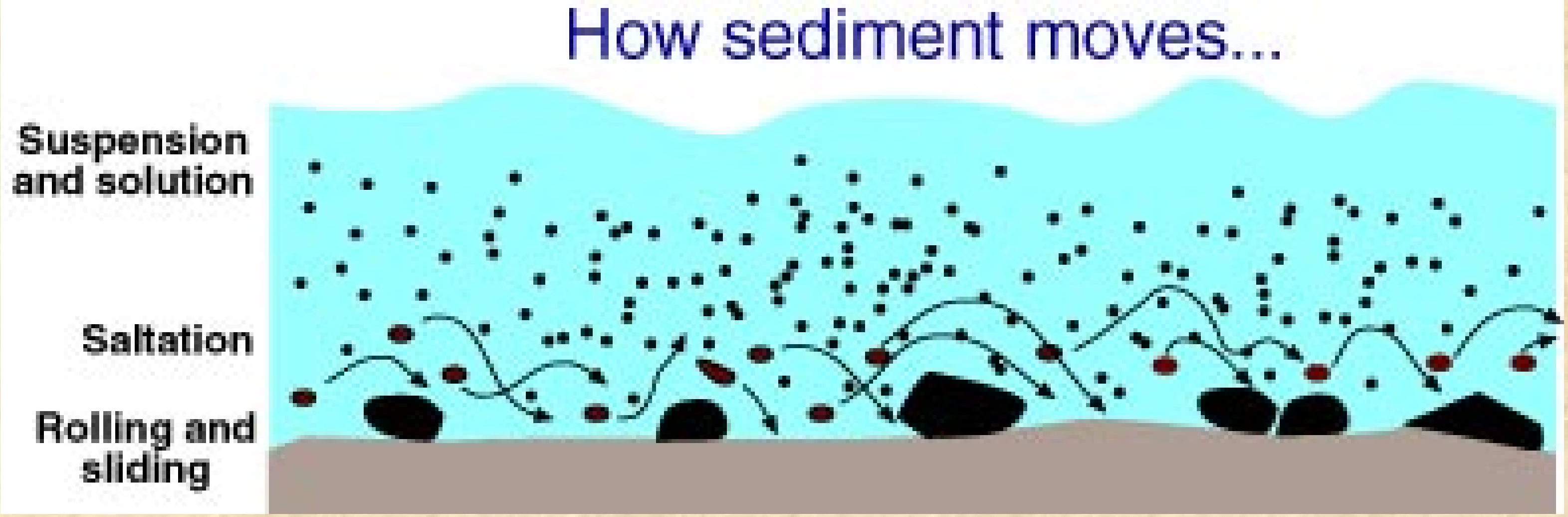
Modeling of suspended sediment transport



From: http://www.usask.ca/geology/classes/geol243/243notes/243week3b.html

What is sediment transport?

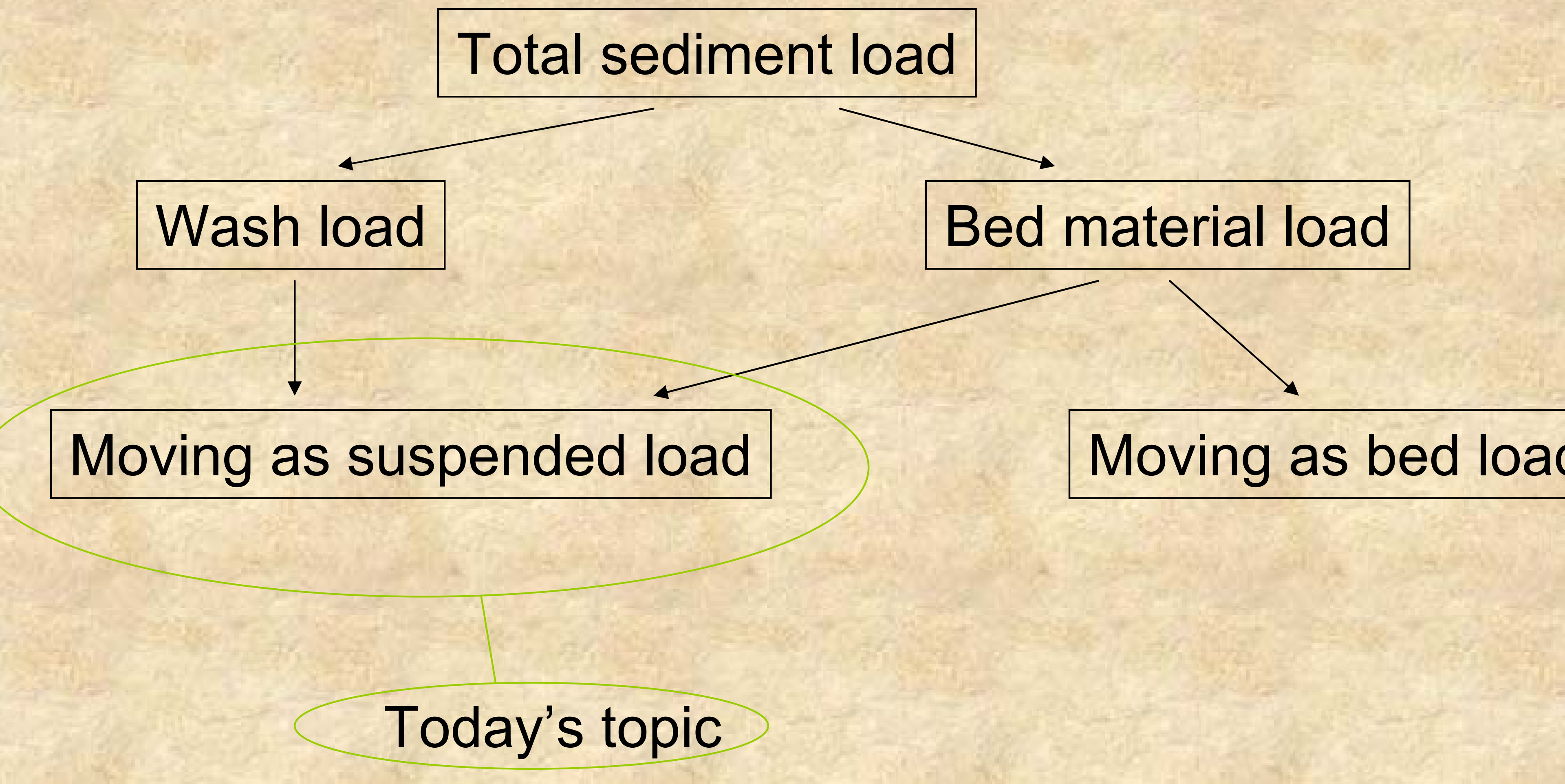
Why does sediment transport take place?

When does sediment transport take place?

Where does sediment transport take place?

Where is the sediment coming from?

Types of sediment transported:



What information do we need to know to model sediment transport?

Flow field ('physics')

Properties of flow away from bottom boundary (wave, mean current), and of the water (e.g. ν).

Particle field ('sedimentology')

Bed and wash material characteristics (e.g. density, size distribution, shape)

Within the BBL the two are coupled:

Stress on bottom due to flow imparts the force that resuspends particles.

Flow is affected by added water density due to suspension of particles.

Flow is affected by settling particles.

Flow is affected by bottom morphology (e.g. ripples).

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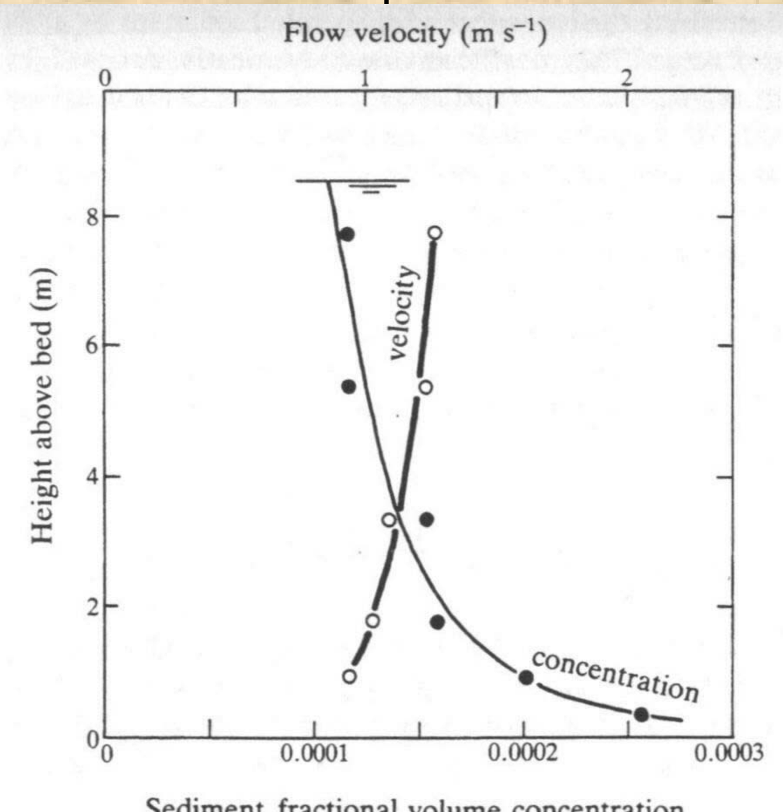
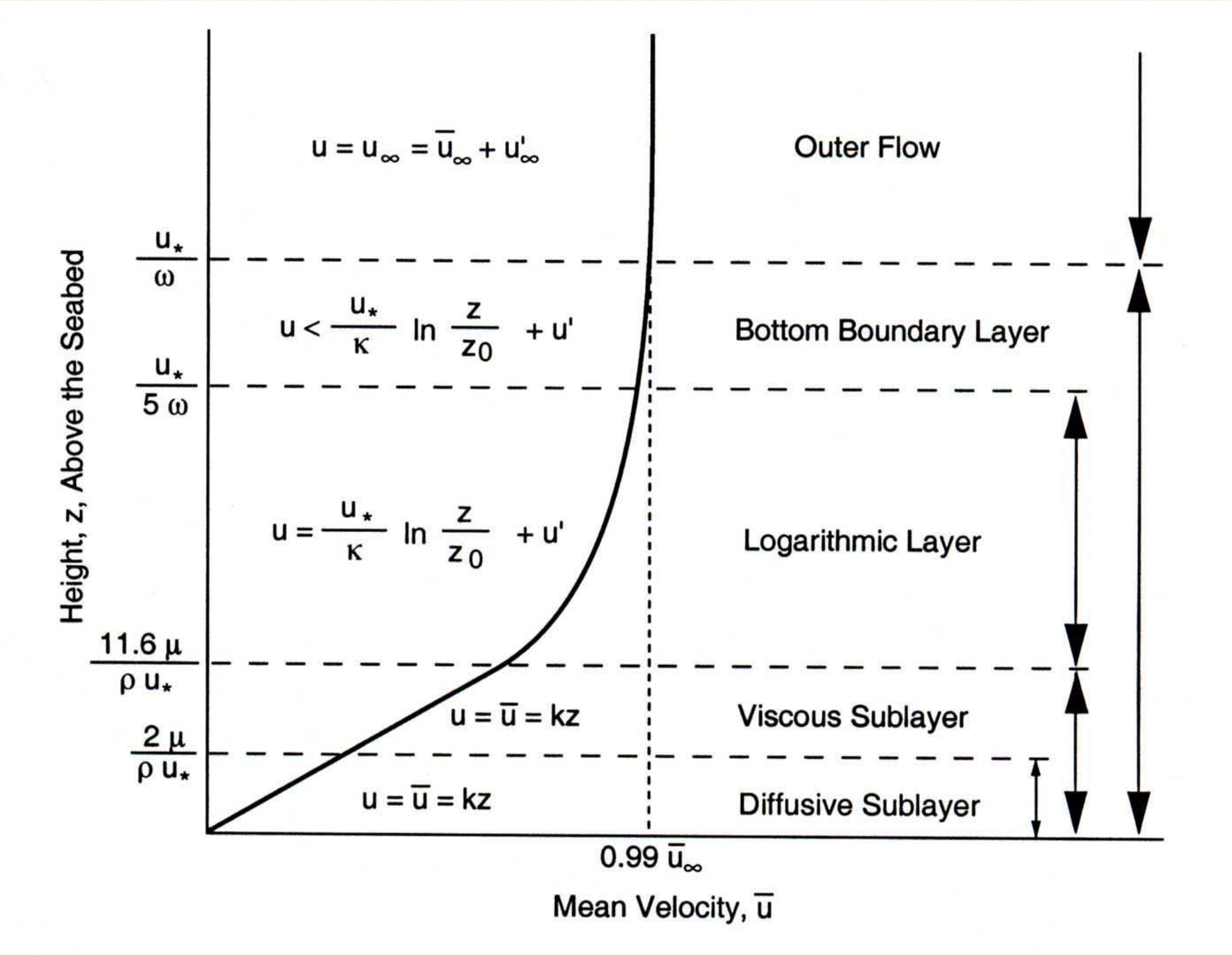


Example: horizontal flow field in a bottom boundary layer

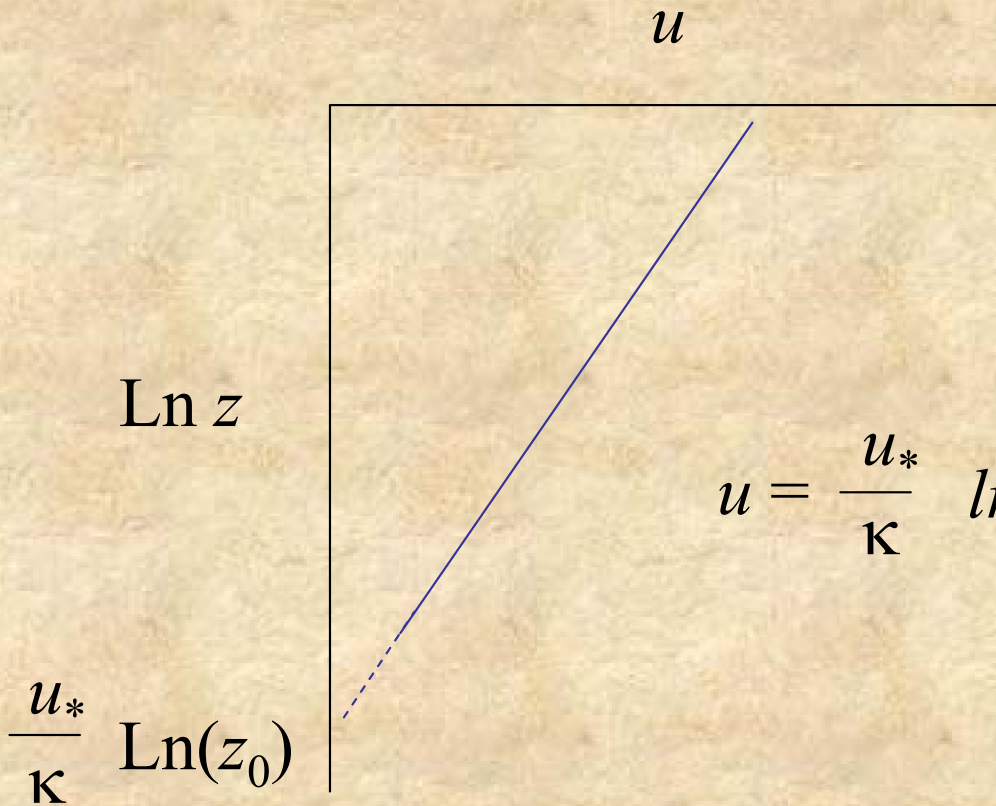


Bottom effect parameterized through z_0 and $u_*=(\tau_0/\rho)^{1/2}$. $\tau_0=\nu du/dz$, $\rho\langle u'w'\rangle$

→ Two parameter fit to velocity profile.

κ~0.41, von Karman's constant.

Example: horizontal flow field in a bottom boundary layer



$$u = \frac{u_*}{\kappa}\; ln\, \frac{z}{z_0} + u'$$

Slope ~ u_*/κ
Intercept ~ $u_* \ln z_0 / \kappa$

Predicting z_0 from D or u_*. Knowing z_0, BBL depth and $u_\infty \rightarrow u_*$.

D-bottom grain size

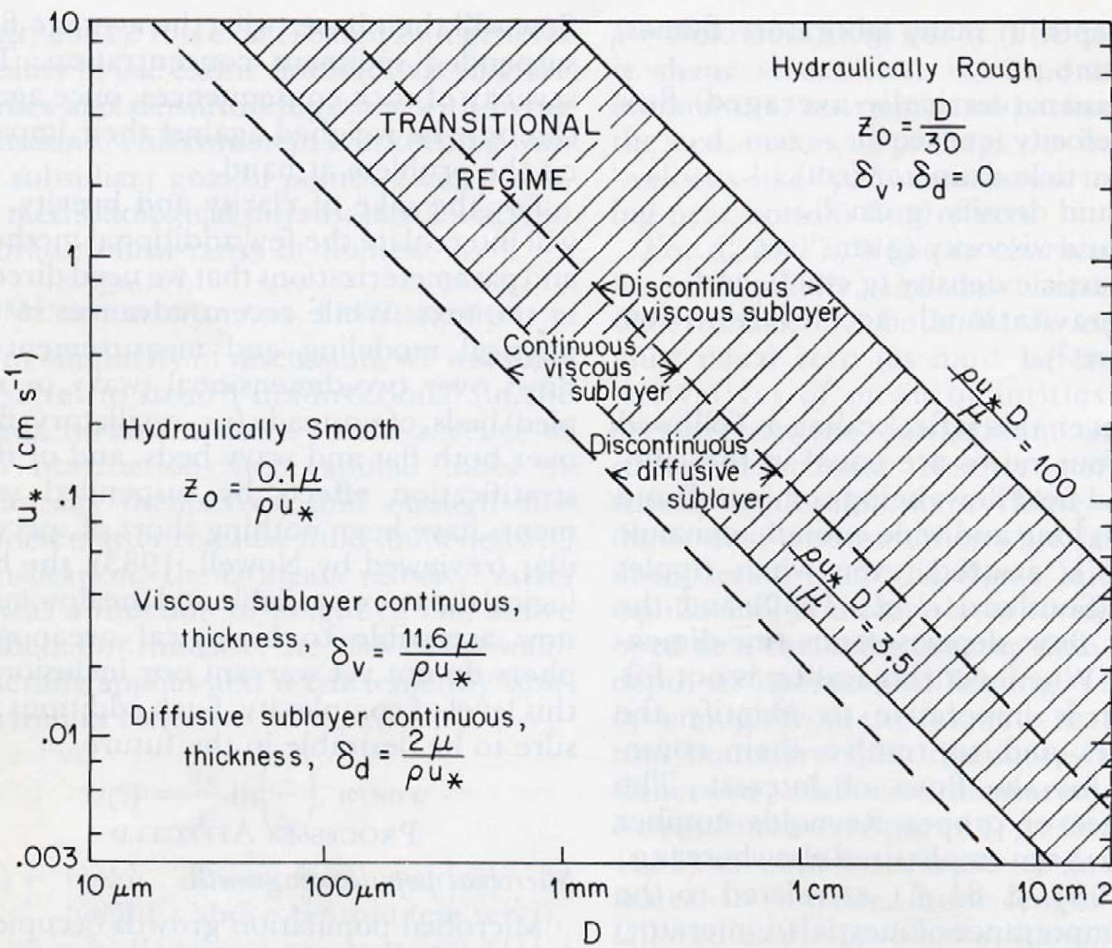
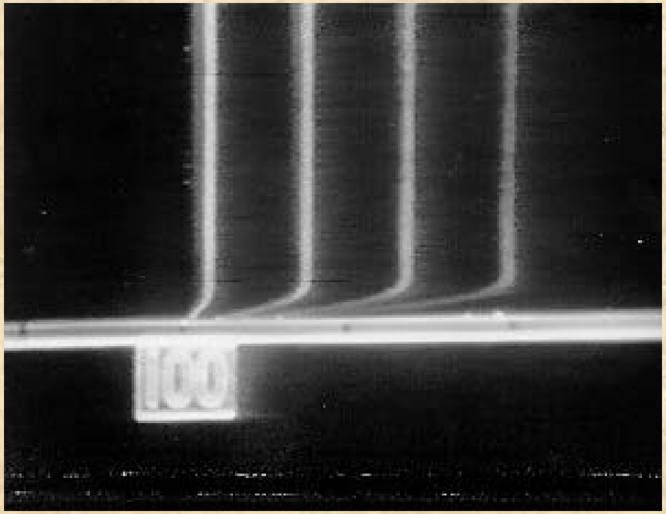
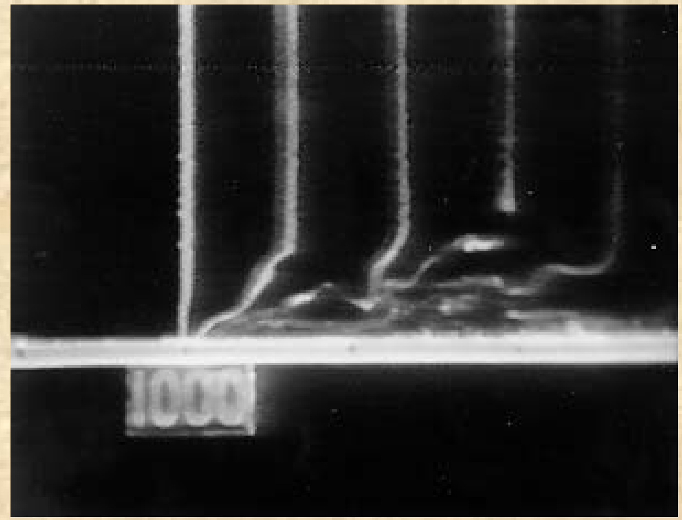


FIG. 1. A plot of the qualitative changes observed in near-bed fluid dynamic behavior (roughness Reynolds number) as a function of shear velocity (u_*) and bed grain diameter (D) for a flat (unrippled) bed of uniform grain size. The diagram encompasses the full range of u_* and D values generally encountered in oceanic and riverine conditions. Viscosity (μ = 0.01 poise [g cm^{-1} sec^{-1}]) and fluid density (ρ = 1.0 g cm^{-3}) are taken to be constants for simplicity of illustration. Their ranges of variation are small in comparison with those of u_* and D.

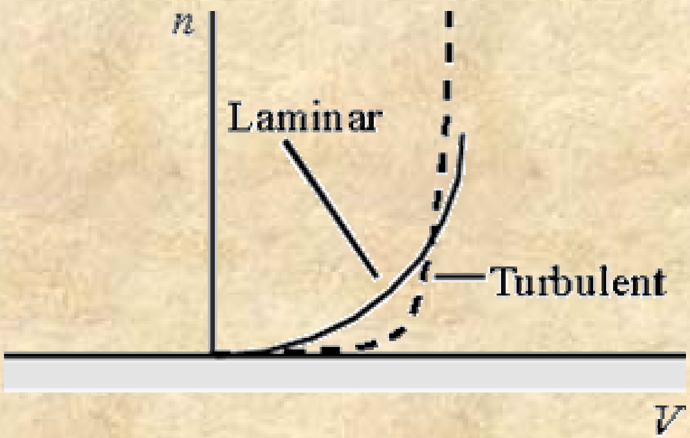
Jumars & Nowell. 1984. *Am. Zool.* **24:** 45-55

Example: horizontal flow field in a bottom boundary layer





http://cvu.strath.ac.uk/courseware/calf/CALF/bl/equations/eq5a.html

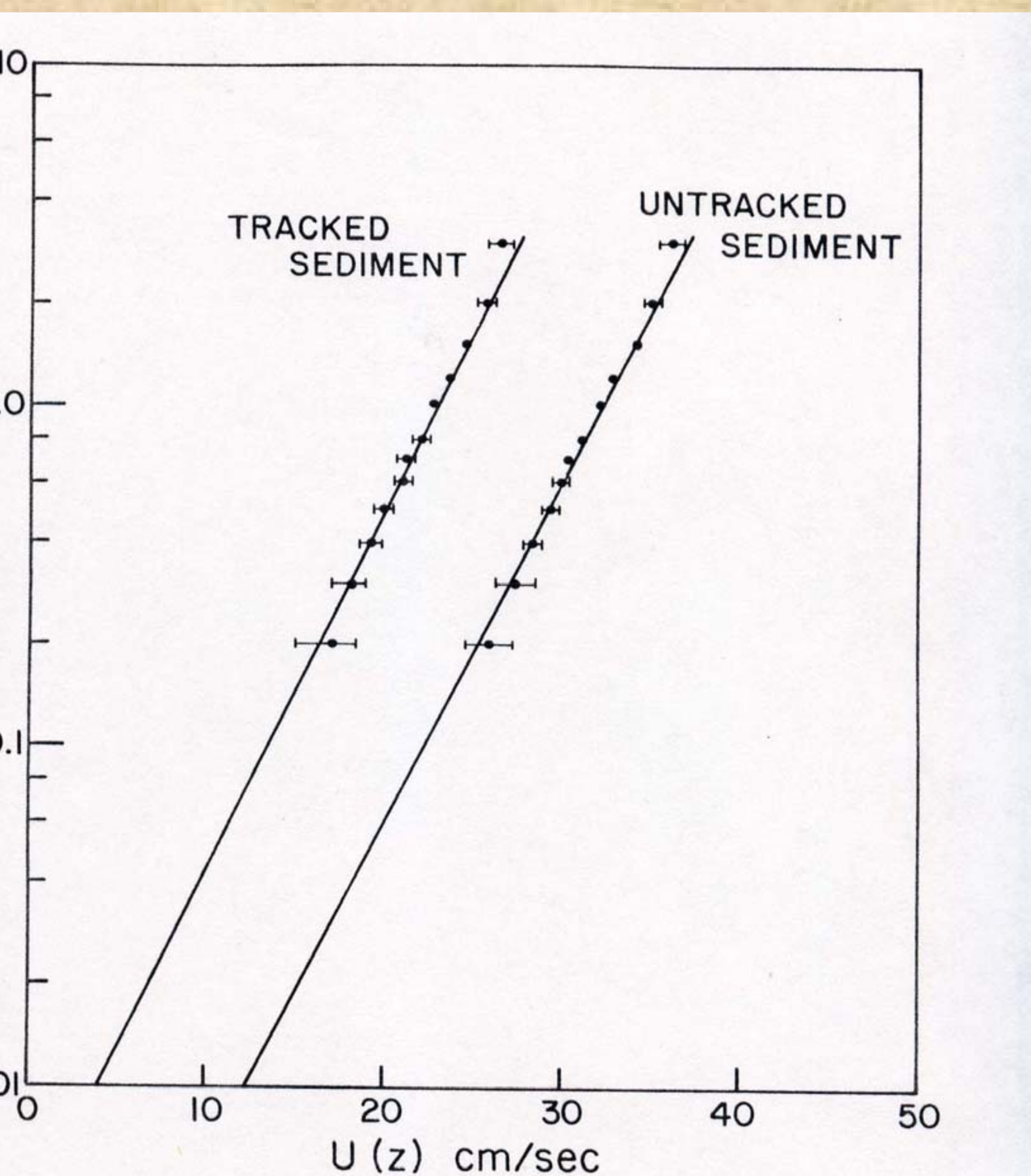


Velocity profiles for Laminar and Turbulent Flow

Turbulent boundary layer is more dissipative;
Applies more resistance to the flow.
Sets up faster.

Example: horizontal flow field in a bottom boundary layer

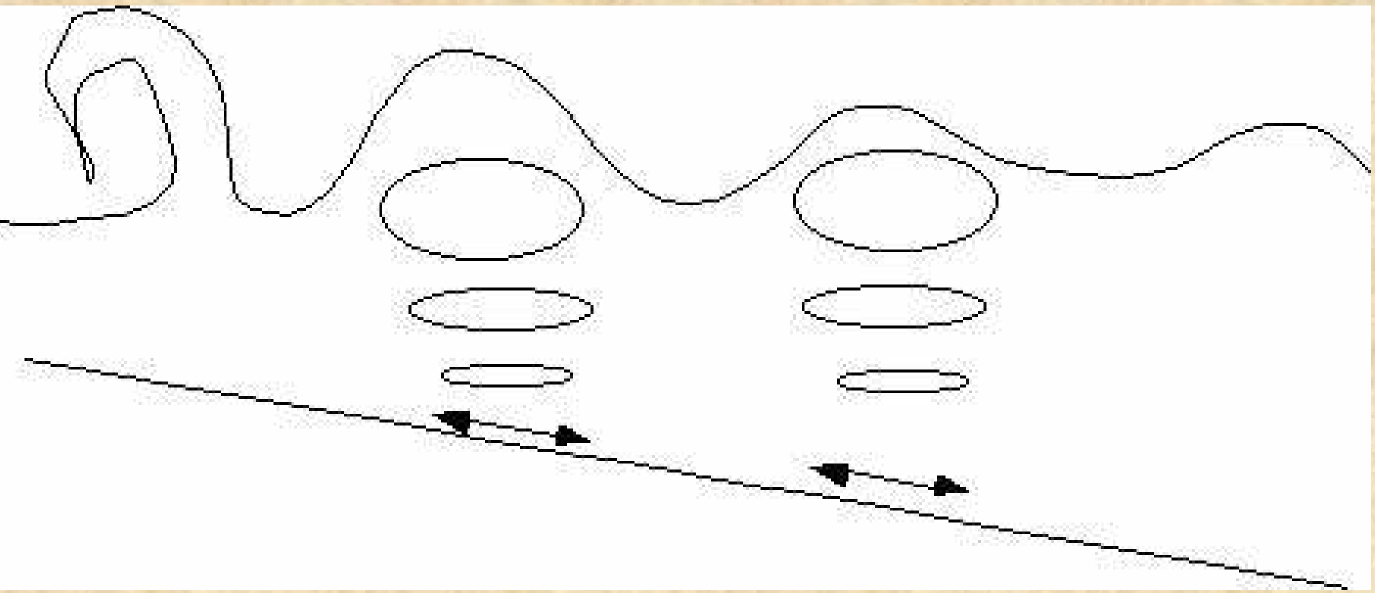
ata for sand tracked by an epifaunal bivalve:



Nowell, A.R.M., P.A. Jumars and J.E. Eckman. 1981. Effect of biological activity on the entrainment of marine sediments. *Mar. Geol.* **42:** 155-172

Gravity waves:

Effects:
Changes the mean flow field.
Change bottom shear stress.



Wave boundary layer is very shallow, $\delta \sim (4\pi\nu T)^{1/2}$, for a 4sec wave, $\delta \sim 0.7$cm.
Orientation relative to mean current is important:

$$u^2{}_{*cw} = \sqrt{u_{*_c}^4 + 2u_{*_c}^2 u_{*_w}^2 \cos\phi_{cw} + u_{*_w}^4}$$

Finally, we get to particles…

Rouse (1937) approach:

Conservation of particle mass (sources and sinks comes as BCs):

$$\frac{\partial C}{\partial t} + \nabla \cdot \left(\vec{u}_s C\right) = \frac{\partial C}{\partial t} + \frac{\partial\left(u_s C\right)}{\partial x} + \frac{\partial\left(v_s C\right)}{\partial y} + \frac{\partial\left(w_s C\right)}{\partial z} = 0$$

Assume no gradient in x and y, and divide into time-mean and fluctuations:

$$C = \bar{C} + C'; \quad w_s = \overline{w_s} + w_s'$$

Equation for mean becomes:

$$\frac{\partial\left(w_s C\right)}{\partial z} = \frac{\partial\left(\overline{w_s}\bar{C} + \overline{w_s' C'}\right)}{\partial z} = 0$$

Rouse's (1937) approach:

Convert Reynolds' stress flux into (eddy-)diffusive flux:

$$K_s \frac{\partial \bar{C}}{\partial z} = -\overline{w_s ' C '}$$

Combining we get:

$$\frac{\partial}{\partial z}\left[\overline{w_s}\bar{C} - K_s \frac{\partial \bar{C}}{\partial z}\right] = 0$$

Issues:

w_s is a function of sediment size, excess weight, and shape.
K_s is not necessarily the same as that of the fluid.
Boundary conditions.

Rouse's (1937) approach;

assume no net flux from top and bottom boundary (reduces to 1st order ODE).

Solution (up to a constant of integration, $C(z_1)$):

Near the bottom:

$$K = \kappa u_* z,\ K_S = \alpha K,\ \ 3 \ge \alpha \ge 0.3:$$

Defining $R = w_s / \kappa u_*$ we get:

$$\ln \frac{\overline{C}(z)}{\overline{C}(z_1)} = -\frac{\overline{w_s}}{\alpha \kappa u_*} \int_{z_1}^{z} \frac{dz}{z} = -\frac{\overline{w_s}}{\alpha \kappa u_*} \ln \frac{z}{z_1} \Rightarrow \frac{\overline{C}(z)}{\overline{C}(z_1)} = \left(\frac{z}{z_1} \right)^{-R/\alpha}$$

This profile fits well lower 30% of BBL.

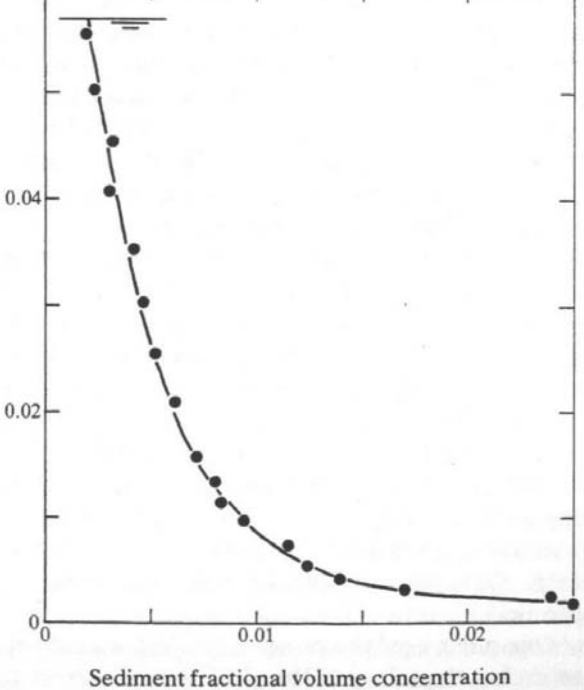
Higher up in the water column:

$$K_s = const. = \alpha \kappa u_* H_{BBL}$$

$$\ln \frac{\overline{C}(z)}{\overline{C}(z_1)} = -\frac{\left|\overline{w_s}\right|}{\alpha \kappa u_* H_{BBL}} \int_{z_1}^{z} dz = -\frac{\left|\overline{w_s}\right|(z - z_1)}{\alpha \kappa u_* H_{BBL}} \rightarrow \overline{C}(z) = \overline{C}(z_1) \exp\left\{ -\frac{R(z - z_1)}{\alpha H_{BBL}} \right\}$$

This profile fits well upper 80% of BBL.

Comparison with laboratory observations (See Allen, 20



re 7.3 Vertical distribution of the total sediment tional volume concentration in a laboratory channel. Data lontes and Ippen (1973).

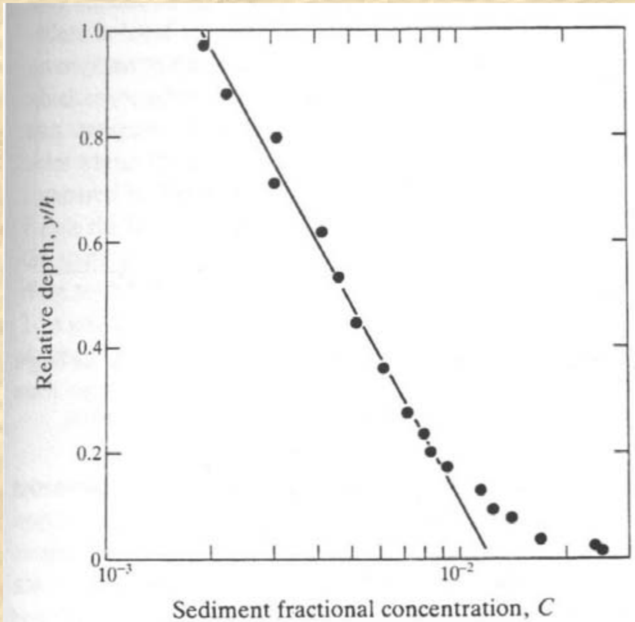


Figure 7.4 Semilogarithmic plot of the data of Figure 7.3.

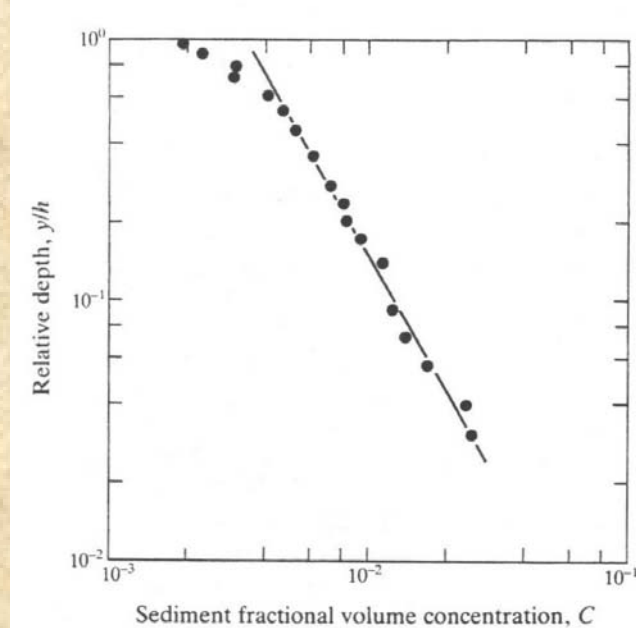


Figure 7.5 Double-logarithmic plot of the data of Figure 7.3

$$\ln\frac{\overline{C}(z)}{\overline{C}(z_1)} = -\frac{\overline{w_s}}{\alpha\kappa u_*}\int_{z_1}^{z}\frac{dz}{z} = -\frac{\overline{w_s}}{\alpha\kappa u_*}\ln\frac{z}{z_1} \Rightarrow \frac{\overline{C}(z)}{\overline{C}(z_1)} = \left(\frac{z}{z_1}\right)^{-R/\alpha}$$

$$\ln\frac{\overline{C}(z)}{\overline{C}(z_1)} = -\frac{\left|\overline{w_s}\right|}{\alpha\kappa u_* H_{BBL}}\int_{z_1}^{z}dz = -\frac{\left|\overline{w_s}\right|(z-z_1)}{\alpha\kappa u_* H_{BBL}} \rightarrow \overline{C}(z) = \overline{C}(z_1)\exp\left\{-\frac{R(z-z_1)}{\alpha H_{BBL}}\right\}$$

Problem with the Rouse equations near the bottom when the sediment concentration is large.

Taylor and Dyer (1977) approach; Add effects of sediment concentration on density. Let's the eddy coefficient vary relative to that of the unstratified fluid.

$$K_s = \frac{K}{1+\beta \frac{z}{L}}, \qquad K_{s,strat} = \frac{K}{\gamma + \beta \frac{z}{L}}$$

Where the Monin-Obukov-length, L, is defined as (based on shear stress and buoyancy flux being the two fundamental processes):

$$L = \frac{u_*^3 \overline{\rho}}{\kappa g \overline{\rho' w'}}$$

β and γ are constant (e.g. 4.7-5.2 and 0.74 respectively, Styles and Glenn, 2000) and z/L is termed the stability parameter.

Taylor and Dyer's (1977) approach; Add effects of sediment concentration on density. Let's the eddy coefficient vary relative to that of the unstratified fluid.

$$\rho' = \sum_n \frac{(\rho_{sn} - \rho_0)}{\rho_0} C'_n \qquad \Rightarrow \overline{\rho' w'} = \sum_n \frac{(\rho_{sn} - \rho_0)}{\rho_0} \overline{w' C'_n}$$

$$\overline{\rho} = \rho_0 \left[1 + \sum_n \frac{(\rho_{sn} - \rho_0)}{\rho_0} \overline{C_n} \right] \approx \rho_0$$

Since:

$$L = \frac{u_*^3 \overline{\rho}}{\kappa g \overline{\overline{\rho' w'}}}$$

Assuming all sediment classes have the same density:

$$\frac{z}{L} = \frac{\kappa g (\rho_s - \rho_0)}{u_*^3 \rho_0 \rho_s} \overline{w_{sn}} \, \overline{C_n(z_0)}$$

Taylor and Dyer's (1977) approach; Add effects of sediment concentration on density. Let's the eddy coefficient vary relative to that of the unstratified fluid.

Denoting by $A = \beta z/L$ (with *β=5.2* and *z/L* from above) and Rouse number $R_n = w_{sn}/\kappa u_*$ and for small roughness length scale z_0, close to the bed, where *K=κu*z*, the analytical solution for the flow and particle concentration is:

$$\ln\left(\frac{C(z)}{C_0}\right) = -R_n\left[\ln\frac{z}{z_0} + \frac{1}{R_n}\ln\left\{1 + \frac{AR_n}{(1-R_n)}\left[\left(\frac{z}{z_0}\right)^{1-R_n} - 1\right]\right\}\right]$$

$$u(z) = \frac{u_*}{\kappa}\left(\ln\frac{z}{z_0} + \frac{1}{R_n}\ln\left\{1 + \frac{AR_n}{(1-R_n)}\left[\left(\frac{z}{z_0}\right)^{1-R_n} - 1\right]\right\}\right)$$

Note that both flow and concentration field is affected and that for an unsorted sediment there is a need to find a way to characterize the effect of all size classes on velocity (through R_n), e.g.:

$$\widetilde{w}_s = \sum C_n w_{sn} \Big/ \sum C_n$$

Boundary conditions (needed when there is no continuous field data)

To solve the particle concentration equations we need to BCs.

$$\frac{\partial}{\partial z}\left[\overline{w_s}\,\overline{C} - K_s\frac{\partial \overline{C}}{\partial z}\right] = 0$$

The top BC is less important (in the limit of infinite ocean, C→0 there. For shallow waters specify no flux. Can incorporate flux from a productive ML if needed.

BC at (near) the bottom:

<u>Concentration BC</u> ($S_n = (\tau_d - \tau_{cn})/\tau_{cn}$):
$$\overline{C_n}\left(\delta^+, t\right) = \begin{cases} \overline{C_n}\left(\delta^-, t\right)\dfrac{\gamma_0 S_n}{1+\gamma_0 S_n} & S_n > 0 \\ 0 & \text{otherwise} \end{cases}$$

A problem with this approach is that γ_0 varies by 3 orders of magnitudes across studies and by 2 orders of magnitude within a single study over a short time.

Boundary conditions

Flux BC:

$$-w_{sn}\overline{C_n} - K_s \frac{\partial C_n}{\partial Z}\bigg|_{z=0^+} = J_{ei}\left(0^-,t\right) + J_{di}\left(0^-,t\right)$$

$$J_{ei}\left(0^-,t\right) = \begin{cases} C_e S_n & S_n > 0 \\ 0 & otherwise \end{cases}, J_{di}\left(0^-,t\right) = -p_n w_{sn} C_n\left(0^+,t\right)$$

with C_e an empirically determined erosion rate coefficient and p_n the probability that a falling particle makes contact with the bed and remains there.
Two models for pn are used:
$p_n=1$ for all shear stresses, in which case erosion balances diffusion in the BC. The second model assumes:

$$p_n = \begin{cases} 1 - \tau_0 / \tau_{dn} & \tau_0 > \tau_{dn} \\ 0 & otherwise \end{cases}$$

The depositional shear stress of class n, τ_{dn}, defines the stress below which sediment is able to remain on the bed after contacting it.

Boundary conditions

Because the eddy coefficient goes to zero at the boundary, there is no mechanism to raise the sediments into the water column, which, for the flux BC, provides physical solutions only when $p_n = 1$ and $J_{di} = 0$.
This problem is addressed in some models by adding a well-mixed near-bed layer of thickness δ_a where the eddy coefficient increases (convenient mathematically but not observed).

Another approach is to incorporate injection of sediments from the bed at various heights above the bottom (which are not resolved in the 1-D case and are the result of averaging in x and y). In this case the conservation equation is

$$w_{sn} \frac{\partial C_n}{\partial z} = \frac{\partial}{\partial z}\left(K_{sn} \frac{\partial C_n}{\partial z} \right) + D_n$$

With, for example:

$$D_n = A_{sn} \exp\left(-\frac{z}{\delta_e} \right)$$

The BC condition in this case is that erosion equals deposition at the bottom.

Summary:

- Momentum and material flux are not mutually independent.

- 1-D steady state equation describe adequately observed data when local data is used in parameter fit.

- Current approaches almost always ignore aggregation dynamics.

- High resolution data (velocity & size fractionated particles) is lacking.

- Current approaches vary from a mix empirical fits to physical approaches tuned with empirical data.

n case you wondered about who cares, and whether there is money o be made:

Some commercial players (based on a simple google search):

Hydrau-Tech, Inc.
Hydraulic Engineering & Software

BOSS INTERNATIONAL

Stillwater Sciences
home

QEA

MOHID
Water Modelling System

HydroQual, Inc.
Environmental Engineers and Scientists

WL | delft hydraulics

Some government agencies in the US funding sediment transport modeling:

UNITED STATES ENVIRONMENTAL PROTECTION AGENCY

USGS
science for a changing world

ONR

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Middleton, G. V. and J. B. Southard, 1984, Mechanics of Sediment Movement, SEPM short course, 2nd edition, Providence.

Taylor, P. A., and K. R. Dyer, 1977. Theoretical models of flow near the bed and their implications for sediment transport. Ch. 13. of *The Sea, Volume 6, Marine modeling*, Edited by E. D. Goldberg, I. N. McCave, J. J. O'Brien, and J. H. Steele, John Wiley & Sons.