
$$\theta_i = \theta_r$$
$$u_{zt}(t) = \frac{p_t(t)}{\rho_2 c_2}\cos\theta_2$$

THEN DEFINE
$$\frac{p_r}{p_i} = R_{12} \quad \text{REFLECTION}$$
$$\frac{p_t}{p_i} = T_{12} \quad \text{TRANSMISSING}$$

$$p_t = p_t(t - z/c_2)$$
$$p_i = p_i(t - z/c_1)$$
$$p_r = p_r(t + z/c_1)$$

LOOK AT VELOCITY
$$u_{zi} + u_{zr} = u_{zt}$$

$$u_i = p_i\cos(kx - wt) \quad \text{OR} \quad p_i f(kx - wt)$$
$$u_r = R_{12} p_i f(kx - wt)$$
$$u_t = T_{12} p_i f(kx - wt)$$

OR $1 + R_{12} = T_{12}$ PRESSURE EQUN

VELOCITY $u_i + u_r = u_t$

$$(\rho_2 c_2 - \rho_2 c_2 R_{12}) \cos\theta_1 = \rho_1 c_1 T_{12} \cos\theta_1$$

SINCE

MOVING $-Z$

$$\frac{\cancel{P_i}}{\rho_1 C_1}\cos\theta_1 + R_{12}\left(\frac{-\cancel{P_i}}{\rho_1 c_1}\right)\cos\theta_1 = T_{12}\frac{\cancel{P_i}}{\rho_2 c_2}\cos\theta_1$$

$$R_{12} = \frac{\rho_2 c_2 \cos\theta_1 - \rho_1 c_1 \cos\theta_2}{\rho_2 c_2 \cos\theta_1 + \rho_1 c_1 \cos\theta_2}$$

$$T_{12} = \frac{2\rho_2 c_2 \cos\theta_1}{\rho_2 c_2 \cos\theta_1 + \rho_1 c_1 \cos\theta_2}$$

$$\theta_2 = \sin^{-1}\left(\frac{c_2}{c_1}\sin\theta_1\right)$$