

Problem set IV:

1. (based on Petty, ex. 7.2) A cloud layer has a vertical profile of the extinction coefficient ($c=0.015\text{m}^{-1}$) that is quadratic with the altitude between its base ($z_{\text{base}}=1000\text{m}$) and its top ($z_{\text{top}}=1200\text{m}$) with a maximum in the middle of the cloud and zero extinction at its base and top.

Compute the total optical path and vertical transmittance through the cloud.

2. (Petty, ex. 7.6) A particular plane parallel cloud has liquid water density $\rho_w=0.1\text{ g m}^{-3}$ and thickness $\Delta z=100\text{m}$. At a certain wavelength, the mass extinction coefficient of the cloud droplets is $c_{\text{water}}^*=150\text{m}^2/\text{kg}$, and the single scatter albedo is $\omega_{\text{water}}=1$. However, the air in which the droplets are suspended is itself absorbing at this wavelength, having an absorption coefficient $a_{\text{air}}=10\text{km}^{-1}$ and $\omega_{\text{air}}=0$.

Compute a , b , c and ω_0 for the mixture (the absorption, scattering, attenuation and single scattering albedo). Compute the total optical thickness of the cloud layer. If the radiation incident on top of the cloud $I_{\lambda,\text{top}}$ is at a zenith angle of sixty degrees, compute the transmitted intensity, $I_{\lambda,\text{bot}}$.

3. (Petty, ex. 7.8) A ground-based radiometer operating at $\lambda=450\text{nm}$ is used to measure the solar intensity $I_{\lambda}(0)$. For a solar zenith angle of $\theta=30^\circ$,

$I_{\lambda}(0)=1.74\times 10^7\text{ Wm}^{-2}\mu\text{m}^{-1}\text{sr}^{-1}$. For $\theta=60^\circ$, $I_{\lambda}(0)=1.14\times 10^7\text{ Wm}^{-2}\mu\text{m}^{-1}\text{sr}^{-1}$. From this information, determine the top-of-the-atmosphere solar intensity S_{λ} and the atmospheric optical thickness τ_{λ} .

4. (Petty, ex. 7.12, using individual particle optical properties to obtain bulk properties) A certain cloud layer has geometric thickness $H=0.1\text{km}$ and liquid water path ($L \equiv \int_{z_{\text{bot}}}^{z_{\text{top}}} \rho_w dz$) $L=0.01\text{kg m}^{-2}$. Assuming $Q_e\sim 2$ (the extinction efficiency of particles larger than the wavelength) and a solar zenith angle of $\theta=60^\circ$, compute the transmittance of a direct light beam for a. $N=100\text{cm}^{-3}$ (characteristic of clean maritime environments), and b. $N=1000\text{cm}^{-3}$

(characteristic of continental environments), where N is the number of spherical drops.

5. (Continuation of problem 3 from problem set 3). Recast the problem in terms of the asymmetry parameter: $g = (T_1 - R_1) / (T_1 + R_1)$ and the single scattering albedo: $\omega_0 = T_1 + R_1 = 1 - A_1$. Investigate the sensitivity of the asymptotic value of R_N to these parameter (as you did in the previous homework).

Extra credit: in the ocean we often assume that $R_N \sim b_b/a$ {or $b_b/(b_b + a)$ }. Translate this expression to g and ω_0 and investigate whether it is consistent with your findings.