Introduction to Acoustical Oceanography

Instructors: Mick Peterson and Emmanuel Boss

Why are we here? Results from student survey:

General:

- Introduction to oceanography, terminology, use of acoustics in oceanography.

Instrumentation:

- SONAR.
- Transducer design and modeling. Signal generations and analysis (e.g. using oscilloscopes).
- Acoustic instrumentation, how they work, advantages and limitations with respect to whale research (e.g. directional arrays).

Sound interaction with matter:

- Scattering from suspended material- theory, relationship to biovolume, scattering from nonspherical targets.
- How does sound behave as function of the physical properties of water.
- Wave propagation and attenuation in sediments and across interfaces.

Sound in water in contrast to air.

- Hull/machinery radiated sound, and noise produced by turbulent flow and cavitation.

Bioacoustics:

- How do whales use sound to communicate and sense their environment.
- Active and passive bio-acoustics with reference to mammals and zooplankton.

Last week, waves:

Point source, far enough from the source, in an unbounded, lossless medium:

$$p(R,t) = p_0(t - R/c)\frac{R_0}{R}$$

$$u(R,t) = \frac{p}{\rho_w c} = \frac{p_0(t - R/c)}{\rho_w c}\frac{R_0}{R}$$

Intensity= energy per unit time per unit area:

$$i_R(R,t) = pu = \frac{p_0^2(t - R/c)}{\rho_w c}\frac{R_0^2}{R^2}$$

Rays

Geometric optics- modeling the structure of the acoustic field as a set of path of sonic energy (rays).
Principles of the method:

•Assumes the wave front >> λ and that changes in sound speed occur on scales >> λ.

•Refraction of the propagation direction due to changes in sound speed following Snell's law.

•Specular reflection at interfaces.

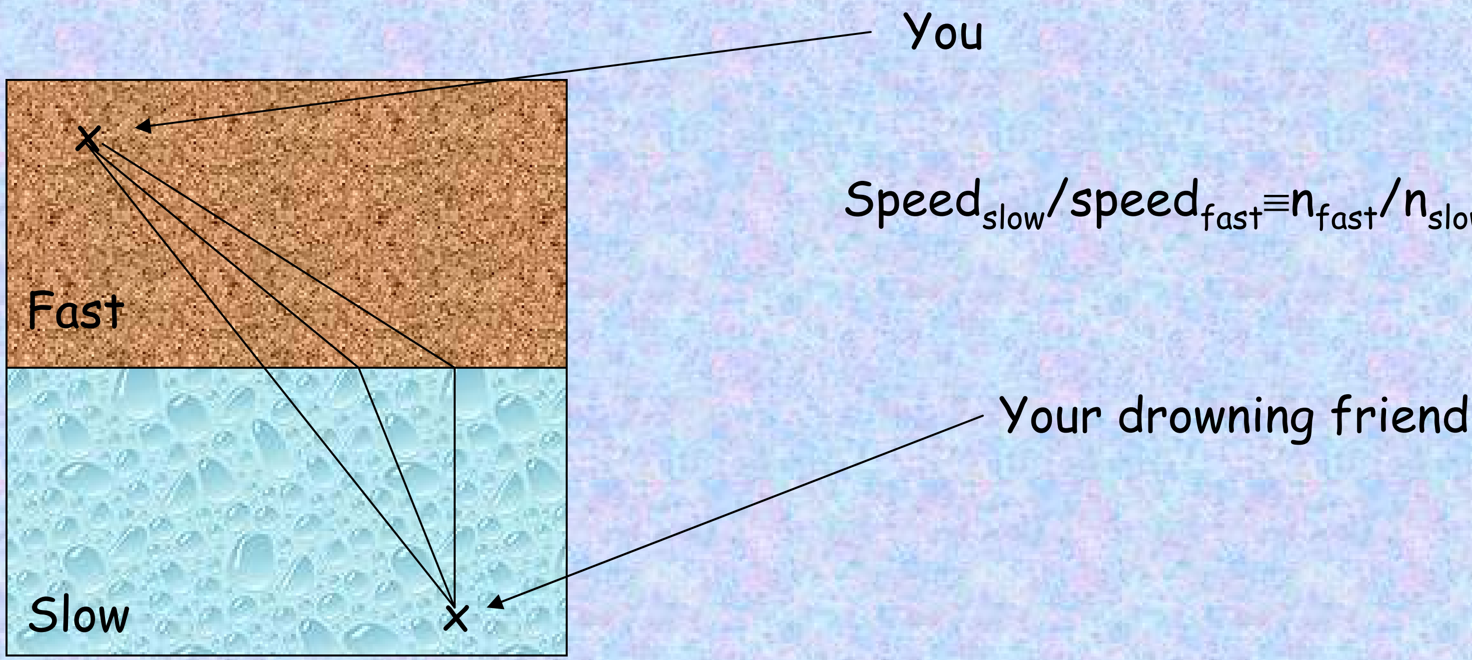
•Intensity losses along rays are taken into account due to geometric divergence, attenuation along the path, and reflection on the interfaces.

•Ignores diffraction effects.

Approximate the full wave solutions.

Much easier than the full wave solutions.

A little discussion of refraction at the interface of two layers (Feynman. Vol I, ch. 26):



$Speed_{slow}/speed_{fast} \equiv n_{fast}/n_{slow}$

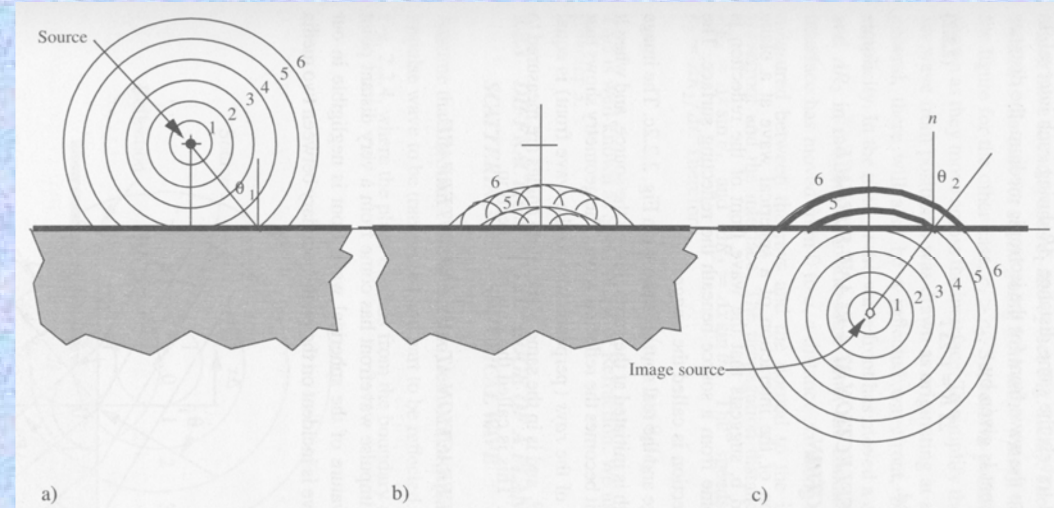
Fermat's principle: out of all possible paths between two points light (sound) will take the path requiring the shortest travel time (an extreme).

➡ $Sin\theta_{fast}/speed_{fast}=sin\theta_{slow}/speed_{slow}$

The same principle explain the fact that $\theta_{reflected}=\theta_{incident}$

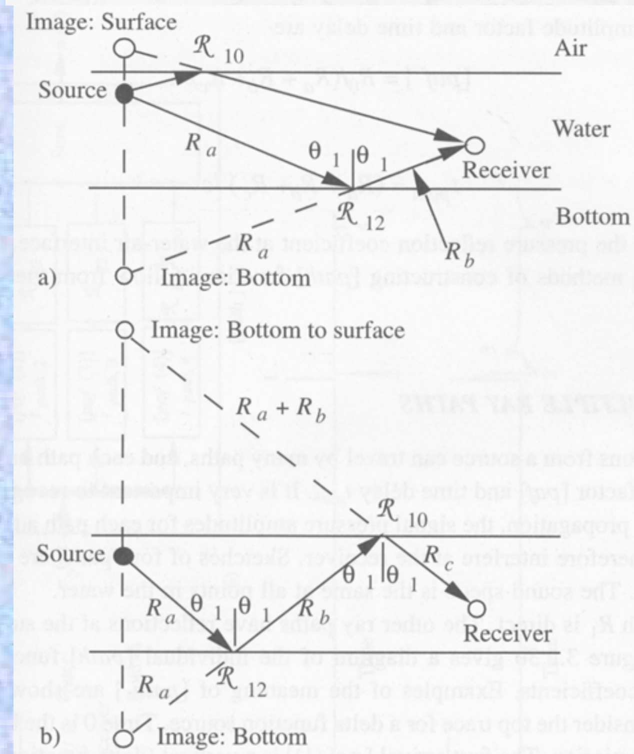
Reflection along a path:

Wave: reflection appears as if coming from an image source with $\theta_1=\theta_2$.

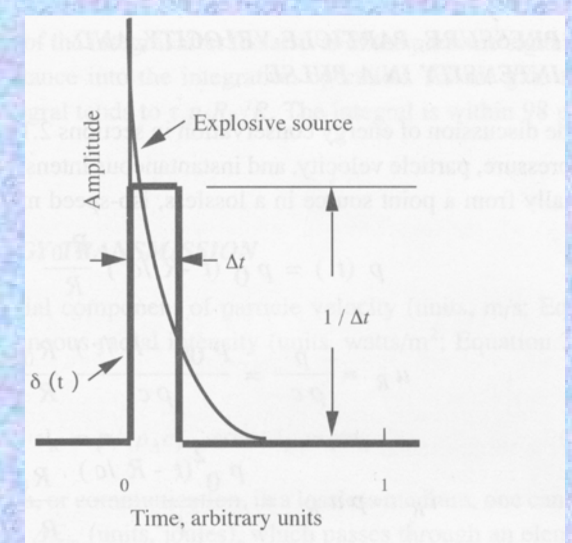


Ray: reflection appears as if coming from an image source with $\theta_1=\theta_2$.

Assumption: ray is reflected locally as if it were a plane wave.



Pulse (transient signal) source (as opposed to CW):



Delta function, definition:

$$\delta(t) = \begin{cases} 1 & t = 0 \\ 0 & t \neq 0 \end{cases} = \begin{cases} 1/\Delta t & \Delta t \geq t \geq 0 \\ 0 & \text{otherwise} \end{cases} \Bigg|_{\Delta t \to 0}$$

Delta function, property:

$$\int_{-\infty}^{\infty} g(t)\delta(t - t_1)dt = g(t_1)$$

Transient signal:

$$p_0(t) = 0 \quad t < 0$$

$$\int_0^{\infty} |p_0(t)| dt < \infty \quad t \geq R/c$$

Q: Examples of sounds in the ocean that are CW or that are pulses.

The energy passing through an area ΔS perpendicular to the sound rays, at distance R and for an interval time t_g following the first arrival is given by:

$$\Delta E_m = \Delta S \int_{\frac{R}{c}}^{\frac{R}{c}+t_g} pu\,dt = \frac{R_0^2 \Delta S}{\rho_w c} \int_{\frac{R}{c}}^{\frac{R}{c}+t_g} \frac{\left[p_0\left(t - R/c\right)\right]^2}{R^2} dt = \frac{R_0^2 \Delta \Omega}{\rho_w c} \int_{\frac{R}{c}}^{\frac{R}{c}+t_g} \left[p_0\left(t - R/c\right)\right]^2 dt$$

Where $\Delta\Omega$ is the spatial angle (=4π over all directions).

Remember:
Intensity= energy per unit time per unit area:

$$i_R\left(R,t\right) = pu = \frac{p_0^2\left(t - R/c\right)}{\rho_w c} \frac{R_0^2}{R^2}$$

Reflection along a path (assume homogeneous medium):

1. Source to boundary

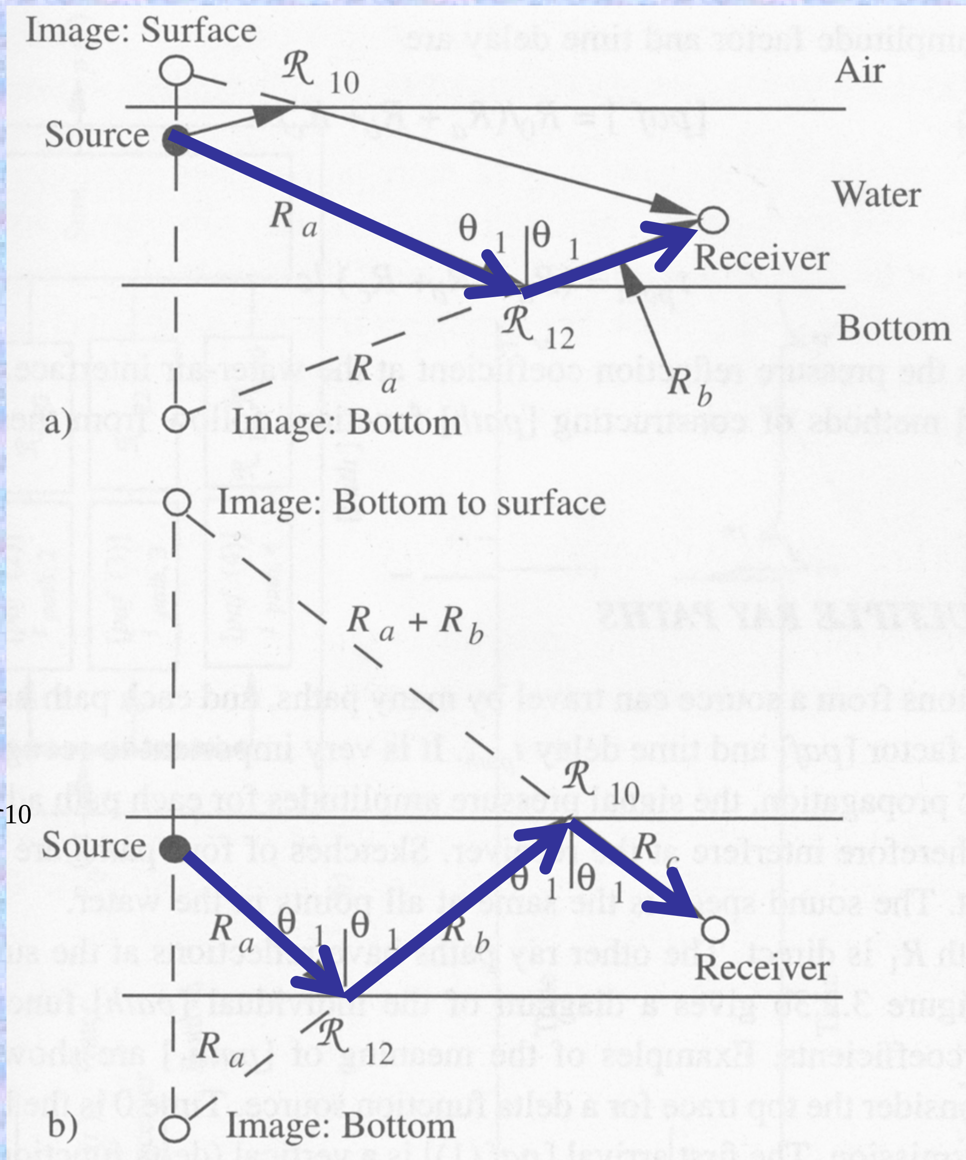
$$p(t) = p_0\left(t - \frac{R_a}{c}\right)\frac{R_0}{R_a}$$

2. Source to receiver ($t_{path}=(R_a+R_b)/c$)

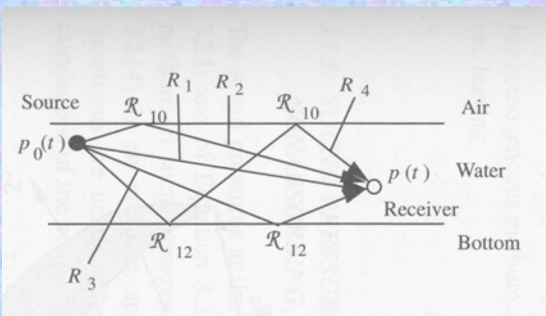
$$p(t) = p_0\left(t - \frac{R_a + R_b}{c}\right)\frac{R_0}{R_a + R_b}\mathbf{R}_{12}$$

3. Two reflections ($t_{path}=(R_a+R_b+R_c)/c$)

$$p(t) = p_0\left(t - \frac{R_a + R_b + R_c}{c}\right)\frac{R_0}{R_a + R_b + R_c}\mathbf{R}_{12}\mathbf{R}_{10}$$



Multiple paths:

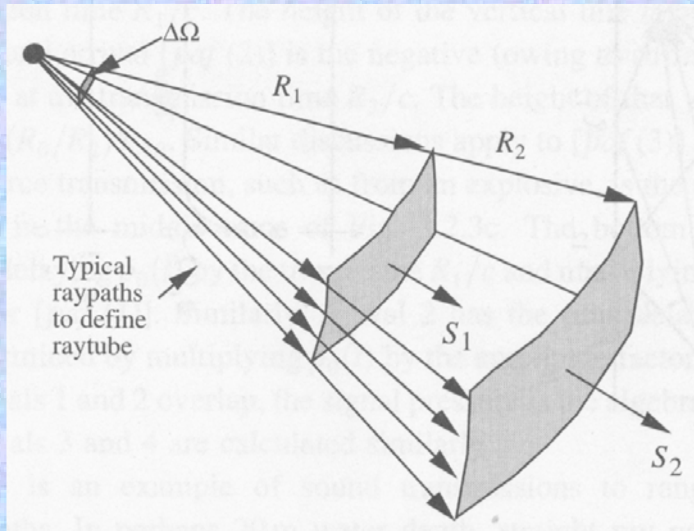


Explosive source

Pressure signal at receiver:

Note the reversal due to reflection at top boundary (R_{10}=-1)

Ray tubes:



Sound travel parallel to sound rays (by construction).

Thus, if a lossless medium, energy is conserved within a ray tube.

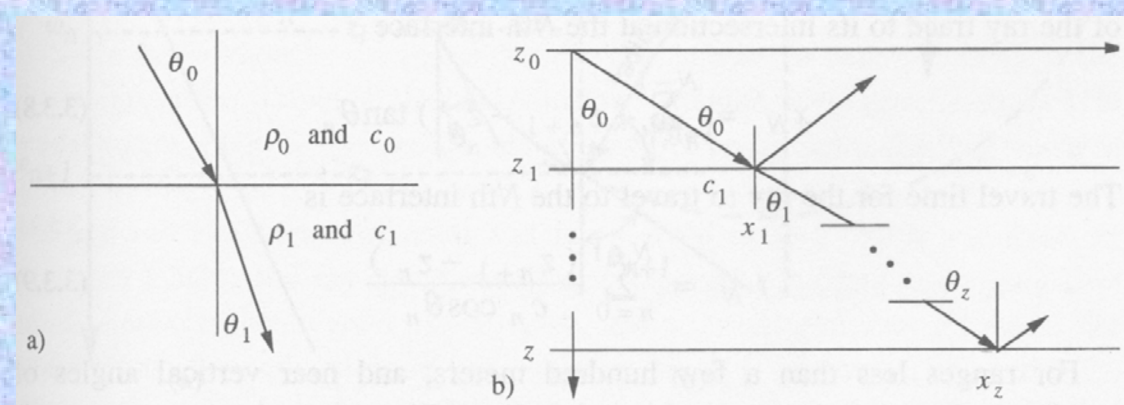
$$S_1 \int_0^{t_g} [p_1(t)]^2 \, dt = S_2 \int_0^{t_g} [p_2(t)]^2 \, dt$$

where t_g is defined such there is no sound for $t > t_g$.

Ray propagation in a lossless refractive medium

Snell's law:

$$\frac{\sin\theta_0}{c_0} = \frac{\sin\theta_1}{c_1} = \frac{\sin\theta_z}{c_z} = a$$



I. Ray through piecewise constant sound speed layers.

$$\frac{x_1 - x_0}{z_1 - z_0} = \tan\theta_0 = \frac{\sin\theta_0}{\cos\theta_0} = \frac{\sin\theta_0}{\sqrt{1-\sin\theta_0^2}} = \frac{ac_0}{\sqrt{1-(ac_0)^2}}$$

$$cdt = ds \rightarrow dt = \frac{dz}{c\cos\theta} \rightarrow t_1 - t_0 = \frac{z_1 - z_0}{c\cos\theta_0}$$

Note:

- When $c_n \sin\theta_{n-1}/c_{n-1} > 1$, total reflection occurs (only when passing from slow to fast).
- for distances < O(1000m) and $\theta_0 \sim 0$ refraction can be ignored.

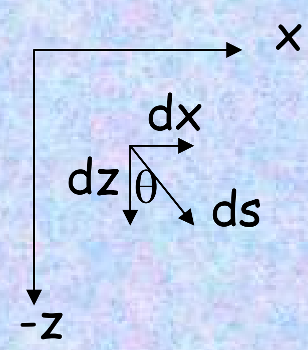
Ray propagation in a lossless refractive medium

II. Ray through an ocean with slowly changing sound speed (c(z)).

Let s denote distance in x-z plane and t time.

$$ds = \frac{dz}{\cos\theta} \quad \text{and} \quad dt = \frac{ds}{c(z)} = \frac{dz}{c(z)\cos\theta}$$

$$dx = dz \tan\theta$$



From Snell's law we can compute the horizontal distance:

$$\frac{dx}{dz} = \frac{ac(z)}{\sqrt{1-(ac(z))^2}} \rightarrow x - x_0 = \int_{z_0}^{z} \frac{ac(z)}{\sqrt{1-(ac(z))^2}} dz$$

And from above we can compute the time it gets to get there:

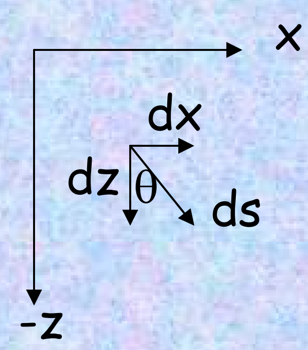
$$dt = \frac{dz}{c(z)\cos\theta} \rightarrow t - t_0 = \int_{z_0}^{z} \frac{1}{c(z)\sqrt{1-(ac(z))^2}} dz$$

Ray propagation in a lossless refractive medium

II. Ray through an ocean with slowly changing sound speed (c(z)).

$$\frac{dx}{dz} = \frac{ac(z)}{\sqrt{1-(ac(z))^2}} \rightarrow x - x_0 = \int_{z_0}^{z} \frac{ac(z)}{\sqrt{1-(ac(z))^2}} dz$$

$$dt = \frac{dz}{c(z)\cos\theta} \rightarrow t - t_0 = \int_{z_0}^{z} \frac{1}{c(z)\sqrt{1-(ac(z))^2}} dz$$



Turning point:

$$(ac(z))^2 = 1 \rightarrow ac(z) = 1$$

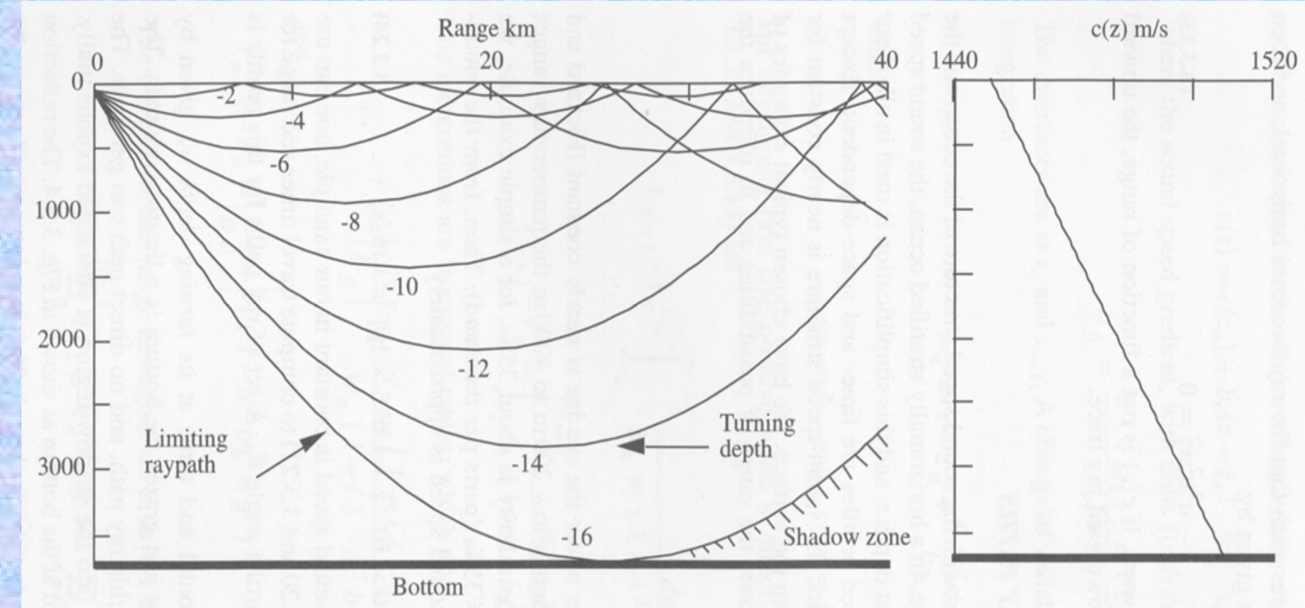
If c≠c(x) the upward ray path is the mirror image of the downward ray's path.

Example of ray paths in the oceans: Arctic Ocean

Arctic Ocean (linear sound speed):

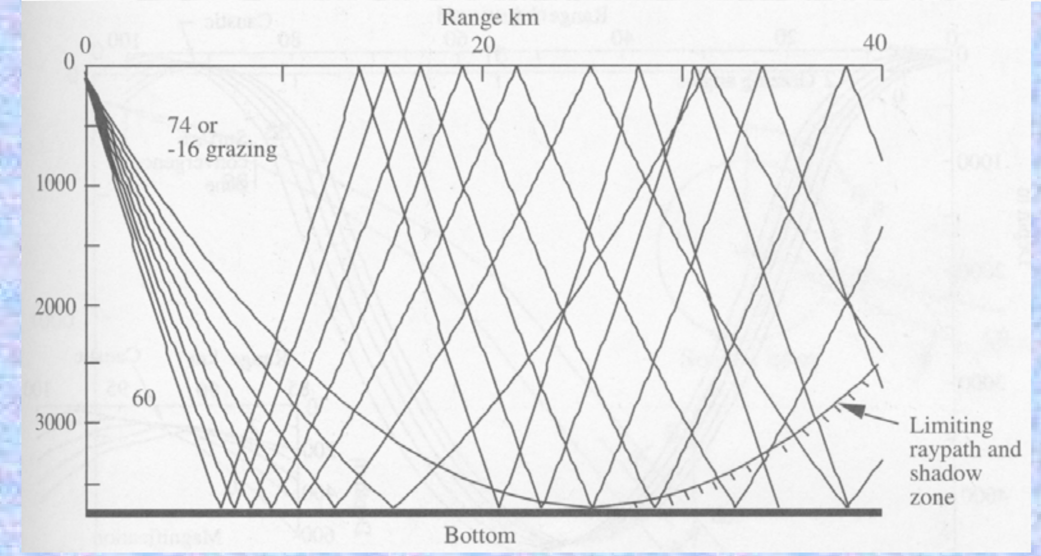
angles: $\phi=\theta-90^\circ$

Rays from $\phi=-2 \rightarrow -16$ all have turning points



1. Shadow zone get sound through multiple reflections

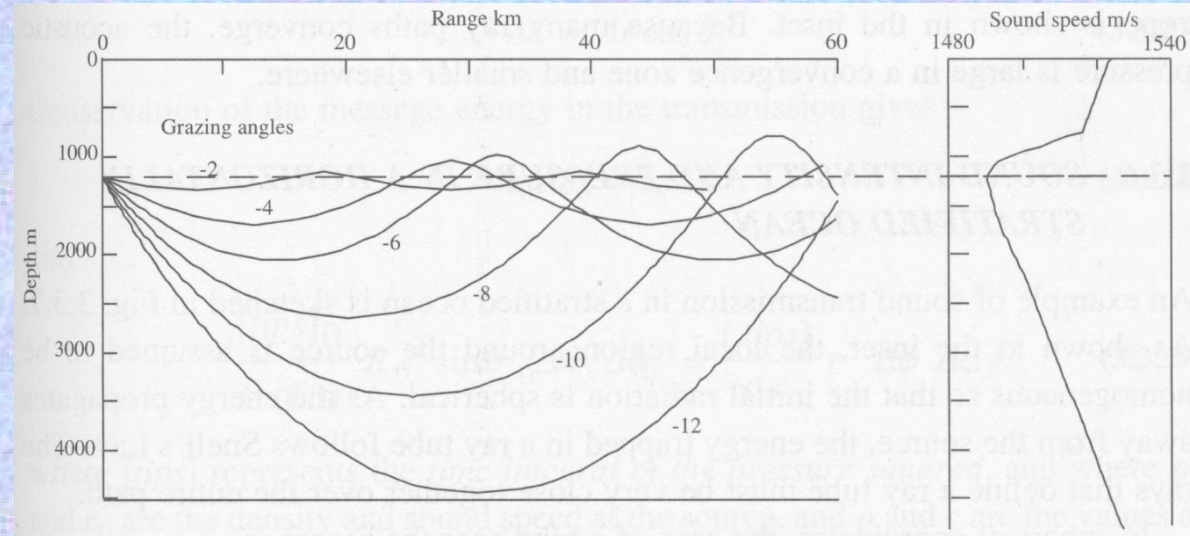
2. Steeper angles have straighter paths.



Example of ray paths in the oceans: North Atlantic

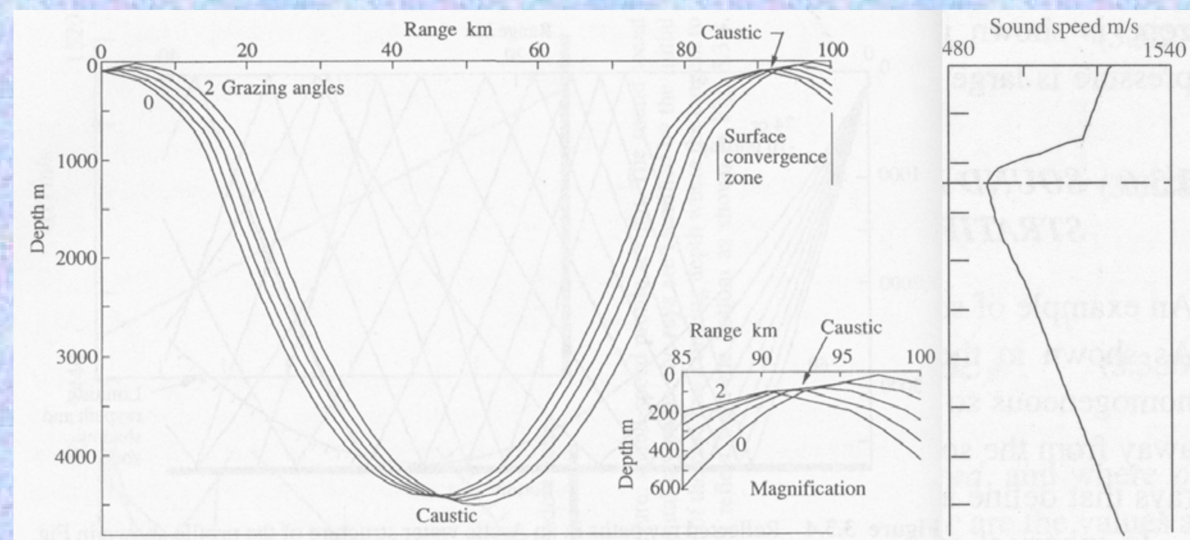
Minima in sound speed creates wave guide to well placed sources.

Rays can propagate far without reflections from boundaries.



1. Source near surface have very different path.

2. Two convergence zones (Caustics, where acoustic energy is focused for CW source) are present.



Transmission losses along a ray (TL):

Wave description of sound from a point source in an unbounded ocean:

$$p(t) = p_0 \cos\left\{2\pi f\left(t - \frac{R_a}{c}\right) + \phi\right\} \frac{R_0}{R_a} \exp(-\gamma R)$$

γ is the attenuation coefficient due to:

- Absorption due to pure water
- Absorption due to MgSO4 for f>100kHz (depends on S, z and T)
- Absorption due to B(OH)3 for f>1kHz (depends on S, z, T, and pH)

- Scattering by marine particles (when abundant enough and with ka~1)

If γ can be assumed constant along the ray:

$$dB \equiv 20\log\left(\frac{p}{P_{ref}}\right) = 20\log\left(\frac{p(R_0)_{rms}}{P_{ref}} \frac{1}{R} \exp(-\gamma R)\right) =$$

$$20\log\left(\frac{p(R_0)_{rms}}{P_{ref}}\right) - 20\log R - \alpha R \quad \alpha \equiv 20\gamma \log_{10} e \approx 8.686\gamma$$

Transmission losses along a ray (TL):

Transmission loss is then:

$$TL(R/R_0) = 20\log(R/R_0) + \alpha(R/R_0)$$

Note: It most often implicitly assumed that R_0=1m and thus only R appears in the TL term.

If γ varies along the ray:

$$dB \equiv 20\log\left(\frac{p}{P_{ref}}\right) = 20\log\left(\frac{p(R_0)_{rms}}{P_{ref}}\frac{1}{R}\exp\left(-\int_{R_0}^{R}\gamma dR\right)\right) =$$

$$20\log\left(\frac{p(R_0)_{rms}}{P_{ref}}\right) - 20\log R - 20\log\int_{R_0}^{R}\gamma dR$$