Introduction to Acoustical Oceanography

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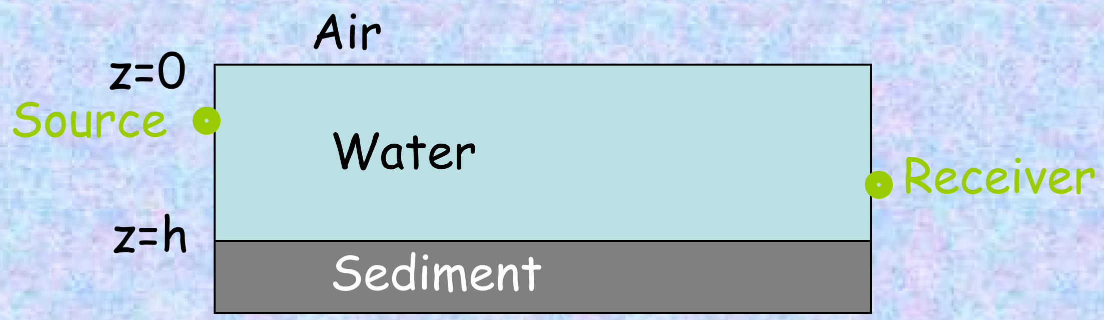
Request for chapters to cover:

Chapter 12, "Active acoustical assessment of plankton and micronekton".
Chapter 13, "Models, measures, and visualizations of fish backscatter".
Chapter 14, "Bioacoustic absorption spectroscopy: a new approach to monitoring the number and lengths of fish in the ocean".
Chapter 15, "Passive acoustics as a key ot the study of marine animals".
Chapter 16, "The acoustical causes of collisions between marine mammals and vessels".
Chapter 17, "Whale monitoring".
Chapter 19, "Acoustic time reversal in the ocean".

Ocean Waveguides – Normal modes

When we are interested in solutions for sound propagation and do not expect geometric acoustics to work well (e.g. shallow water, low frequency, when diffraction may be important). Resolution close to that of the wavelength is needed. CW problem.



Start with the wave equation:

$$\frac{\partial^2 p}{\partial r^2} + \frac{\partial p}{r \partial r} + \frac{\partial^2 p}{\partial z^2} = \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2}$$

Assume a solution in which we can separate the temporal vertical and horizontal dependencies:

$$p(r, z, t) = U(r) Z(z) T(t)$$

Substitute into the wave equation and divide by p:

$$\frac{1}{U(r)}\left(\frac{\partial^2 U}{\partial r^2}+\frac{\partial U}{r\partial r}\right)+\frac{1}{Z(z)}\frac{\partial^2 Z}{\partial z^2}=\frac{1}{c^2T(t)}\frac{\partial^2 T(t)}{\partial t^2}$$

Since every function depends on a different variable, they each are equal to a constant:

$$\frac{1}{U(r)}\left(\frac{\partial^2 U}{\partial r^2}+\frac{\partial U}{r\partial r}\right)=-\kappa^2$$

$$\frac{1}{Z(z)}\frac{\partial^2 Z}{\partial z^2}=-\gamma^2$$

Note: κ is not a function of z. k and γ are.

$$\frac{1}{c^2T(t)}\frac{\partial^2 T(t)}{\partial t^2}=-k^2$$

$$\kappa = k\sin\theta$$

$$\gamma = k\cos\theta$$

Indeed, for a continuous harmonic source $T(t) \sim e^{(i\omega t)}$:

$$\frac{1}{c^2T(t)}\frac{\partial^2 T(t)}{\partial t^2}=-\frac{\omega^2}{c^2}=-k^2 \;\rightarrow\; k^2=\gamma^2+\kappa^2$$

Solutions:

II. The radial dependence for wave spreading from the source and decaying at infinity:

$$\frac{\partial^2 U}{\partial r^2}+\frac{\partial U}{r\partial r}+\kappa^2 U=0\rightarrow U(r)=h_0^2(\kappa r)=j_0(\kappa r)-in_0(\kappa r)$$

The asymptotic behavior of which (as soon as $\kappa r>1$):

$$h_0^1(\kappa r)\underset{kr>1}{\rightarrow}\sqrt{\frac{2}{\pi\kappa r}}\exp\left(-i\left(\kappa r-\frac{\pi}{4}\right)\right)$$

Note: in 2-D cylindrical spread in a waveguide $p\propto 1/r^{0.5}$ while in 3-D spherical, $p\propto 1/r$.

Solutions:

II. The depth dependence (assuming constant sound speed):

$$\frac{\partial^2 Z}{\partial z^2} + \gamma^2 Z = 0 \rightarrow Z(z) = e^{i\gamma z}$$

BCs:

1. Pressure release at the top, $Z(z=0)=0$ (air-sea interface).
2. Pressure release at the bottom, $Z(z=h)=0$ (sea-sediment interface at angles steeper than critical.

$$Z_m(z) = \sin(\gamma_m z), \gamma_m = \frac{m\pi}{h}, m = 1,2,...$$

Where m is the mode number, γ_m the eigenvalue, and $Z_m(z)$ the eigenmode.

For the modes to propagate horizontally (why?):

$$\kappa_m = \sqrt{k^2 - \gamma_m^2} \geq 0$$

The low modes propagate while the higher modes attenuate close to the source.

Solutions:

II. The depth dependence.

Solution, far enough from source:

$$p(r,z,t;z_s) = A_s \sum_{m=1}^{N_m} \frac{Z_m(z_s)Z_m(z)}{\sqrt{\kappa_m r}} e^{i(\omega t - \kappa_m r + \pi/4)}$$

More general BCs:

1. Pressure release at the top, $Z(z=0)=0$ (air-sea interface).
2. $F\{Z(z=h), dZ/dz(z=h)\}=0$ (sediment bottom interface)

When $c=c(z,r)$ we can still solve the equations numerically with the appropriate BCs.

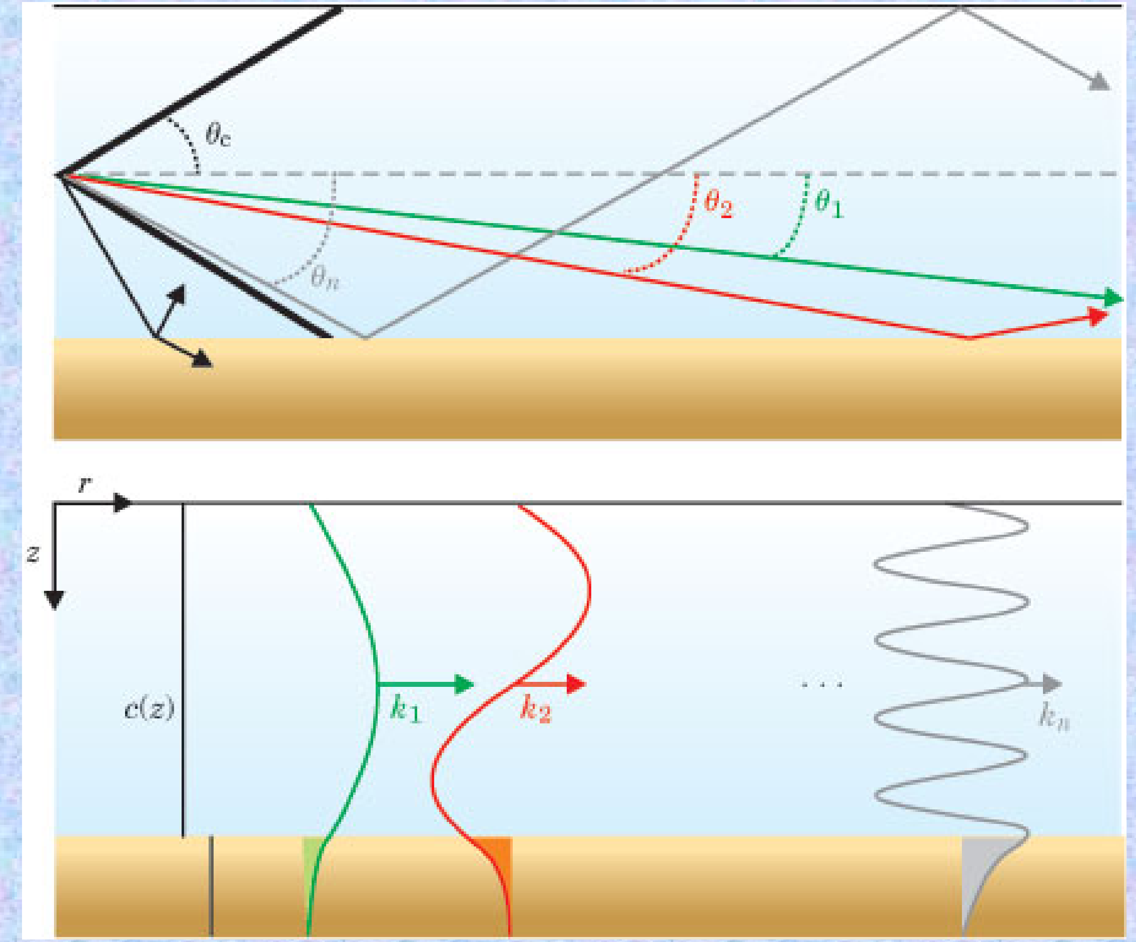
The number of nondecaying mode $N_m \sim 2h/\lambda$.

If $N_m > 50$ use ray techniques.

In the general case, some modes penetrate the sediment and 'leak' energy into it. Some parameterize it as an additional attenuation.

Normal Modes in Shallow Water:

The canonical (Pekeris) shallow-water acoustic waveguide has a constant sound speed, mirror reflection at the surface, and a grazing-angle-dependent reflectivity at the ocean bottom. The interface has a critical angle θ_c—typically about 15°, depending on the material there.



As shown in the upper panel of the figure, a source in such a waveguide produces a sound field that propagates at angles confined to a cone of $2\theta c$. Within that cone, constructive interference selects discrete propagation angles; outside the cone, waves disappear into the bottom after a few reflections.

From Kuperman and Lynch, Shallow water acoustics, October 2004, Physics Today.

The modes propagate at different speeds (not necessarily the sound speed).

Phase: $\phi_m = \omega t - \kappa_m r + \pi / 4$

A line of constant phase travels at speed (the phase speed):

$$c_{p,m} = r / t = \omega / \kappa_m$$

Energy travels at the group speed:

$$c_{g,m} = d\omega / d\kappa_m$$

Group velocity and phase velocity changes with mode → dispersion of waves of different frequencies.

Downslope propagation of normal modes in a shallow water wedge

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~500λ long, 5λ deep (80kHz), same as 80Hz H=100m and L=10km.

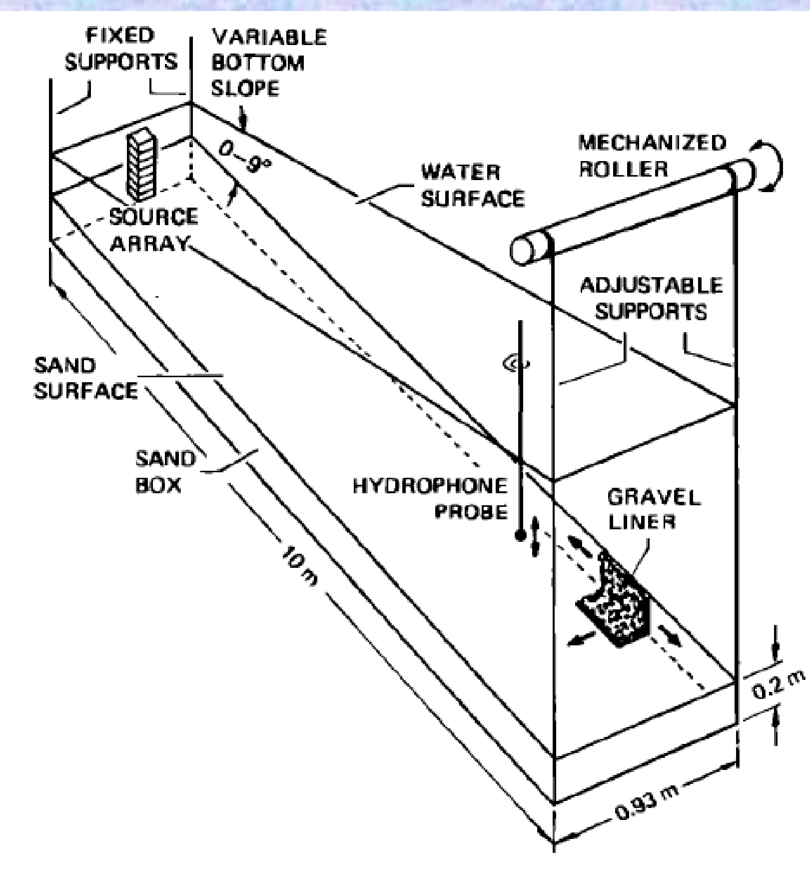

FIG. 1. Experimental arrangement.

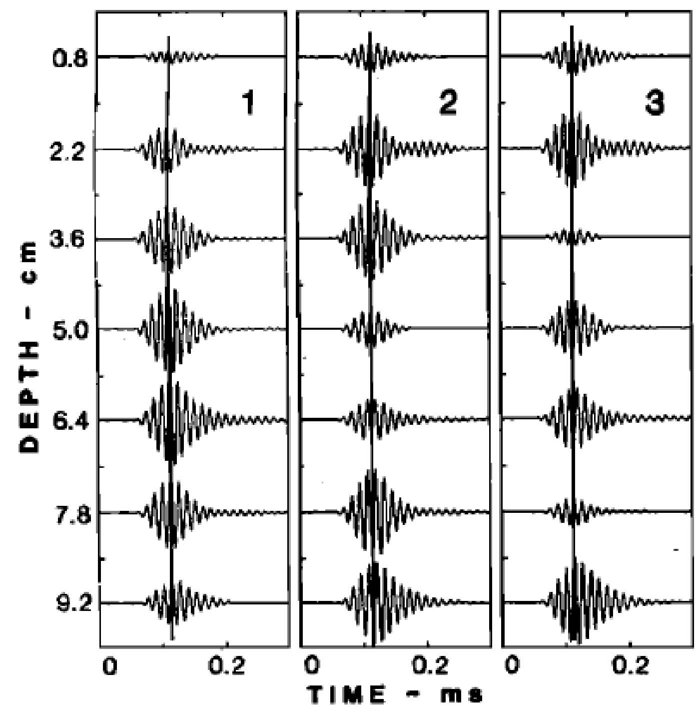

FIG. 2. Source waveforms for modes 1–3. The waveforms were recorded singly in free-field conditions. When used in the waveguide, the sources are at the depths indicated.

Adiabatic approximation: normal modes adapt to slight changes in depth without coupling.

Tindle et al., 1987

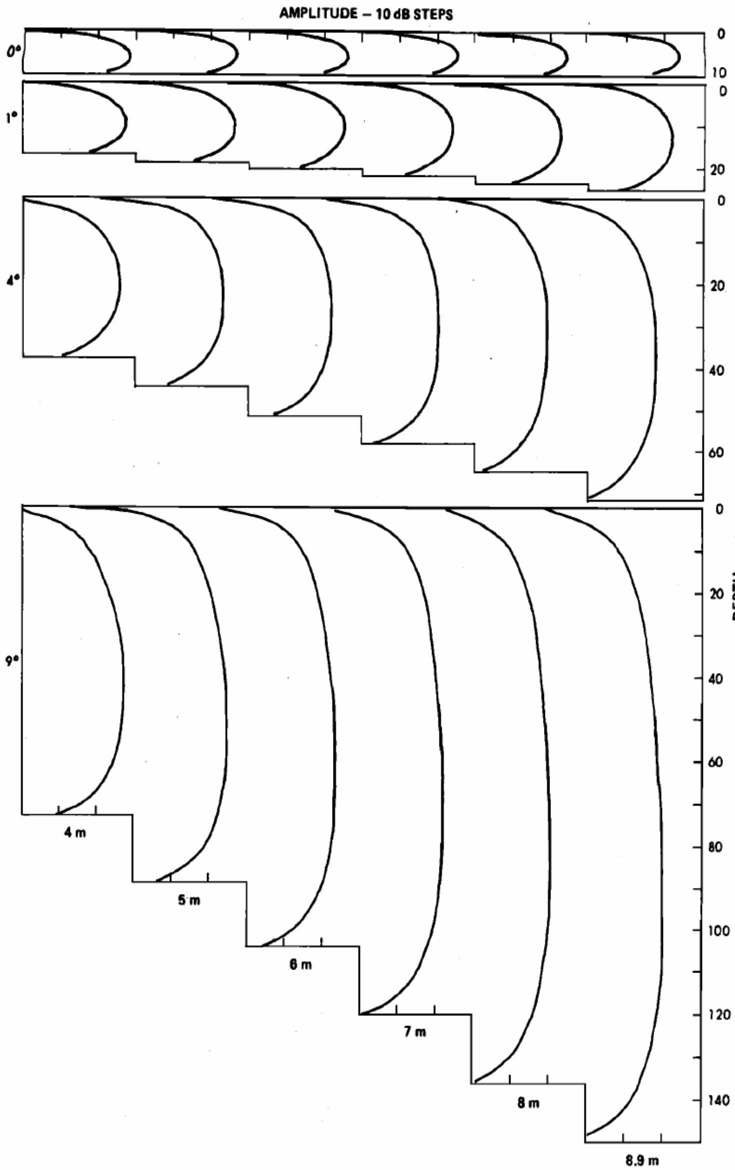

FIG. 4. Analog records of mode 1 amplitude as a function of depth at the slopes and ranges shown. The source array is at zero range in 10-cm water depth.

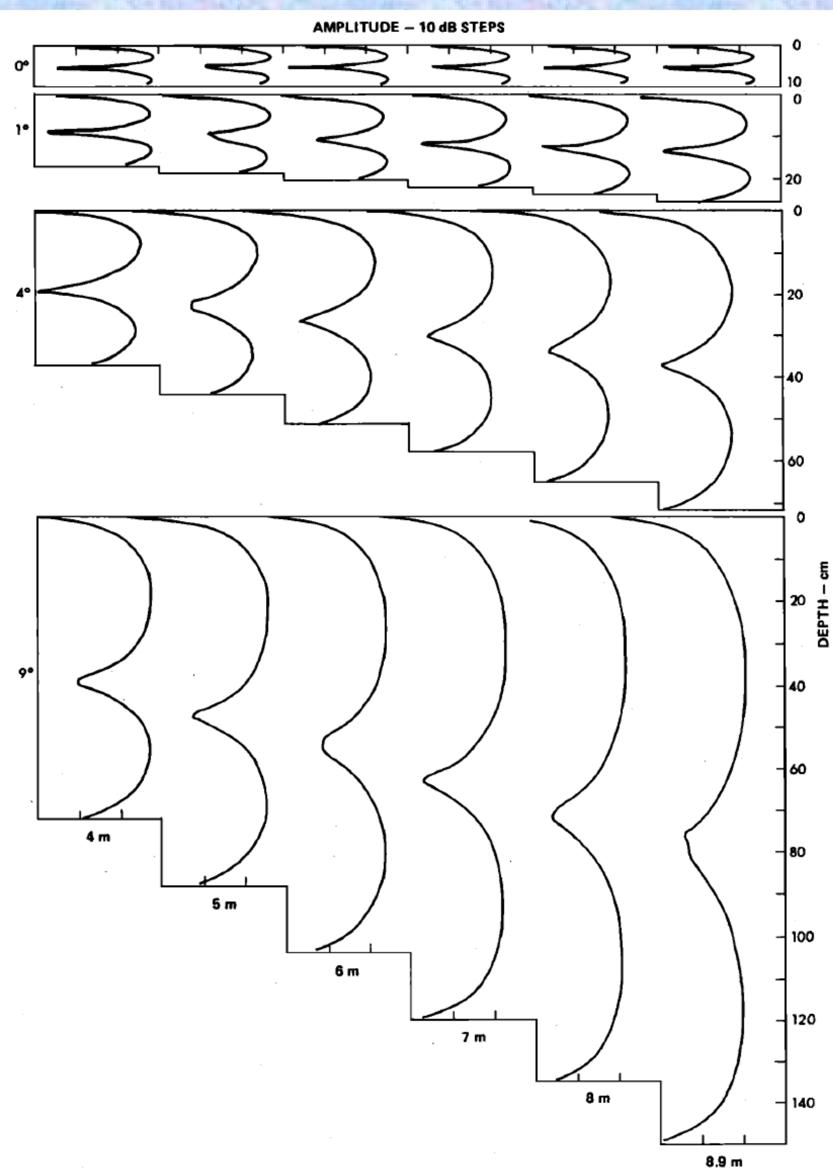

FIG. 5. Analog records of mode 2 amplitude as a function of depth at the slopes and ranges shown. The source array is at zero range in 10-cm water depth.

Tindle et al., 1987

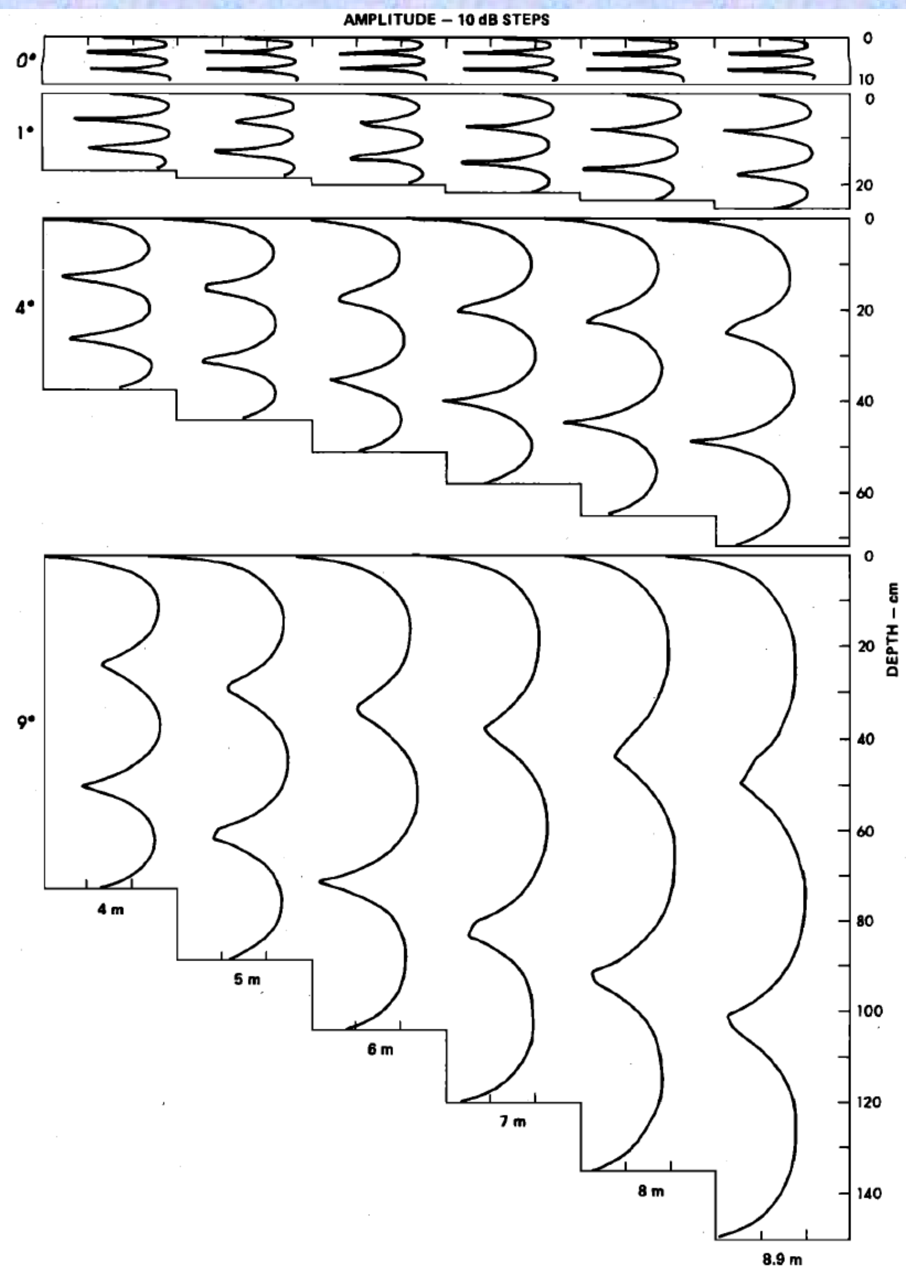

FIG. 6. Analog records of mode 3 amplitude as a function of depth at the slopes and ranges shown. The source array is at zero range in 10-cm water depth.

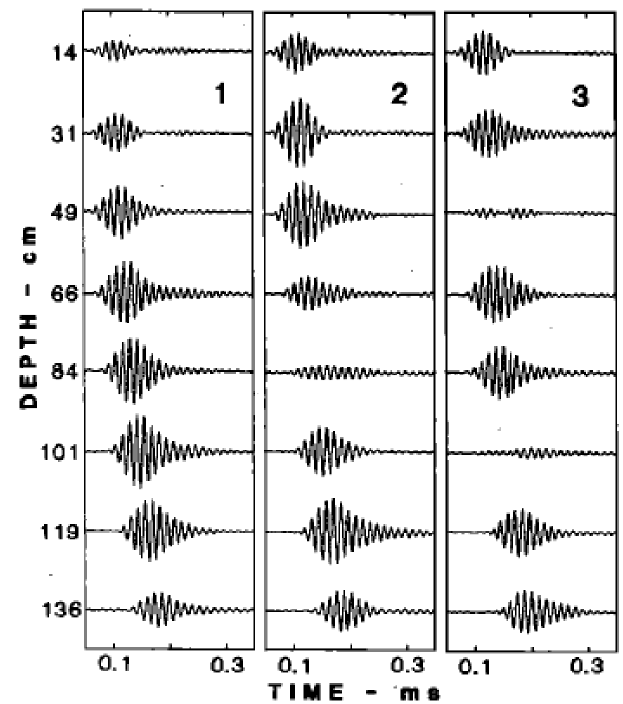

FIG. 8. Waveforms of modes 1–3 at the depths shown. The range is 8.9 m; the bottom slope is 9°. The water depth at the source array is 10 cm.

Tindle et al., 1987

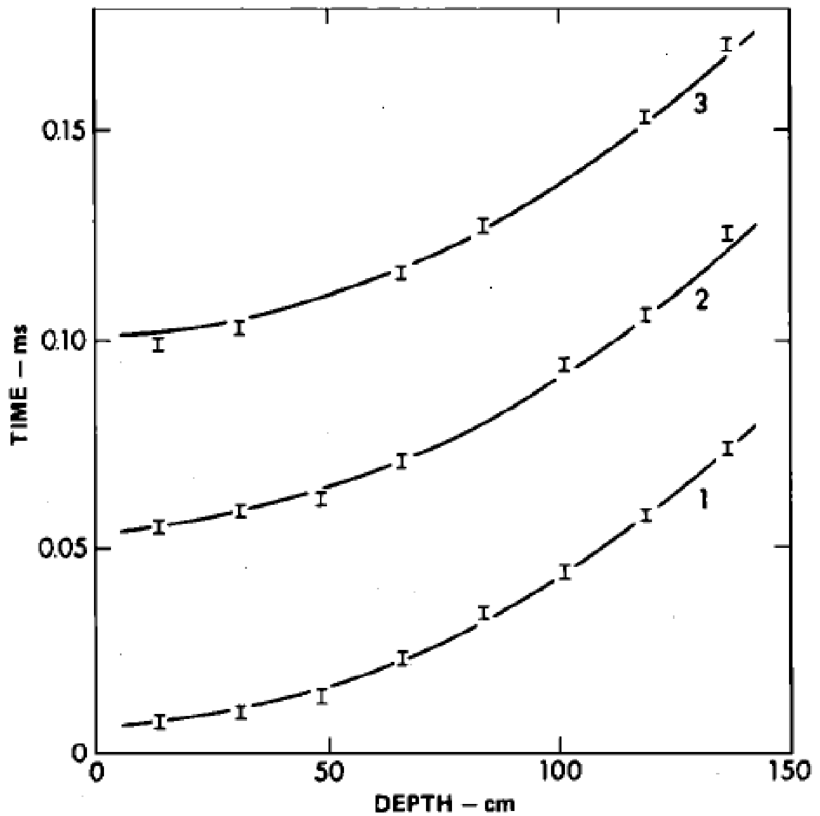

FIG. 9. Arrival time as a function of depth for modes 1–3. The modes are offset 0.04 ms for clarity. The solid curves are theoretical results assuming curved wave fronts centered on the wedge apex.

The inverse problem; Reconstructing the source from the signal:

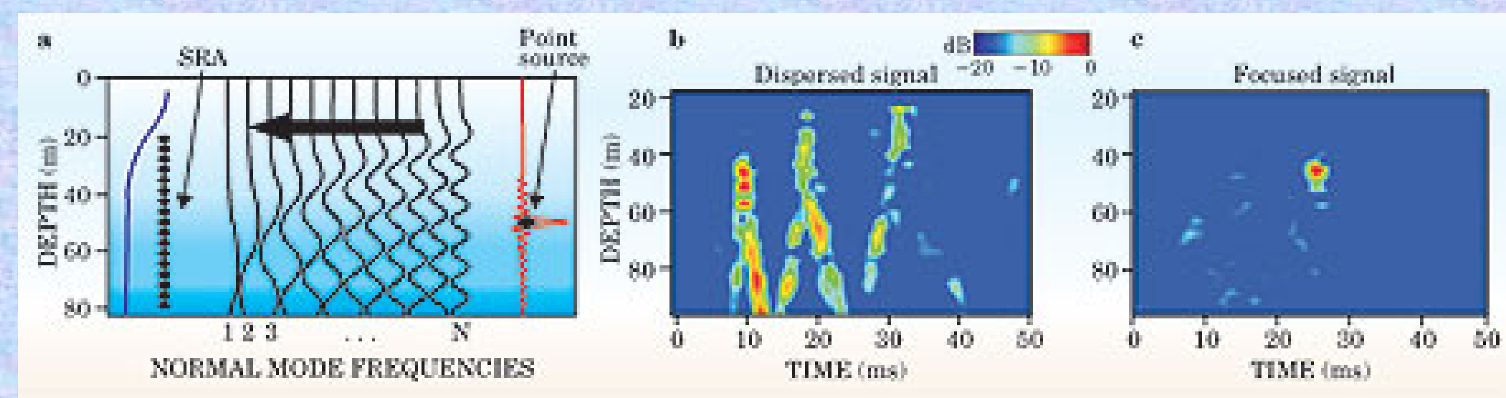

Figure 2. Shallow-water waveguide. (a) A point source launches a 2-millisecond acoustic pulse that excites a series of normal modes that propagate in an ocean with a summer sound-speed profile (the purple line). The lower modes are trapped below the thermocline. **(b)** Because of modal dispersion, the signal arrives at the source-receiver array (SRA)—8 km from where it started—with more than a 40 ms spread. **(c)** Time reversal of the pulse at the SRA (retransmitting the last arrival first, and so on, back through the waveguide) produces a recompressed focus at the original point source position. The pulse's focal size is commensurate with the shortest wavelength of the highest surviving mode. (S. Kim et al., *J. Acoust. Soc. Am.* **114**, 145, 2003)

From Kuperman and Lynch, Shallow water acoustics, October 2004, Physics Today.