What is Monte Carlo Modeling*?

Monte Carlo Modeling is a statisitcal method used here to simulate radiative transfer by simulating photon (or more exactly light rays/beams) interaction with a medium.

MC models can be built such that increasing complexity can be added as the model develops.

*based partially on a ppt file found at: www.ece.utexas.edu/bell/MonteCarlo.ppt

Program

Some steps and concepts needed to implement a Monte Carlo model of photon propagation in a medium:

- Random Number Generation
- Attenuation of beam in the medium
- Scattering of Photons
- Variable weight photon
- Index Mismatch of Boundary Layers
- Grid Structure
- Detector
- Inverse MC (from detector → source)

Basic concepts:

The "fundamental principle": we can map any probability distribution to a uniform distribution from 0 to 1:

$$\xi = \int_{-\infty}^{x_0} p(x)dx = P(x_0)$$

Example: the probability of a photon travel to a distance l (or an optical depth $\tau = cl$) without an interaction is $exp(-\tau)$.

$$\xi = \int_{-\infty}^{l} \exp(-\tau)d\tau = 1 - \exp(-\tau)$$

$$\rightarrow \tau = -\log(1-\xi) = -\log(\xi)$$

For a uniform medium: $\rightarrow l = -\log(\xi)/c$

Random Number Generation

- The first step in building a Monte Carlo model is to have a method to create random numbers.
- In Matlab the function 'rand.m' generates a uniform distribution of random numbers (r.n.) in (0,1). ← see: gen_rand.m
- Random number generators produce 'pseudo' random numbers.

Essential Properties of a Random Number Generator*

- **Repeatability** -- the same sequence should be produced with the same initial values (or *seed*). This is vital for debugging.

- **Randomness** -- should produce independent uniformly distributed random variables that pass all statistical tests for randomness.

- **Long period** -- a pseudo-random number sequence uses finite precision arithmetic, so the sequence must repeat itself with a finite period. This period should be much longer than the amount of random numbers needed for the simulation.

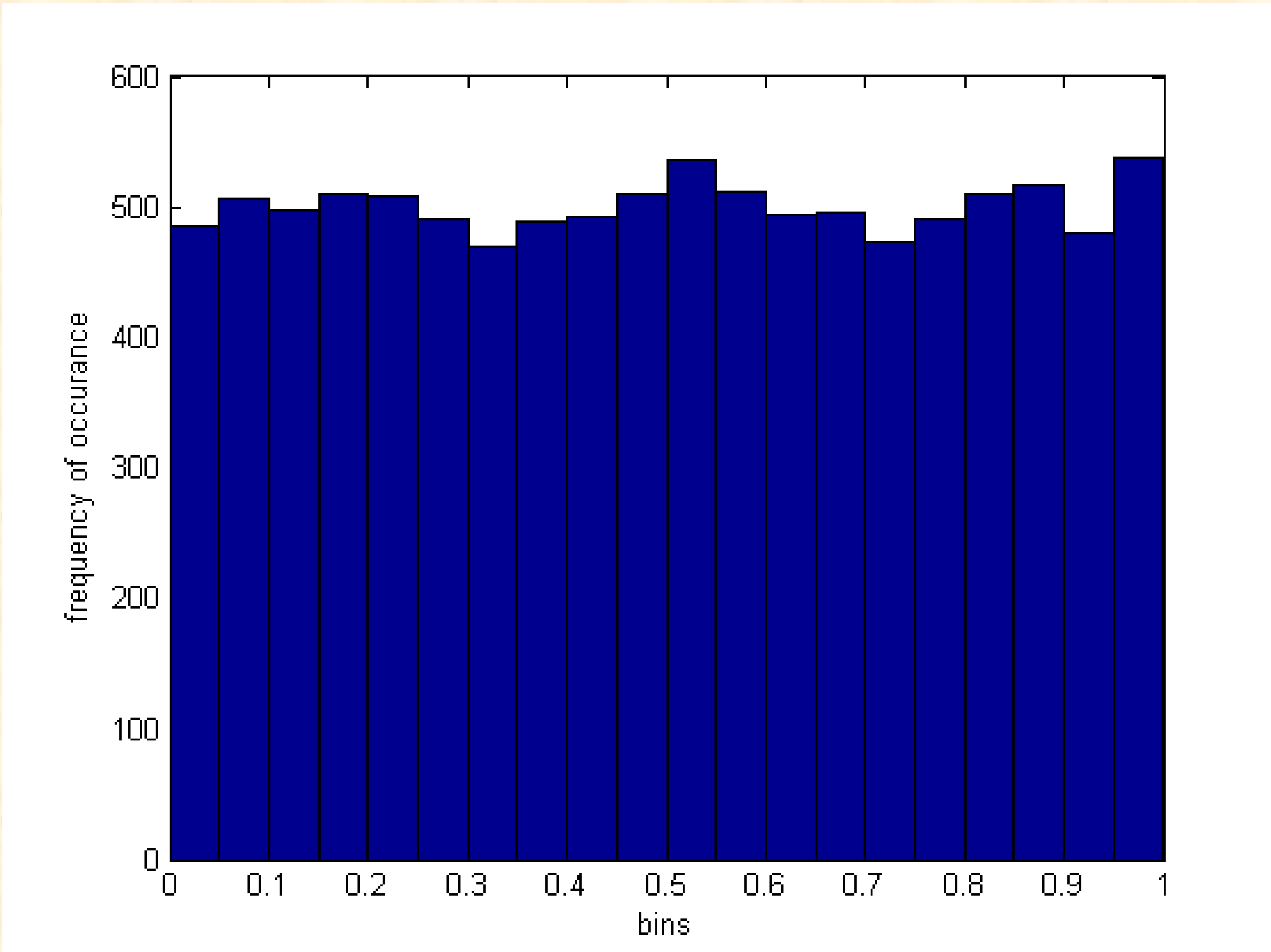
- **Insensitive to seeds** -- period and randomness properties should not depend on the initial seeds.

*http://www.npac.syr.edu/users/paulc/lectures/montecarlo/p_montecarlo.html

Concepts

- Test if r.n. is "uniformly", i.e., equally distributed
- Generate 10,000 r.n. and divide the (0,1) axis into 20 equal intervals.
- Observe the frequency of occurrence of r.n. in each interval. The ideal mean value is 500.
- Observe the standard deviation. The standard deviation should be less than 36.

Results



Std=18.05

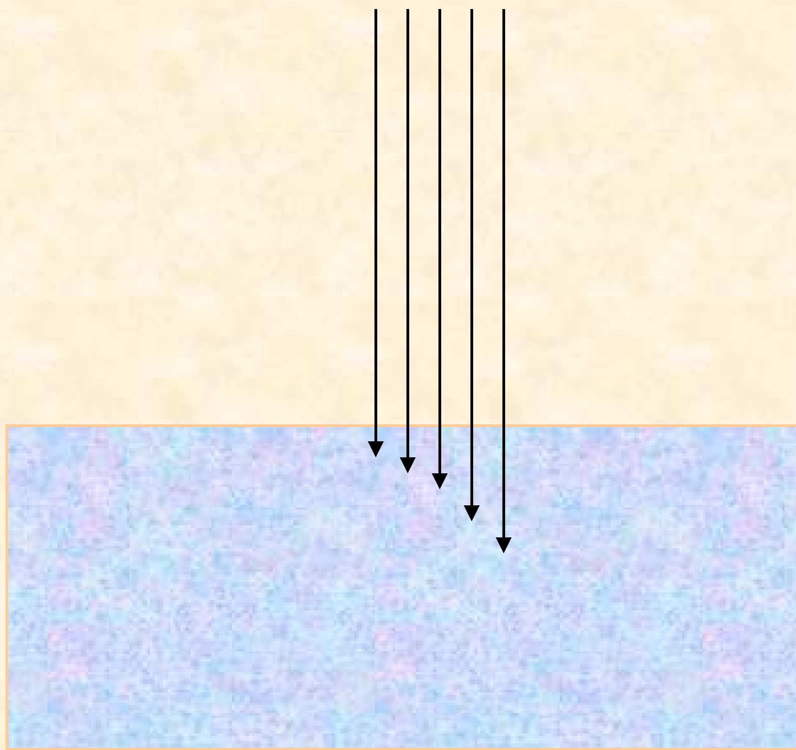
Attenuation of a Collimated Beam

- The goal of this step is to figure out how photons are attenuated in the medium.

- Attenuation coefficient is c with units m^{-1}.

- Attenuation obeys Beer-Lambert-Bouguer's Law:

  $L = L_0 e^{-cz}$, where z is the depth within the medium.



➔The probability of attenuation within the medium in the interval [z, z+ dz] is $c dz$, for $dz \ll 1/c$.

Concepts

- After moving a step, a photon is either absorbed, scattered or transmitted. However, to simplify the problem, we only consider absorption in this step, that is , no scattering occurs. Therefore, $c = a = 1\ m^{-1}$ in this program.

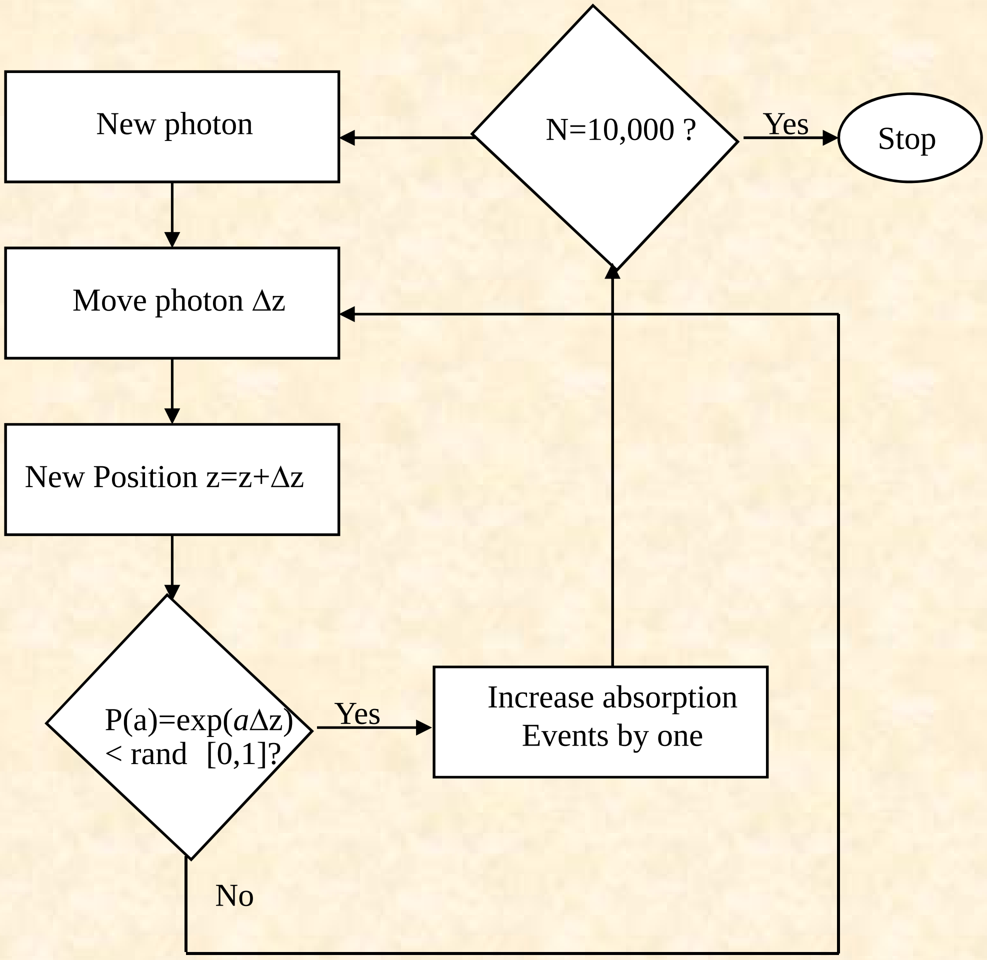
- To simplify further, assume index matched boundary ($n_1 = n_2$), fixed step, and fixed weight photon.

- Index matching means that no reflection occurs at the boundary.

- Fixed step means that absorption only happens at the end of a fixed step Δz. In the following program, $\Delta z = 20$ cm.

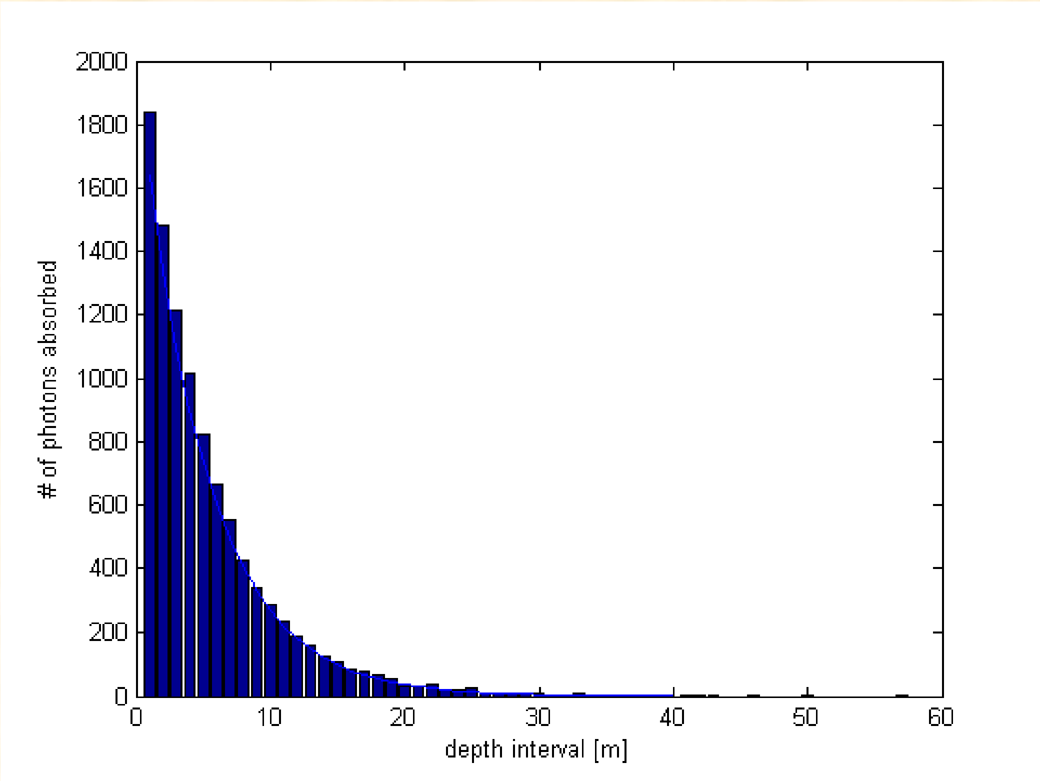
- Fixed weight photon means that a photon is treated as an integral particle. A photon cannot be partially absorbed.

Flow Chart



New photon
N=10,000 ?
Yes
Stop
Move photon Δz
New Position z=z+Δz
P(a)=exp(aΔz)
< rand [0,1]?
Yes
Increase absorption
Events by one
No

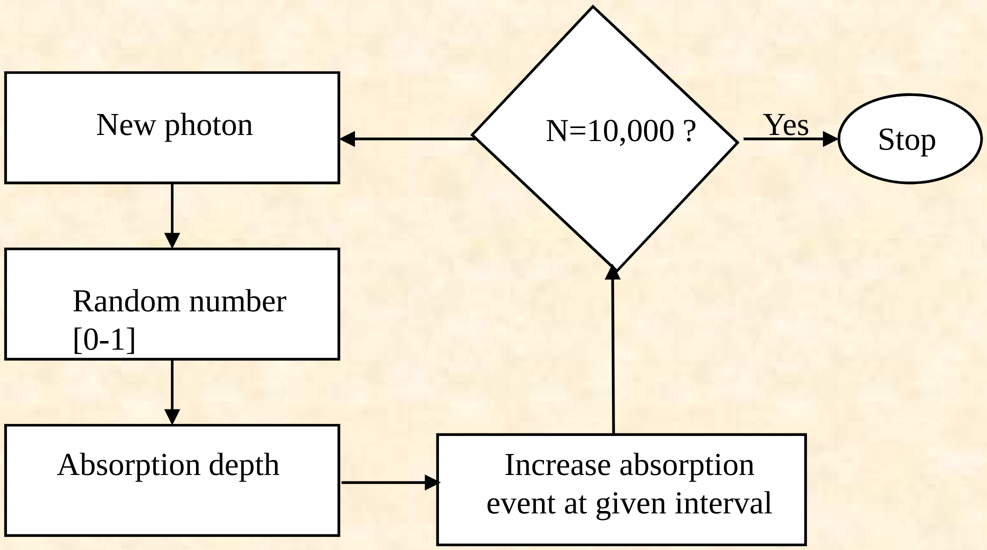
Results



Matlab program (code MC_abs.m):

```
a=1; %absorption [m^-1];
dz=0.2; %depth interval;
depths=[0:dz:20/a]; %depth discritization
interval=zeros(1,length(depths));
for i=1:10000
    k=1;
    while rand(1)>exp(a*dz)
        k=k+1;
    end
    interval(k)=interval(k)+1;
end
bar(interval);
hold on
plot([1:1:40],dz*10000*exp(-a*[1:1:40]*dz));
hold off
xlabel('depth interval')
ylabel('# of photons absorbed')
```

A better algorithm



Probability of absorption at depth between z and z+dz:

$\exp(-az) \times (1-\exp(-a\,dz))$

We can map [0,1] to the depths photon reach:

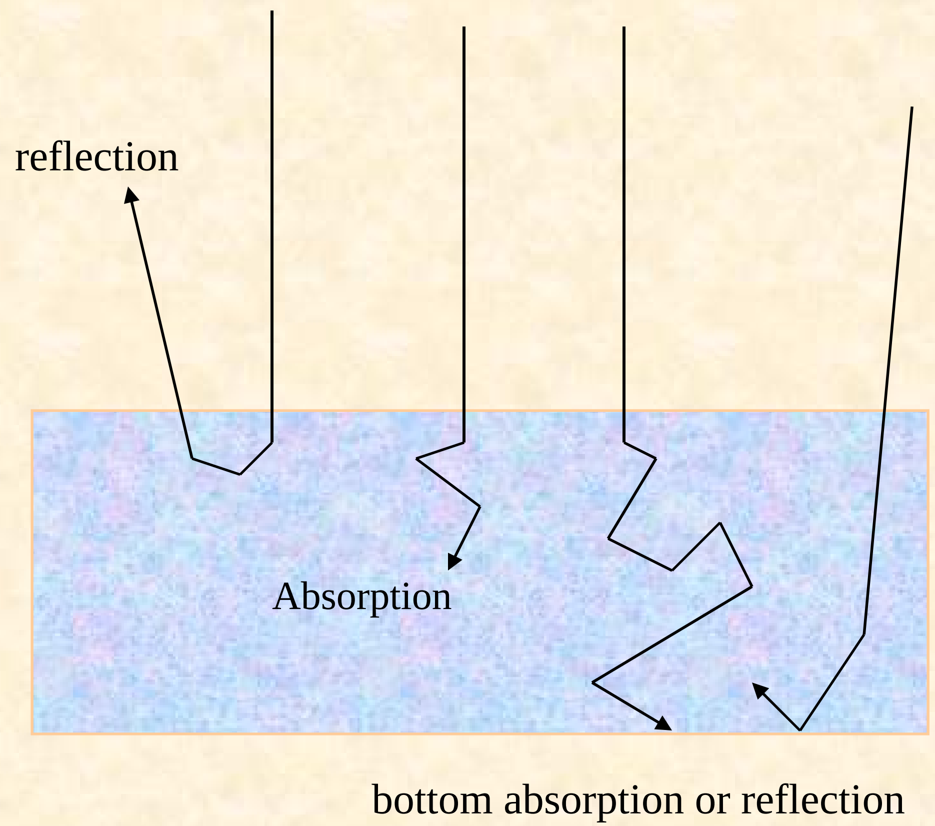
$z(abs) = -\ln(1-r.n)/a = -\ln(r.n)/a$

A single calculation per photon.

Code: MC_abs.m (2nd part)

Scattering of Photons in medium

- photons are either absorbed, transmitted through the medium or reflected back from the medium.
- We keep track of each photon and tally where all the photons terminate.
- Boundaries are index matched (can do it differently if needed).
- The step length of each photon is variable.



Concepts

- In this step we will simulate the scattering of a photon in the medium.
- Scattering is accomplished by accounting for the position and direction of each photon.
- If a photon exits the medium ($z<0$), it has been reflected. Determining which depends on the z position at the exit point. At this point the photon is terminated (could be reflected back).
- Termination of the photon can also occur due to absorption in the medium or bottom.
- Step length is no longer a fixed value. It is calculated as:

$$z = -\log(\text{r.n.})/c$$

Deflection Angle

- Deflection angle (θ) : When a photon is scattered it is deflected by this angle relative to its propagation direction:

$$\int_0^{\theta} \beta(\theta')\sin(\theta')\,d\theta' = \frac{b}{2\pi}\cdot \mathrm{r.n.}$$

For empirical VSF use a lookup table and interpolation to save time on computations.

For certain idealized functions we have analytical expressions:

For Rayleigh, Raman and Pure-water scattering

$$-\frac{f}{6+2f}\mu^3-\frac{3}{6+2f}\mu^2+\frac{1}{2}-\xi=0$$

Where f=0.835 for pure water 0.55 for Raman at 490nm and 1 for Rayleigh scattering. μ is the only positive root for a given ξ.

Henyey-Greenstein:

$$\mu_s=\frac{1}{2g}\left[1+g^2-\left(\frac{1-g^2}{1+g-2g\xi}\right)\right]$$

Where *g*>0 is the asymmetry parameter

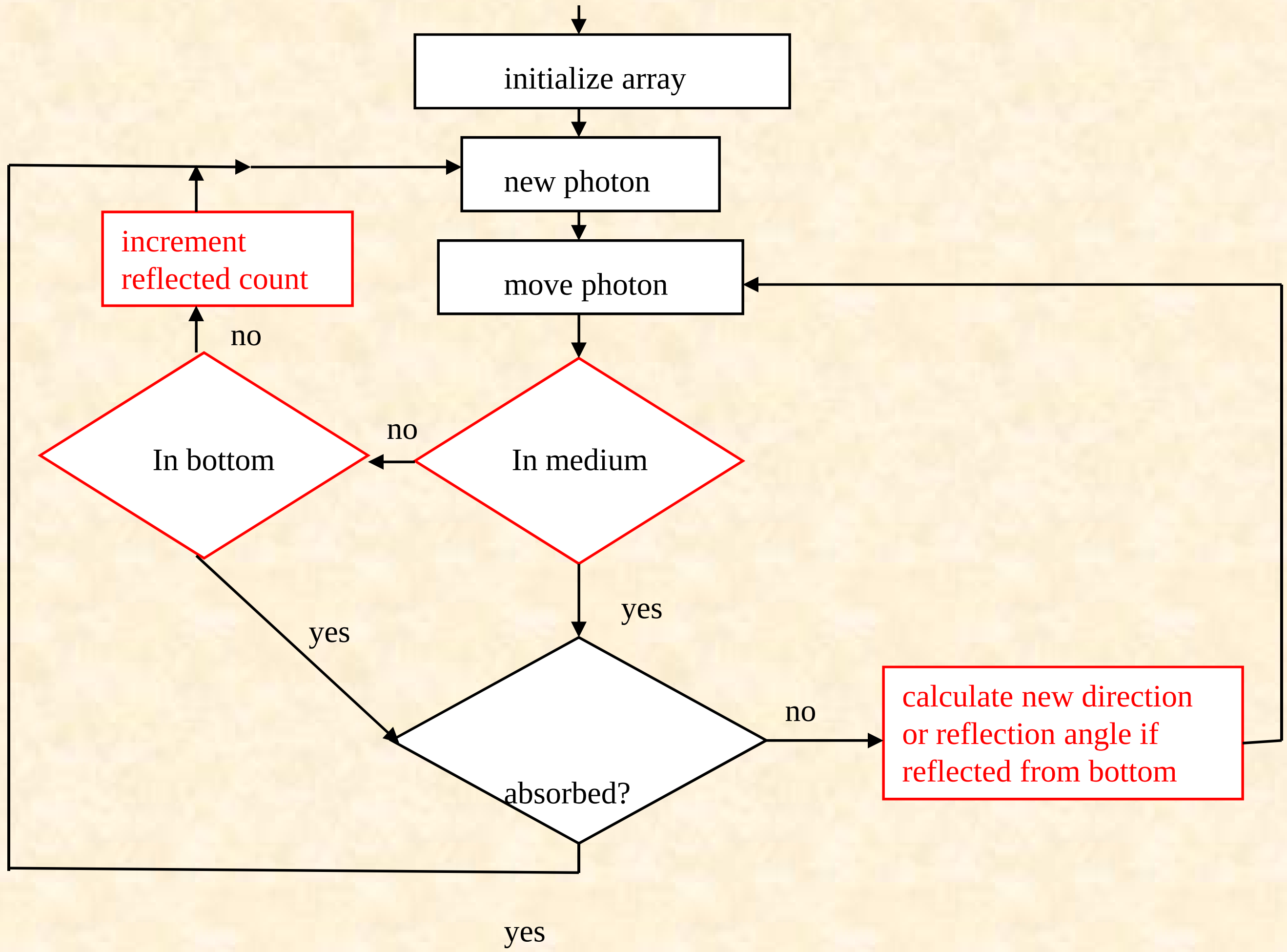
Program: Single_photon.m

Azimuthal Angle

- Azimuthal angle (ϕ) : When deflected by the Deflection angle, it is also deflected with respect to the orthogonal plane of propagation.

$$\phi = 2 * \pi * r.n.$$

Flow Chart


initialize array
new photon
increment
reflected count
move photon
no
no
In bottom
In medium
yes
yes
calculate new direction
or reflection angle if
reflected from bottom
no
absorbed?
yes

Variable Weight Photons

- Now, as a photon encounters the medium it's weight is decremented.

- Once the weight is small enough the photon will either terminate or be re-energized. A threshold is set to determine if the weight is significantly small.

- A photon is only re-energize every 1/threshold photons to keep the energy balanced. This concept is called roulette.

```
if(r.n. < threshold)
  weight = weight * 20;
else
  weight = 0;
```

Plane-Parallel semi infinite scattering medium illuminated from above:

```
c = 1; % attenuation coefficient (1/m)
w0 = 0.8; % single-scattering albedo
N = 1e5; % number of photons to trace
ns = 100; % max number of scatters per photon
E = 0; % initialize detector
for i=1:N
    z = 0; % initial position depth
    muz = 1; % incident light direction (collimated)
    w = 1; % initial photon weight
    for j=1:ns
        s = - log(rand)/c; % geometric path length
        z = z + muz*s; % move photon
        if (z<0) E=E+w; break, end % count photons leaving out top
        w=w*w0; % absorb fraction of photon packet
        muz= 1 - 2*rand; % isotopic scattering
    end
end
E = E/N % normalize result
```

See program: semi-infinite.m

Index Mismatch at boundary

- In this step, the index-mismatched boundary between air and water is taken into effect. When light enters water, the total reflectance R_t should include both the specular reflection R_{sp} and the remitted diffuse reflectance R_d.

Develop a Grid Structure

- To keep track of the locations at which photons are absorbed within the water.
- To accomplish this develop a two/three-dimensional array of bins corresponding to the depth/location of the photon.
- With each absorption event, the photon's weight will be added into the bin corresponding to the photon's current location.

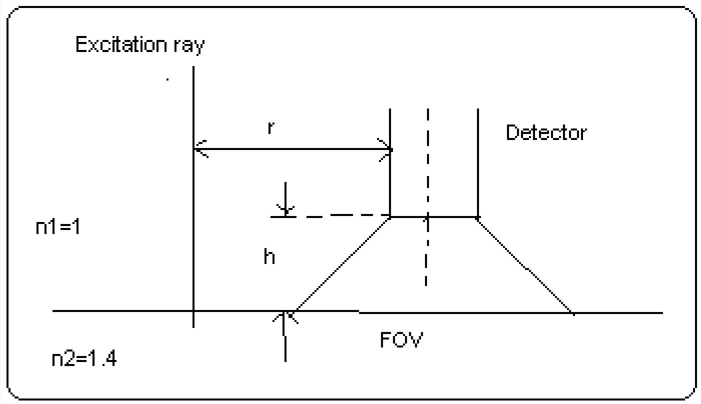
Detection of photons

Including a detector to detect the number of photons which enter the detector field of view.

- The detector is placed within the previously defined grid boundaries.

Detector Variables

- Detector position (Lateral distance from ray and the height above water)
- The detector radius
- The field of view of the detector (FOV)



Inverse Monte Carlo

- Use the principle of reciprocity: if a photon makes it from a to b, a photon can make it from b to a along the same path.
- Only simmulates photons that made it to the detector (as opposed to the whole medium).

Monte Carlo vs. other RT

- MC is the most flexible of all RT can accommodate any arrangement of source, receiver and medium properties.
- MC is simple to conceptualize.
- MC is slow. Need to simmulate millions of photons to reduce statistical error.

Monte Carlo Programming

A tutorial to Monte Carlo methods:
http://people.revoledu.com/kardi/tutorial/Simulation/index.html
List of tutorials:
http://www.cooper.edu/engineering/chemechem/MMC/tutor.html

Monte Carlo Simulation for Statistical Physics by Paul Coddington at Northeast Parallel Architectures Center at Syracuse University: An electronic book covering Monte Carlo programming in depth – includes discussion on Monte Carlo methods, specific algorithms, and problems and subtleties in Monte Carlo programming.
http://www.npac.syr.edu/users/paulc/lectures/montecarlo/p_montecarlo.html

Radiative transfer codes and documentation can also be found at:
http://www-star.st-and.ac.uk/~kw25/research/montecarlo/montecarlo.html