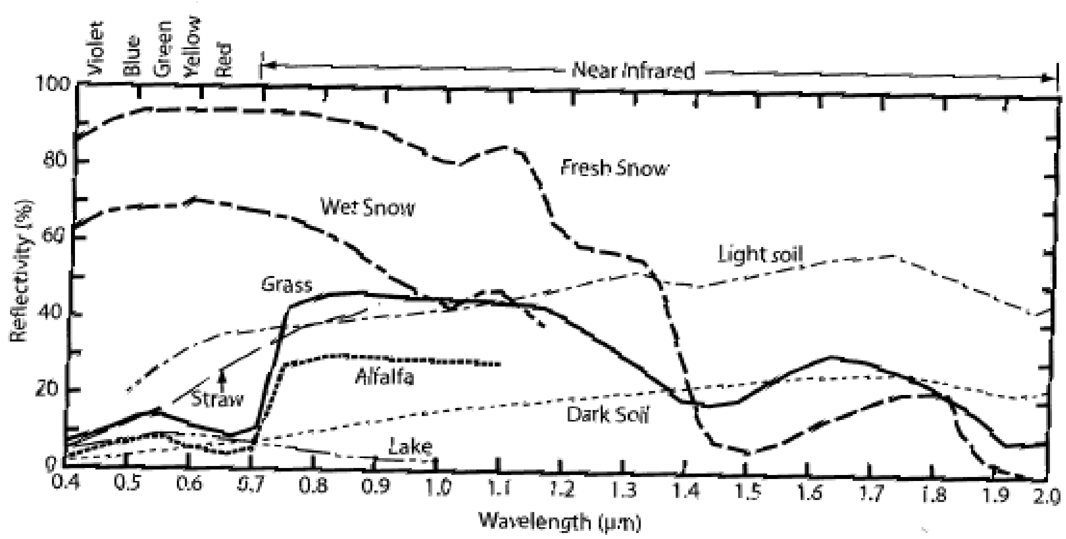
Radiative transfer in the environment. Weitzmann, fall 2008.

Answers to proem set II:




1. (based on Petty, 2006, pr. 5.5, p.112) Above is a figure from Petty, 2006, describing the reflectivity of different surfaces. A particular satellite sensor is being designed to measure reflectivities at two wavelengths. Based on this figure, choose to wavelengths so as to optimize the ability of the satellite to discriminate different surface types (assume the atmosphere is transparent). Start by drawing $R(\lambda_1)$ as function of $R(\lambda_2)$ for different surfaces. Plot points corresponding to each surface type and label them. Try to develop a simple mathematical algorithm (e.g. a series of tests based on inequalities) that would allow you to correctly classify the surface as snow, soil, growing vegetation, dry vegetation, or water based on the observations of $R(\lambda_1)$ and $R(\lambda_2)$. You may want to consider differences in reflectivities and/or ratios and/or the value of the reflectivities in

one wavelength, as possible variable in the algorithm. Whatever criteria depict them graphically as curves separating surface types on your plot of $R(\lambda_1)$ as function of $R(\lambda_2)$. Not that in order for an algorithm to be successful the classifier should be spread enough on your plot that uncertainties in R will not compromise your scheme. There are multiple solutions to this problem - be creative!

*There is a big difference between $\lambda$ <1.1 and $\lambda$>1.4$\mu$m so it seems logical to pick one wavelength of each of the ranges. To separate grass from dark soil I decided to use 0.65um in the visible and 2.0um in the infra red. Dry vegetation has no NIR data and a value in the VIS (0.3) at 0.65um wavelength that is quite distinct in absolute value from the rest. Water is black in the NIR while snow has a very high VIS/NIR ratio. The NIR value alone can separate light soil and dark soil which line on a similar wavelength ratio line. Growing vegetation as a low and similar value of NIR and VIS.*

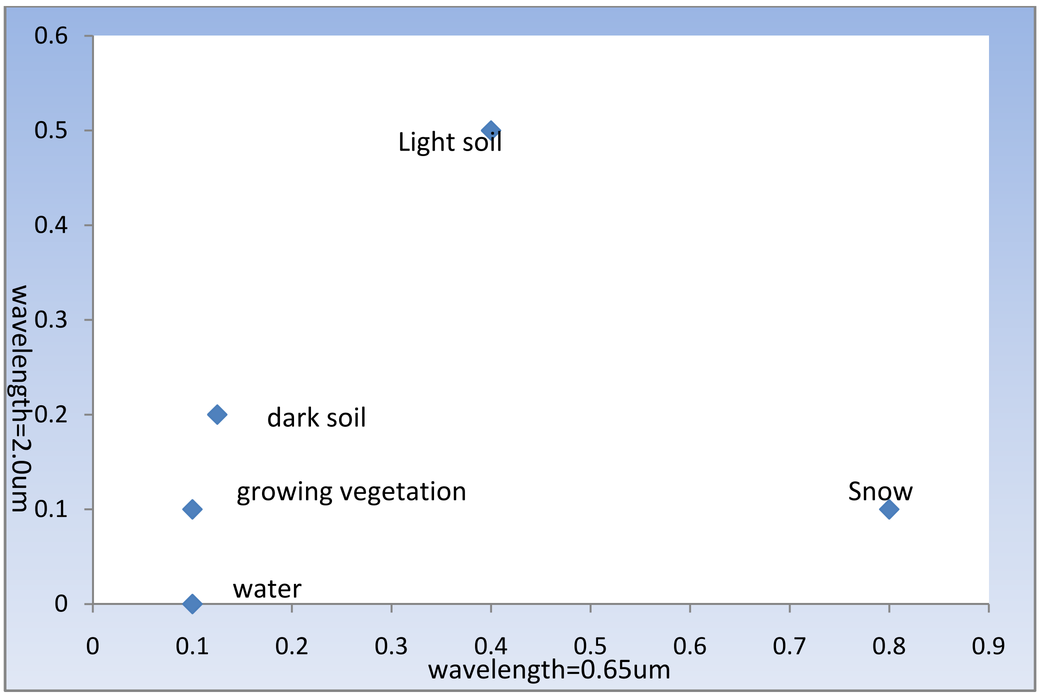



*Figure 1. Values of reflectance at 0.65$\mu$m and 2.0$\mu$m associated with different substrate.*

2. (based on Petty, 2006, pr. 6.2) Sometimes Planck's function (B(T) may be expressed as function of frequency instead of wavelength (see last class for expression of $B_\lambda(T)$). Find the correct expression for $B_\nu(T)$. Hint: think about energy conservation and watch for units.

The best way to go about it (and not have problems with units) is to start from the integral of Plank's function, which represents energy:

$$\int_{\lambda_1}^{\lambda_1} B_\lambda d\lambda = \int_{\lambda_1}^{\lambda_2} \frac{2\pi hc^2}{\lambda^5} \frac{1}{\exp(hc/\lambda\, k_B T) - 1} d\lambda$$

We substitute:

$$\nu = \frac{c}{\lambda} \rightarrow \lambda = \frac{c}{\nu} \rightarrow d\lambda = -\frac{cd\nu}{\nu^2}$$

$$\int_{\nu_2}^{\nu_1} \frac{2\pi h\nu^3}{c^2} \frac{1}{\exp(h\nu/k_B T) - 1} d\nu = \int_{\nu_2}^{\nu_1} B_\nu d\nu \rightarrow B_\nu = \frac{2\pi h\nu^3}{c^2} \frac{1}{\exp(h\nu/k_B T) - 1}$$

Note that we inverted the limits on the integral as smaller wavelengths are associated with larger frequencies and vice-versa.

3. The simplest model of the greenhouse effect involves a slab encircling the air assumed to be in thermal equilibrium. This slab (the atmosphere) is assumed to be transparent to the sun's radiation yet absorbing to the radiation that emitted by the Earth with transmissivity $\tau$ for long wave radiation. This layer emits radiation according to its own temperature to both the Earth and outer space. a. assuming S to be the solar irradiance absorbed by the Earth's surface (its value you derived in the previous homework), and that the atmosphere behave as a blackbody, derive and expression for the Earth's temperature as function of S and $\tau$. Using S you derived in the previous homework plot the Earth temperature as function of $\tau$. Are they reasonable?

*From the previous homework we know that the absorbed radiation in the visible is $S=243W/m^2$. I assume the reflected radiation is not absorbed by the atmosphere. The Earth warms and emits NIR radiation based on its own temperature. The Earth has a temperature of Te while the atmosphere Ta.*
*At equilibrium the radiation absorbed by the Earth is balanced by that radiated to space:*
$S=\sigma Te^4\tau + \sigma Ta^4$
*Since at equilibrium the atmospheric layer emits the same amount as it absorbs both to outer space and back to Earth {that is $\sigma Ta^4 =(1-\tau)\sigma Te^4 /2$}. Hence:*
$S= \sigma Te^4(1+\tau)/2$ *or*
$Te^4=2S\{(\sigma(1+\tau)\}$
*Values on graph (Fig. 2 below) are reasonable, warmer the more absorbing the layer is.*

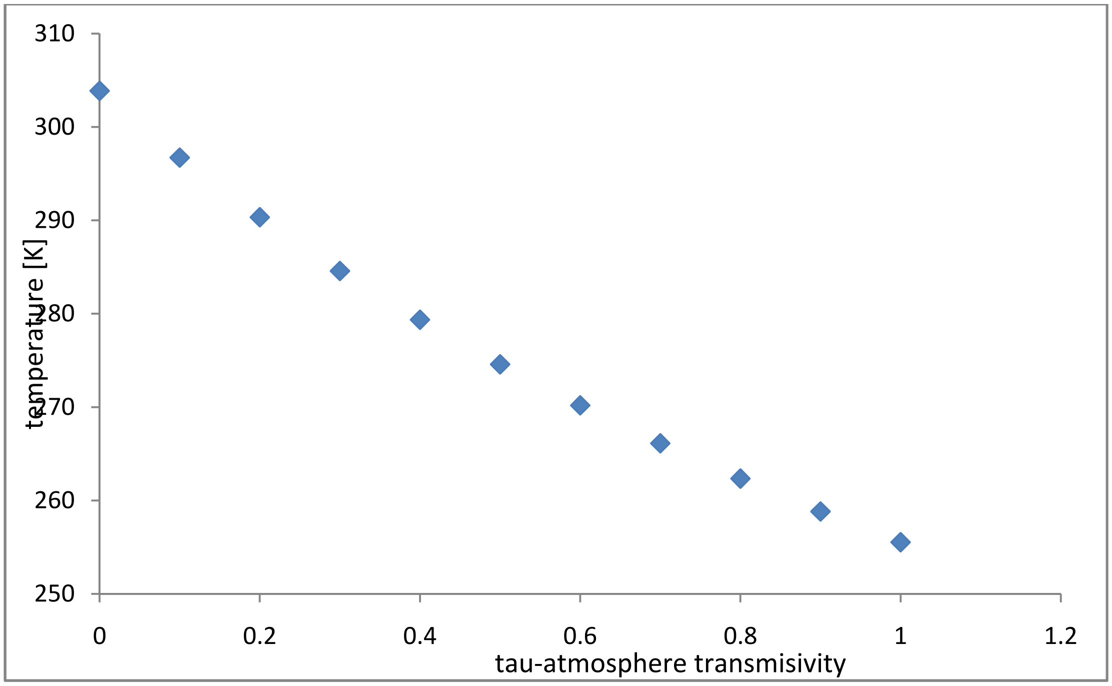



*Figure 2. temperature of the hypothetical Earth for a given atmospheric layer transmission in the NIR, assuming a single slab.*

*Analysis of BLB data:*

*Fitting an exponential to the data (I used Excel, which for now is good enough. DO NOT USE it for your research data unless you know that your relative uncertainties are constant. Excel simply log(y-data) and fits a line with a linear-least-square fit. Better to use a non-linear fit, weigh the fit with your measured uncertainties and when uncertainties in both dependent and independent parameter are important, use type-II regression) provide us an equation where the exponent is equal to the specific attenuation for a drop times the length of the tank.*

➔ *Attenuation per drop (in the volume of our tank) = 0.111/0.34=0.326 tank volumes/ drop /m.*

*Drop/tank volume are the units of concentration of the drops in our experiment.*

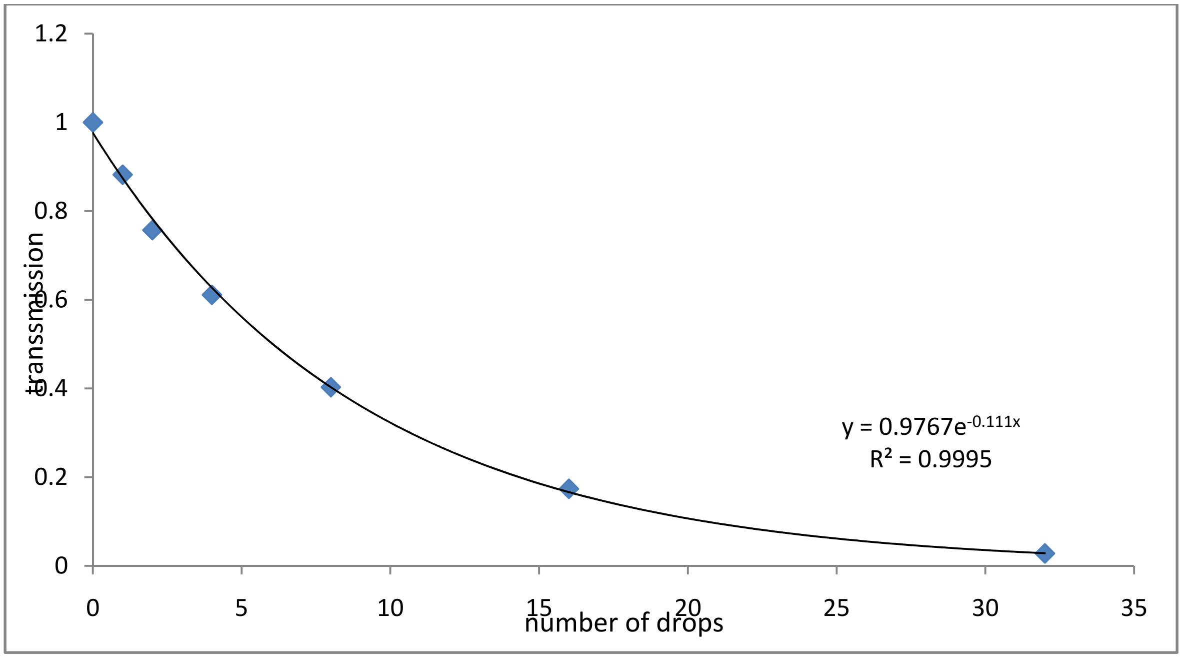


*Figure 3. Data and fit for the BLB experiment.*