Radiative transfer in the environment. Weitzmann, fall 2008.

Problem set IV, Answers:

1. (based on Petty, ex. 7.2) A cloud layer has a vertical profile of the extinction coefficient ($c=0.015m^{-1}$) that is quadratic with the altitude between its base (z_{base}=1000m) and its top (z_{top}=1200m) with a maximum in the middle of the cloud and zero extinction at its base and top.

Compute the total optical path and vertical transmittance through the cloud.

The equation for the quadratic c profile with a zero value at 1000 and 1200 and an intermediate maxima is: $c(z)=0.015(z-1000)(1200-z)/10000=-1.5\times10^{-6}(z^2-2200z+1.2\times10^6)$.

Integrating this profile we get:

$$\tau = \int_{1000}^{1200} -1.5\cdot10^{-6}(z^2-2200z+1.2\cdot10^6)\,dz = -1.5\cdot10^{-6}\left(\frac{z^3}{3}-1100z^2+1.2\cdot10^6 z\right)\Big]_{1000}^{1200} = 2.$$

$$Transmittance = e^{-\tau} = 0.1353$$

2. (Petty, ex. 7.6) A particular plane parallel cloud has liquid water density ρ_w=0.1 g m^{-3} and thickness Δz=100m. At a certain wavelength, the mass extinction coefficient of the cloud droplets is $c_{water}^*=150m^2/kg$, and the single scatter albedo is $\omega_{water}=1$. However, the air in which the droplets are suspended is itself absorbing at this wavelength, having an absorption coefficient $a_{air}=10km^{-1}$ and $\omega_{air}=0$.

Compute a, b, c and ω_0 for the mixture (the absorption, scattering, attenuation and single scattering albedo). Compute the total optical thinkcness of the cloud layer. If the radiation incident on top of the cloud $I_{\lambda,top}$ is at a zenith angle of sixty degrees, compute the transmitted intensity, $I_{\lambda,bot}$.

$$a = a_{air} + a_{water} = 10km^{-1} + 0 = 0.01m^{-1}$$
$$b = b_{air} + b_{water} = 0 + \rho_w\cdot\omega_0 c^* = 0.015m^{-1}$$
$$c = c_{air} + c_{water} = 0.01m^{-1} + 0.015m^{-1} = 0.025m^{-1}$$

$$\omega_0 = \frac{b_{air} + b_{water}}{c_{air} + c_{water}} = \frac{0.015m^{-1}}{0.025m^{-1}} = 0.6$$

$$\tau = \int_{\Delta z} cdz = 100m \cdot 0.025m^{-1} = 2.5$$

$$I_{\lambda,Bot} = I_{\lambda,Top} e^{-\tau/cos\theta} = I_{\lambda,Top} e^{-2\tau} = I_{\lambda,Top} e^{-5} = 0.0067 \cdot I_{\lambda,Top}$$

3. (Petty, ex. 7.8) A ground-based radiometer operating at λ=450nm is used to measure the solar intensity $I_\lambda(0)$. For a solar zenith angle of θ=30°, $I_\lambda(0)$=1.74x10^7 Wm$^{-2}\mu$m^{-1}sr^{-1}. For θ=60°, $I_\lambda(0)$=1.14x10^7 Wm$^{-2}\mu$m^{-1}sr^{-1}. From this information, determine the top-of-the-atmosphere solar intensity S_λ and the atmospheric optical thickness τ_λ.

$$I_{\lambda,\theta}(0) = S_\lambda e^{-\tau/\cos\theta}$$

$$\rightarrow \log\left(I_{\lambda,\theta}(0)\right) = \log(S_\lambda) - \frac{\tau}{\cos\theta}$$

With two angles we have two equations for two unknowns (τ, S_λ):

$$\log(S_\lambda) = \frac{\cos\theta_1 \log\left(I_{\lambda,\theta_1}(0)\right) - \cos\theta_2 \log\left(I_{\lambda,\theta_2}(0)\right)}{\cos\theta_1 - \cos\theta_2} \rightarrow S_\lambda$$

$$= 3.1 \cdot 10^7 Wm^{-2}\mu m^{-1} Sr^{-1} \rightarrow \tau_\lambda = 0.5$$

4. (Petty, ex. 7.12, using individual particle optical properties to obtain bulk properties) A certain cloud layer has geometric thickness H=0.1km and liquid water path ($L \equiv \int_{z_{bot}}^{z_{top}} \rho_w \, dz$) L=0.01kg m^{-2}. Assuming Qe~2 (the extinction efficiency of particles larger than the wavelength) and a solar zenith angle of θ=60°, compute the transmittance of a direct light beam for a. N=100cm^{-3} (characteristic of clean maritime environments), and b. N=1000cm^{-3} (characteristic of continental environments), where N is the number of spherical drops.

Calculating the volume of each water droplets: the amount of water in air=L/H=0.0001kgm^{-3}. For a given number of droplets N, the mass of a single drop=L/H/N. For N=100cm^{-3}=10^{8}m^{-3}, Mass of single drop=10^{-12}kg. For

N=1000cm^{-3}, mass of a single drop=10^{-13}kg. Given a density of water=1000kg/m^3 we can compute the volume of the drop in both cases:V_{100}=10^{-15} m^3, V_{1000}=10^{-16} m^3. From the volumes we can calculate the radius of each drop: $R = \sqrt[3]{\frac{3V}{4\pi}}$ → *R_{100}=6.20·10^{-6} m, R_{1000}=2.88·10^{-6} m.*

Calculations of the beam attenuation:

$$c = NQ_e\pi R^2 \rightarrow c_{100} = 0.024m^{-1}, c_{1000} = 0.052m^{-1}$$

Transmittance of a direct beam through H in a zenith angle of 60°:

$$T = e^{-\tau/\cos\theta} = e^{-200c} \rightarrow T_{100} = 0.008, T_{1000} = 0.00003$$

For all practical purpose no light makes it through this kind of cloud.

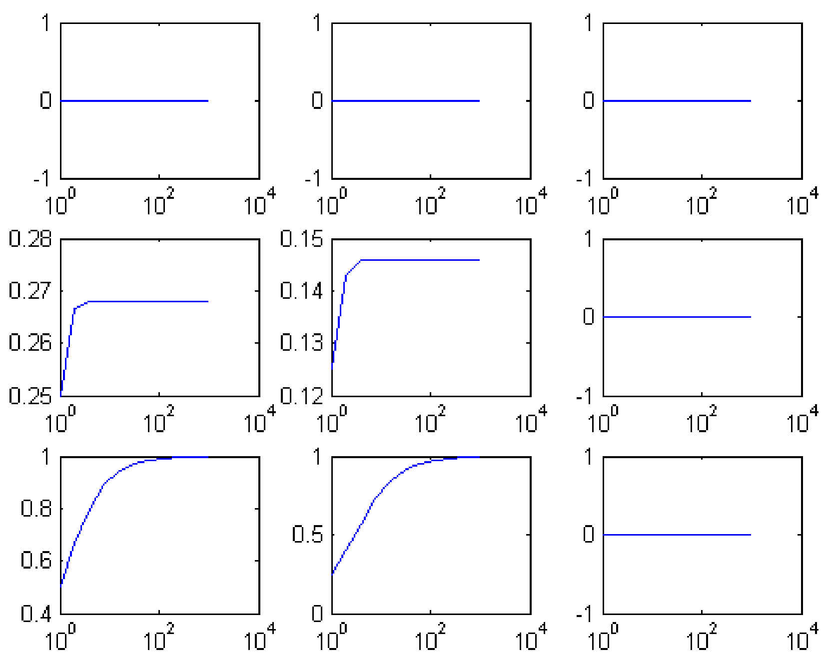
5. (Continuation of problem 3 from problem set 3). Recast the problem in terms of the asymmetry parameter: $g=(T_1-R_1)/(T_1+R_1)$ and the single scattering albedo: $\omega_0=T_1+R_1=1-A_1$. Investigate the sensitivity of the asymptotic value of R_N to these parameter (as you did in the previous homework).

R_N

w$_0$=0

w$_0$=0.5

w$_0$=1



g=0 g=0.5 g=1

Extra credit: in the ocean we often assume that $R_N \sim b_b/a$ {or $b_b/(b_b+a)$}. Translate this expression to g and ω_0 and investigate whether it is consistent with your findings.

$$If\ R_\infty \propto \frac{b_b}{a} \propto \frac{R}{A} = \frac{\omega_0(1-g)}{2(1-\omega_0)}$$

This is obviously not consistent with our result for ω_0=0.

$$If\ R_\infty \propto \frac{b_b}{a+b_b} \propto \frac{R}{A+R} = \frac{2\omega_0(1-g)}{4(1-\omega_0)+\omega_0(1-g)}$$

This is consistent for our results with both ω_0=0 and g=1. When ω_0=1 we get a result that is insensitive to g (good) but that equals 2. Thus, maybe:

$$R_\infty \sim \frac{\omega_0(1-g)}{4(1-\omega_0)+\omega_0(1-g)}$$

Substituting for the cases when ω_0=0.5 and g=0,0.5 we get for g=0, R_∞ =0.2 and for g=0.5, R_∞=0.11. These are lower than the results in the graph but in the right ballpark. Indeed, it is found that approximating R_∞ as a series gives better results.