Particle dynamics class, SMS 618, (Emmanuel Boss 11/19/2003)
Van Rijn's TRANSPOR lab: computation of sediment transport in current and wave direction.

Handout: Appendix A in van Rijn, 1993, Principles of sediment transport in rivers, estuaries, and coastal seas, Aqua Publications.

Note: in this write-up log is $\log_{10}$, while ln is $\log_e$.

Inputs (see Fig. A.1 in appendix for directions):
h-water depth. [m]
$\bar{v}_r$ - depth-averaged velocity in main current direction. [m/s]
$\bar{u}_r$ - time-averaged and depth-averaged return velocity below wave trough compensating the mass transport between wave crest and trough. [m/s]
u_b- time-averaged near-bed velocity due to waves, wind or lateral density gradient. [m/s]
Hs- significant wave height (significant wave height, Hs, is approximately equal to the average of the highest one-third of the waves. Hs is calculated using: Hs = 4.0 * sqrt(m_0) where m_0 is the variance of the wave displacement time series acquired during the wave acquisition period. Variance can also be calculated using the nondirectional wave spectrum according to the following relationship: m_0 = sum(S(f)*df) where the summation of spectral density, S(f), is over all frequency bands, f, of the nondirectional wave spectrum and df is the bandwidth of each band. Wave analysis systems typically sum over the range from 0.03 to 0.40 Hz with frequency bandwidths of 0.01 Hz. (see http://www.ndbc.noaa.gov/wavecalc.shtml). [m]
Tp- Dominant, or peak, wave period. (the period corresponding to the frequency band with the maximum value of spectral density in the nondirectional wave spectrum. It is the reciprocal of the peak frequency, fp: Tp = 1/fp. Dominant period is representative of the higher waves encountered during the wave sampling period). [s]
ϕ-angle between the directions of waves propagation and main current direction. [°]
d_{50}- median diameter of bed material. [m]
d_{90}- 90th percentile diameter of bed material. [m]
d_s-representative diameter of suspended material (will be in the range of (0.6→1) d_{50}). [m]
$k_{s,c}$- current-related bed roughness height (minimum 0.01m). (likely range 0.01→1m). [m]
$k_{s,w}$- wave-related bed roughness height (minimum 0.01m) (likely range 0.01→1m, for sheet flow, 0.01m). [m]
Te- temperature of fluid. [°C]
SA- salinity of fluid. [ppt]

Assumptions:
ρ_s=2650kg/m^3 – sediment's density.
κ=0.4 – Von Karman's constant.

<u>General parameters computed:</u>
Chlorinity: CL=(SA-0.03)/1.805
Fluid density: $\rho=1000+1.455CL-0.0065(Te-4+0.4CL)^2$ [Kg/m^3]
Kinematic viscosity: $\nu=(4/(20+Te))10^{-5}$ [m^2/s]

Sinking velocity: $w_s = \begin{cases} \dfrac{(s-1)gd^2}{18\nu} & 1 < d < 100\mu m \\ \dfrac{10\nu}{d}\left[\sqrt{1+\dfrac{0.01(s-1)gd^3}{\nu^2}}-1\right] & 100 < d < 1000\mu m \\ 1.1\sqrt{(s-1)gd} & 1000\mu m < d \end{cases}$; s= ρ_s/ρ

<u>Sediment characteristics computed:</u>
s= ρ_s/ρ – relative density.
$D_* = d_{50}[(s-1)g/\nu^2]^{1/3}$ - Particle parameter.

Shields parameter: $\theta_{cr} = \begin{cases} 0.24D_*^{-1} & 1 < D_* < 4 \\ 0.14D_*^{-0.64} & 4 < D_* < 10 \\ 0.04D_*^{-0.1} & 10 < D_* < 20 \\ 0.013D_*^{0.29} & 20 < D_* < 150 \\ 0.055 & 150 < D_* \end{cases}$

$\tau_{cr} = (\rho_s - \rho)gd_{50}\theta_{cr}$ – critical shear stress.
$\bar{u}_{cr} = 5.75\sqrt{(s-1)gd_{50}}\sqrt{\theta_{cr}}\log(4h/d_{90})$ – Critical depth-averaged velocity.

$\hat{U}_{cr} = \begin{cases} \left(0.12(s-1)g\sqrt{d_{50}}\sqrt{T_p}\right)^{2/3} & d_{50} < 0.0005m \\ \left(1.09(s-1)gd^{0.75}{}_{50}\sqrt{T_p}\right)^{0.571} & 0.0005m < d_{50} \end{cases}$ -- Critical peak orbital velocity.

<u>Wave parameter computed</u>
L'–Doppler shifted wavelength (change in wavelength due to presence of mean flow):
find L', the solution of: $\left[\dfrac{L'}{T_p} - \bar{v}_R\cos\phi\right]^2 = \left[\dfrac{gL'}{2\pi}\right]\tanh\left[\dfrac{2\pi h}{L'}\right]$.

T_p'- Doppler shifted wave period: $T_p' = \dfrac{T_p}{1-(\bar{v}_R T_p \cos\phi)/L'}$.

Near-bed peak orbital excursion: $\hat{A}_\delta = \dfrac{H_s}{2\sinh(2\pi h/L')}$.

Near-bed peak orbital velocity: $\hat{U}_\delta = \dfrac{\pi H_s}{T'_p\, 2\sinh(2\pi h/L')}$.

Wave-boundary layer thickness: $\delta_w = 0.072\hat{A}_\delta\left(\hat{A}_\delta/k_{s,w}\right)^{0.25}$

Near-bed peak orbital velocity in forward direction:

$$\hat{U}_{\delta,f} = \begin{cases} \hat{U}_\delta + \dfrac{3\pi^2 H_s^2}{4T_p' L' \left(\sinh\left(2\pi h/L'\right)\right)^4} & 0.01gT_p^2 < h \\ \left(1+0.3\dfrac{H_s}{h}\right)\hat{U}_\delta & 0.01gT_p^2 > h \end{cases}.$$

Near-bed peak orbital velocity in backward direction:

$$\hat{U}_{\delta,b} = \begin{cases} \hat{U}_\delta - \dfrac{3\pi^2 H_s^2}{4T_p' L' \left(\sinh\left(2\pi h/L'\right)\right)^4} & 0.01gT_p^2 < h \\ \left(1-0.3\dfrac{H_s}{h}\right)\hat{U}_\delta & 0.01gT_p^2 > h \end{cases}.$$

Return velocity mass transport: $\overline{u}_r = -\dfrac{0.125\sqrt{g}H_s^2}{\sqrt{h}\left(0.95h-0.35H_s\right)}$.

Near bed wave induced mean velocity: $u_b = \left(0.05-0.5\dfrac{\hat{U}_{\delta,f}-\hat{U}_{\delta,b}}{\hat{U}_{\delta,f}+\hat{U}_{\delta,b}}\right)\hat{U}_\delta$

The last two are input parameter. To have them computed by the program insert 9 at the input line.

<u>Bed parameters computed:</u>

Apparent bed roughness:

$$k_a = \begin{cases} k_{s,c}\exp\left[\gamma\dfrac{\hat{U}_\delta}{\sqrt{\left(v_R^2+u_R^2\right)}}\right] & k_a < 10k_{s,c} \\ 10k_{s,c} & \text{otherwise} \end{cases}, \quad \gamma=0.8+\beta-0.3\beta^2,\ \beta = \frac{2\pi}{360^\circ}\phi \quad .$$

<u>Effective time-averaged bed-shear stresses computed:</u>

Efficiency factor for current: $\mu_c = \left\{\dfrac{\log\left(12h/3d_{90}\right)}{\log\left(12h/k_{s,c}\right)}\right\}^2$.

Efficiency factor for waves: $\mu_w = \dfrac{\exp\left[-6+5.2\left(\hat{A}_\delta/3d_{90}\right)^{-0.19}\right]}{\exp\left[-6+5.2\left(\hat{A}_\delta/k_{s,w}\right)^{-0.19}\right]}$.

Wave-current interaction coefficient: $\alpha_{cw} = \left[\dfrac{\ln\left(90\,\delta_w/k_a\right)}{\ln\left(90\,\delta_w/k_{s,c}\right)}\right]^2\left[\dfrac{-1+\ln\left(30\,h/k_{s,c}\right)}{-1+\ln\left(30\,h/k_a\right)}\right]^2 \le 1$.

Bed shear stress due to current: $\tau_c = \frac{1}{8}\rho\left(\frac{0.24}{\left(\log\left(12h/k_{s,c}\right)\right)^2}\right)\left(\overline{v}_R^2 + \overline{u}_R^2\right)$

Bed shear stress due to waves: $\tau_w = \frac{1}{8}\rho\left(\exp\left[-6+5.2\left(\hat{A}_\delta/k_{s,w}\right)^{-0.19}\right]\right)\hat{U}_\delta^2$

The total shear is the sum of the shears: $\tau_{wc} = \tau_c + \tau_w$.

Effective bed shear velocity current: $u'_{*,c} = \sqrt{\left(\alpha_{cw}\mu_c\right)\tau_c/\rho}$

<u>Bed shear stress parameters computations:</u>

Dimensionless bed-shear stress for bed load transport: $T = \frac{\left(\alpha_{cw}\mu_c\tau_c + \mu_w\tau_w\right) - \tau_{cr}}{\tau_{cr}}$.

Dimensionless bed-shear stress for reference concentration at z=a:

$$T_a = \frac{\left(\alpha_{cw}\mu_c\tau_c + \left(0.6/D_*\right)\tau_w\right) - \tau_{cr}}{\tau_{cr}}.$$

Both are zero if negative.

<u>Current velocity profile computations:</u>

$$\left(u_{R,z}, v_{R,z}\right) = \begin{cases} \dfrac{\left(\overline{u}_R, \overline{v}_R\right)\ln\left[30\,z/k_a\right]}{-1+\ln\left[30\,h/k_a\right]} & z \geq 3\delta_w \\ \dfrac{\left(\overline{u}_R, \overline{v}_R\right)\ln\left[90\,\delta_w/k_a\right]}{-1+\ln\left[30\,h/k_a\right]}\dfrac{\ln\left[30\,z/k_{s.c}\right]}{\ln\left[90\,\delta_w/k_{s.c}\right]} & z < 3\delta_w \end{cases}$$

<u>Sediment eddy diffusion profile computations:</u>

Eddy diffusion due to mean current:

$$\varepsilon_{s,c} = \begin{cases} \kappa\beta u_{*,c} z\left(1 - z/h\right) & z < 0.5h \\ 0.25\kappa\beta u_{*,c} h & z \geq 0.5h \end{cases};$$

where: $\beta = \begin{cases} 1+2\left(w_s/u_{*,c}\right) & \beta < 1.5 \\ 1.5 & otherwise \end{cases}$; $u_{*,c} = \dfrac{\sqrt{g\left(\overline{u}_r^2 + \overline{v}_r^2\right)}}{18\log\left(4h/d_{90}\right)}$

Eddy diffusion due to waves:

$$\varepsilon_{s,w} = \begin{cases} \varepsilon_{s,bed} = 0.004 D_* \delta_s \hat{U}_\delta & z < \delta_s \\ \varepsilon_{s,\max} = 0.035h\,H_s/T_p & z \geq 0.5h \\ \varepsilon_{s,bed} + \left[\varepsilon_{s,\max} - \varepsilon_{s,bed}\right]\left[\dfrac{z-\delta_s}{0.5h-\delta_s}\right] & \delta_s < z < 0.5h \end{cases};$$

$\delta_s = 0.3\sqrt{\mathrm{H_s} h}$ but $\delta_{s,\max} = 0.2\text{m}$ & $\delta_{s,\min} = 0.05\text{m}$

Combined current and wave eddy diffusion: $\varepsilon_{s,wc} = [\varepsilon^2_{s,c} + \varepsilon^2_{s,w}]^{0.5}$.

Sediment volume concentration computations:

Reference level: a=maximum($k_{s,c}$, $k_{s,w}$).

Bed concentration (for z≤a): $C_a = 0.015 \frac{d_{50}}{a} \frac{T_a^{1.5}}{D_*^{0.3}}$.

For z≥a the concentration gradient is give by:

$$\frac{dC}{dz} = -\frac{(1-c)^5 c w_s}{\varepsilon_{s,cw}\left(1+(c/0.65)^{0.8}-2(c/0.65)^{0.4}\right)}.$$

Obtain C by integration to the surface from a (where concentration is known) to h.

Time averaged *suspended* load transport computations:

$$q_s = \begin{cases} \rho_s \int_a^h v_{R,z} C dz & \text{current direction} \\ \rho_s \int_a^h u_{R,z} C dz & \text{wave direction} \end{cases}$$

Time averaged and instantaneous *bed* load transport rate computations:

Velocities are computed at δ=max(3δ_w,$k_{s,c}$): $\begin{matrix} v_{R,\delta} \\ u_{R,\delta} \end{matrix} = \begin{matrix} \dfrac{\bar{v}_R \ln[30\delta/k_a]}{-1+\ln[30h/k_a]} \\ \left(\dfrac{\bar{u}_r}{\bar{v}_r}\right)\dfrac{\bar{v}_R \ln[30\delta/k_a]}{-1+\ln[30h/k_a]} \end{matrix}$.

Instantaneous wave velocity: $U_\delta = \hat{U}_\delta \cos(\varphi)$, φ is the phase of the wave.

Instantaneous velocity along current: $U_{\delta,x} = U_\delta \cos\phi + v_{R,\delta} + (u_b + u_{R,\delta})\cos\phi$.

Instantaneous velocity across current: $U_{\delta,y} = U_\delta \sin\phi + (u_b + u_{R,\delta})\sin\phi$.

Instantaneous velocity: $U_{\delta,R} = \sqrt{U_{\delta,x}^2 + U_{\delta,y}^2}$

Instantaneous friction coefficient: $\alpha = \frac{|v_{R,\delta}|}{|v_{R,\delta}| + |\hat{U}_\delta|}$.

Instantaneous bed shear stress:

$$\tau'_{b,cw} = 0.5\rho\left\{0.25\alpha\left[\frac{-1+\ln(30h/k_{s,c})}{\ln(30h/k_{s,c})}\right]^2 \frac{0.24}{(\log(4h/d_{90}))^2} + (1-\alpha)\exp\left[-6+5.2\left(\hat{A}_\delta/3d_{90}\right)^{-0.19}\right]\right\}U_{\delta,R}^2$$

Instantaneous bed-load transport:

$$q_b = \begin{cases} 0.25\gamma\rho_s d_{50} D_*^{-0.3}\sqrt{\dfrac{\tau'_{b,cw}}{\rho}}\left[\dfrac{\tau'_{b,cw}-\tau_{b,cr}}{\tau_{b,cr}}\right] & \tau'_{b,cw} > \tau_{b,cr} \\ 0 & \text{otherwise} \end{cases}, \quad \gamma = \max\left(0.3, 1-\sqrt{\frac{H_S}{h}}\right)$$

Time averaged values are obtained by averaging over a wave period (e.g. over its phase, φ).

Bed form calculations:
This is beyond the scope of our class.

Class assignment: How waves affect the total transport.

Use the TRANSPOR model with the following inputs (maybe appropriate to the Dock in front of the Darling Marine center):

h=12m
d_{50}=150µm
d_{90}=400µm
d_s=0.8 d_{50}.
Te=12°C
SA=30psu
$\bar{u}_r = 0$
$\bar{v}_r = 0.5 m/\mathrm{s}$
$k_{s,c} = k_{s,w} = 0.02$

How do you expect the transport to change with wave parameters (Hs, Tp and ϕ) compared to having no waves? Write down your predictions.

Investigate how the total transport varies with changes in the significant wave height (0→2m), wave period (1→8s), and angle (0→90) between wave and current.

How do the results compare with prediction?

How do they (qualitatively) compare with van-Rijn calculations attached to the handout?

For one case, vary d_s from 0.6d_{50} to d_{50}. How would you expect the total transport to change? How much change do you observe in the total transport?