INTRODUCTION

If you are uncomfortable with the math required to solve the word problems in this class, we strongly encourage you to take a day to look through the following links and notes. Some of them contain exercises - they are the best way to understand and learn / remember the math, so solve them on your own!

- A useful set of rules and exercises related to the scientific notation: http://www.docstoc.com/docs/2479096/English-%EF%BF%BD-%EF%BF%BD-Algebra-Cheat-Sheet – the website asks you to buy the document if you want to download it, but you can easily read it online, and even print it out.

- Operations with positive and negative numbers - see first chapter below.

- Rules related to working with fractions - see second chapter below

- Basic rules about solving equations - see third chapter below

- Some essential pre-algebra (properties and order of basic mathematical operations) - fourth chapter of this document

- Finally, an assortment of algebra rules. This is an edited version of http://tutorial.math.lamar.edu/pdf/Algebra_Cheat_Sheet.pdf. My opinion is that you need NOT learn everything there by heart. I highlighted those rules that are a 'must', then some that are useful to know, but are just derived from the main definitions, and I cut out those 'rules' that were just particular cases of the main ones, and that you really don't need to remember as separate entities. - see the algebra cheat sheet at the end of this document.

OPERATIONS WITH POSITIVE AND NEGATIVE NUMBERS

1. **Adding Rules:**

Positive + Positive = Positive: 5 + 4 = 9
Negative + Negative = Negative: (- 7) + (- 2) = - 9

Sum of a negative and a positive number: Use the sign of the larger number and subtract

(- 7) + 4 = -3
6 + (-9) = - 3
(- 3) + 7 = 4
5 + (-3) = 2

2. **Subtracting Rules:**

Negative - Positive = Negative: (- 5) - 3 = -5 + (-3) = -8
Positive - Negative = Positive + Positive = Positive: 5 - (-3) = 5 + 3 = 8
Negative - Negative = Negative + Positive = Use the sign of the larger number and subtract *(Change double negatives to a positive)*

(-5) - (-3) = (-5) + 3 = -2
(-3) - (-5) = (-3) + 5 = 2

3. **Multiplying Rules:**

Positive x Positive = Positive: 3 x 2 = 6
Negative x Negative = Positive: (-2) x (-8) = 16
Negative x Positive = Negative: (-3) x 4 = -12
Positive x Negative = Negative: 3 x (-4) = -12

4. **Dividing Rules:**

Positive ÷ Positive = Positive: 12 ÷ 3 = 4
Negative ÷ Negative = Positive: (-12) ÷ (-3) = 4
Negative ÷ Positive = Negative: (-12) ÷ 3 = -4
Positive ÷ Negative = Negative: 12 ÷ (-3) = -4

Tips:

1. When working with rules for positive and negative numbers, try and think of weight loss or poker games to help solidify 'what this works'.
2. Using a number line showing both sides of 0 is very helpful to help develop the understanding of working with positive and negative numbers/integers.
Source: http://math.about.com/od/prealgebra/ht/PostiveNeg.htm

FRACTIONS

Like fractions have the same denominator (2/3 and 1/3 are like fractions). You can add and subtract like fractions easily—simply add or subtract the numerators and write the sum over the common denominator.

$$1/3 + 2/3 = 3/3$$
$$5/7 - 2/7 = 3/7$$

Before you can add or subtract fractions with different denominators, you must first find equivalent fractions with the same denominator, or the **least common denominator (LCM)**. Here's how:

1. Find the smallest multiple (LCM) of both numbers.
2. Rewrite the fractions as equivalent fractions with the LCM as the denominator.

$$1/5 + 1/3 = \frac{1 \cdot 3}{5 \cdot 3} + \frac{1 \cdot 5}{3 \cdot 5} = (3/15) + (5/15) = 8/15$$

The same rules apply for subtracting fractions with different denominators.

Multiplying and Dividing Fractions:

Multiplication Rule: $a/b \cdot c/d = ac/bd$

Multiply the two numerators over the two denominators.

$$1/3 \cdot 4/5 = \frac{1 \cdot 4}{3 \cdot 5} = 4/15$$

Division Rule: Multiply the dividend by the reciprocal of the divisor.

$$\frac{2/5}{3/4} = 2/5 \cdot 4/3 = 8/15$$

Practice exercises:

6.) Are 4/5 and 8/10 equal?
7.) 4/5 – 2/3 =
8.) ½ + 3/8 =
9.) 3/10 + 4/15 =
10.) 11/56 + 3/7 =
11.) 2/7 x 3/8 =
12.) 2/9 x 4/9 =
13.) 1/8 ÷ ½ =
14.) 2/9 ÷ 6/8 =
15.) 2/3 ÷ 3/8 =

Answers:
6.) Yes; 4 x 10 = 8 x 5; 7.) 2/15; 8.) 7/8; 9.) 17/30; 10.) 5/8; 11.) 3/28; 12.) 8/81; 13.) ¼; 14.) 8/27; 15.) 16/9

Source: http://tutoring.sylvanlearning.com/newsletter/0103/cheat68.cfm

SOLVING EQUATIONS

Keep It Simple!

When a problem can be simplified, you should simplify before substituting numbers for the variables. This will make your job a lot easier. Here's How:

2(3 + x) + x(1-4x) + 5

1. Simplify the parentheses.
   $6 + 2x + x - 4x^2 + 5$
2. Combine like terms by adding coefficients.
   $6 + 3x - 4x^2 + 5$
3. Combine the constants.
   $11 + 3x - 4x^2$

To keep it clear, here are a few rules on the order of operations.

1. First, do all operations that lie inside parentheses.
2. Next, do any work with exponents or radicals.
3. Working from left to right, do all multiplication and division.
4. Finally, working from left to right, do all addition and subtraction.

Here's an example to work through.

$8 \cdot 2^2 + 7y\ (4 + 1) = 32+35y$

1. First add the elements in parentheses. (4 + 1) = 5.
2. Next, carry out the exponent. $2^2 = 4$.
3. Multiply $8 \cdot 2^2$ or $8 \cdot 4 = 32$.
4. Multiply $7y \cdot 5 = 35y$.
5. Add the remaining elements from left to right: 32 + 35y

Isolate the variables (if you are dealing with linear equations) or put the equation in polynomial form (if you are dealing with higher-order equations), by making the <u>same changes to the left side and the right side of the equation</u>.

Example: 16*x = x+30 |-x (subtract x from both sides of the equation) <=>
<=> 16*x –x = x+30-x <=>
<=> (16-1)*x = 30+(x-x) <=>
<=> 15*x = 30 |:15 (divide both sides to 15) <=>
<=> x = 30/15 <=>
<=> x=2

Practice exercises:
<u>Solve for x (x=?):</u>
1.) -6 = x – 2
2.) -3x – 5 = 7
3.) -106 = 4x + 6x + 4
4.) 4 (7 + 6x) = -260
5.) -6x – 3 = 5x -47
Answers: 1.) x = -4; 2.) x = -4; 3.) x = -11; 4.) x = -12; 5.) x = 4
Source: http://tutoring.sylvanlearning.com/newsletter/0103/cheat68.cfm

PRE-ALGEBRA

Addition and Subtraction

First off: Subtraction is nothing but a particular case of addition.
Writing a-b is the same as writing a+(-b) where -b is the opposite (or additive inverse) of b. If you know this, and the properties of addition (listed below), then you can immediately derive the properties of subtraction.

Properties of addition
Commutativity: **a+b=b+a** (Note: in the case of subtraction, by applying this rule you get: a-b = a+(-b) = -b+a = -(b-a). You do NOT get a-b=b-a!)
Associativity: **(a+b)+c = a+(b+c)**. Note: if you also use the property of commutativity, then you get (a+b)+c = a+(b+c) = a+(c+b) = (a+c)+b (This means that you can change the order of the added numbers, you can group them differently, and you will still obtain the same result.)
Identity element: 0. **a+0=a** (the addition of 0 does not change the result)
Additive inverse: **a+(-a)=0** (-a is the additive inverse of a; adding a number to its additive inverse results in the identity element).

Multiplication and Division

Just like subtraction is a particular case of addition, division is a particular case of multiplication. Writing a/b is the same as a*(1/b), where 1/b is the reciprocal (or multiplicative inverse) of b.

Properties of multiplication
Commutativity: **a*b = b*a** (Note: in the case of division you get: a/b = a*1/b = [1/(1/a)]*(1/b) = 1/(b/a))
Associativity: **(a*b)*c = a*(b*c)**. Just like with addition, you can use the associative and commutative properties together to get: (a*b)*c = a*(b*c) = a*(c*b) = (a*c)*b .
Identity element: 1. **a*1=a** (multiplying by 1 does not change the result). Note that the identity element for multiplication is different than that of addition!
Multiplicative inverse: **a*(1/a)=1** (1/a is the multiplicative inverse of a; multiplying a number by its multiplicative inverse results in the identity element).

Common property of addition and multiplication: Distributivity

a*(b+c) = a*b + a*c

Order of Operations

1. Work within parentheses ( ), brackets [ ], and braces { }, from the innermost outward.
2. Powers (and roots, which are just particular cases of powers)
3. Multiplications (and divisions)
4. Additions (and subtractions)

Algebra cheat sheet legend

Whatever is in a red rectangle you need to know!

Whatever is crossed with a dashed line you need NOT learn by heart – it can be easily derived. It IS correct, however! ☺

These you have to know at a glance, but need not learn them by heart (just think about the definition, and you will see that they are all derived from it.

Algebra Cheat Sheet

Basic Properties & Facts

Arithmetic Operations

$ab + ac = a(b+c)$

$a\left(\frac{b}{c}\right) = \frac{ab}{c}$

$\frac{\left(\frac{a}{b}\right)}{c} = \frac{a}{bc}$

$\frac{a}{\left(\frac{b}{c}\right)} = \frac{ac}{b}$

$\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$

$\frac{a}{b} - \frac{c}{d} = \frac{ad-bc}{bd}$

$\frac{a-b}{c-d} = \frac{b-a}{d-c}$

$\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$

$\frac{ab+ac}{a} = b+c, \quad a \neq 0$

$\frac{\left(\frac{a}{b}\right)}{\left(\frac{c}{d}\right)} = \frac{ad}{bc}$

Exponent Properties

$a^n a^m = a^{n+m}$

$\left(a^n\right)^m = a^{nm}$

$(ab)^n = a^n b^n$

$a^{-n} = \frac{1}{a^n}$

$\frac{a^n}{a^m} = a^{n-m} = \frac{1}{a^{m-n}}$

$a^0 = 1, \quad a \neq 0$

$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

$\frac{1}{a^{-n}} = a^n$

$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n = \frac{b^n}{a^n}$

$a^{\frac{n}{m}} = \left(a^{\frac{1}{m}}\right)^n = \left(a^n\right)^{\frac{1}{m}}$

Also:

0^0 does NOT exist!

Properties of Radicals

$\sqrt[n]{a} = a^{\frac{1}{n}}$

$\sqrt[n]{ab} = \sqrt[n]{a}\sqrt[n]{b}$

$\sqrt[m]{\sqrt[n]{a}} = \sqrt[nm]{a}$

$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

$\sqrt[n]{a^n} = a$, if n is odd

$\sqrt[n]{a^n} = |a|$, if n is even

Properties of Inequalities

If $a < b$ then $a + c < b + c$ and $a - c < b - c$

If $a < b$ and $c > 0$ then $ac < bc$ and $\frac{a}{c} < \frac{b}{c}$

If $a < b$ and $c < 0$ then $ac > bc$ and $\frac{a}{c} > \frac{b}{c}$

Properties of Absolute Value

$|a| = \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a < 0 \end{cases}$

$|a| \geq 0$ $\qquad |-a| = |a|$

$|ab| = |a||b|$ $\qquad \left|\frac{a}{b}\right| = \frac{|a|}{|b|}$

$|a+b| \leq |a| + |b|$ Triangle Inequality

Distance Formula

If $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$ are two points the distance between them is

$$d(P_1, P_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

This is geometry.

Complex Numbers

$i = \sqrt{-1} \qquad i^2 = -1 \qquad \sqrt{-a} = i\sqrt{a}, \quad a \geq 0$

$(a+bi) + (c+di) = a + c + (b+d)i$

$(a+bi) - (c+di) = a - c + (b-d)i$

$(a+bi)(c+di) = ac - bd + (ad+bc)i$

$(a+bi)(a-bi) = a^2 + b^2$

$|a+bi| = \sqrt{a^2+b^2}$ Complex Modulus

$\overline{(a+bi)} = a - bi$ Complex Conjugate

$\overline{(a+bi)}(a+bi) = |a+bi|^2$

Don't worry about complex numbers for now.

Logarithms and Log Properties

Definition

$y = \log_b x$ is equivalent to $x = b^y$

Example

$\log_5 125 = 3$ because $5^3 = 125$

Special Logarithms

$\ln x = \log_e x$ natural log

$\log x = \log_{10} x$ common log

where $e = 2.718281828\ldots$

Logarithm Properties

$\log_b b = 1$ $\log_b 1 = 0$

$\log_b b^x = x$ $b^{\log_b x} = x$

$\log_b\left(x^r\right) = r\log_b x$

$\log_b\left(xy\right) = \log_b x + \log_b y$

$\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$

The domain of $\log_b x$ is $x > 0$

Factoring and Solving

These are useful, but not essential.

Factoring Formulas

$x^2 - a^2 = (x+a)(x-a)$

$x^2 + 2ax + a^2 = (x+a)^2$

$x^2 - 2ax + a^2 = (x-a)^2$

$x^2 + (a+b)x + ab = (x+a)(x+b)$

$x^3 + 3ax^2 + 3a^2x + a^3 = (x+a)^3$

$x^3 - 3ax^2 + 3a^2x - a^3 = (x-a)^3$

$x^3 + a^3 = (x+a)(x^2 - ax + a^2)$

$x^3 - a^3 = (x-a)(x^2 + ax + a^2)$

$x^{2n} - a^{2n} = (x^n - a^n)(x^n + a^n)$

If n is odd then,

$x^n - a^n = (x-a)(x^{n-1} + ax^{n-2} + \cdots + a^{n-1})$

$x^n + a^n$

$= (x+a)(x^{n-1} - ax^{n-2} + a^2x^{n-3} - \cdots + a^{n-1})$

Quadratic Formula

Solve $ax^2 + bx + c = 0$, $a \neq 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

If $b^2 - 4ac > 0$ - Two real unequal solns.

If $b^2 - 4ac = 0$ - Repeated real solution.

If $b^2 - 4ac < 0$ - Two complex solutions.

Square Root Property

If $x^2 = p$ then $x = \pm\sqrt{p}$

Absolute Value Equations/Inequalities

If b is a positive number

$|p| = b \quad \Rightarrow \quad p = -b$ or $p = b$

$|p| < b \quad \Rightarrow \quad -b < p < b$

$|p| > b \quad \Rightarrow \quad p < -b$ or $p > b$

Completing the Square

Solve $2x^2 - 6x - 10 = 0$

(1) Divide by the coefficient of the x^2

$$x^2 - 3x - 5 = 0$$

(2) Move the constant to the other side.

$$x^2 - 3x = 5$$

(3) Take half the coefficient of x, square it and add it to both sides

$$x^2 - 3x + \left(-\frac{3}{2}\right)^2 = 5 + \left(-\frac{3}{2}\right)^2 = 5 + \frac{9}{4} = \frac{29}{4}$$

(4) Factor the left side

$$\left(x - \frac{3}{2}\right)^2 = \frac{29}{4}$$

(5) Use Square Root Property

$$x - \frac{3}{2} = \pm\sqrt{\frac{29}{4}} = \pm\frac{\sqrt{29}}{2}$$

(6) Solve for x

$$x = \frac{3}{2} \pm \frac{\sqrt{29}}{2}$$

This is a more advanced chapter.
Make sure you are comfortable with the previous concepts before getting into this!

Functions and Graphs

Constant Function

$y = a$ or $f(x) = a$

Graph is a horizontal line passing through the point $(0, a)$.

Line/Linear Function

$y = mx + b$ or $f(x) = mx + b$

Graph is a line with point $(0, b)$ and slope m.

Slope

Slope of the line containing the two points (x_1, y_1) and (x_2, y_2) is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{rise}}{\text{run}}$$

Slope – intercept form

The equation of the line with slope m and y-intercept $(0, b)$ is

$$y = mx + b$$

Point – Slope form

The equation of the line with slope m and passing through the point (x_1, y_1) is

$$y = y_1 + m(x - x_1)$$

Parabola/Quadratic Function

$y = a(x-h)^2 + k$ $f(x) = a(x-h)^2 + k$

The graph is a parabola that opens up if $a > 0$ or down if $a < 0$ and has a vertex at (h, k).

Parabola/Quadratic Function

$y = ax^2 + bx + c$ $f(x) = ax^2 + bx + c$

The graph is a parabola that opens up if $a > 0$ or down if $a < 0$ and has a vertex at $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$.

Parabola/Quadratic Function

$x = ay^2 + by + c$ $g(y) = ay^2 + by + c$

The graph is a parabola that opens right if $a > 0$ or left if $a < 0$ and has a vertex at $\left(g\left(-\frac{b}{2a}\right), -\frac{b}{2a}\right)$.

Circle

$(x-h)^2 + (y-k)^2 = r^2$

Graph is a circle with radius r and center (h, k).

Ellipse

$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$

Graph is an ellipse with center (h, k) with vertices a units right/left from the center and vertices b units up/down from the center.

Hyperbola

$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

Graph is a hyperbola that opens left and right, has a center at (h, k), vertices a units left/right of center and asymptotes that pass through center with slope $\pm\frac{b}{a}$.

Hyperbola

$\frac{(y-k)^2}{b^2} - \frac{(x-h)^2}{a^2} = 1$

Graph is a hyperbola that opens up and down, has a center at (h, k), vertices b units up/down from the center and asymptotes that pass through center with slope $\pm\frac{b}{a}$.

Useful!
Look over these and understand what the error is in each example.

Common Algebraic Errors

Error	Reason/Correct/Justification/Example
$\frac{2}{0} \neq 0$ and $\frac{2}{0} \neq 2$	Division by zero is undefined!
$-3^2 \neq 9$	$-3^2 = -9$, $(-3)^2 = 9$ Watch parenthesis!
$(x^2)^3 \neq x^5$	$(x^2)^3 = x^2 x^2 x^2 = x^6$
$\frac{a}{b+c} \neq \frac{a}{b} + \frac{a}{c}$	$\frac{1}{2} = \frac{1}{1+1} \neq \frac{1}{1} + \frac{1}{1} = 2$
$\frac{1}{x^2 + x^3} \neq x^{-2} + x^{-3}$	A more complex version of the previous error.
$\frac{\not{a} + bx}{\not{a}} \neq 1 + bx$	$\frac{a+bx}{a} = \frac{a}{a} + \frac{bx}{a} = 1 + \frac{bx}{a}$ Beware of incorrect canceling!
$-a(x-1) \neq -ax - a$	$-a(x-1) = -ax + a$ Make sure you distribute the "-"!
$(x+a)^2 \neq x^2 + a^2$	$(x+a)^2 = (x+a)(x+a) = x^2 + 2ax + a^2$
$\sqrt{x^2 + a^2} \neq x + a$ $\sqrt{x+a} \neq \sqrt{x} + \sqrt{a}$	$5 = \sqrt{25} = \sqrt{3^2 + 4^2} \neq \sqrt{3^2} + \sqrt{4^2} = 3 + 4 = 7$ See previous error.
$(x+a)^n \neq x^n + a^n$ and $\sqrt[n]{x+a} \neq \sqrt[n]{x} + \sqrt[n]{a}$	More general versions of previous three errors.
$2(x+1)^2 \neq (2x+2)^2$	$2(x+1)^2 = 2(x^2 + 2x + 1) = 2x^2 + 4x + 2$ $(2x+2)^2 = 4x^2 + 8x + 4$ Square first then distribute!
$(2x+2)^2 \neq 2(x+1)^2$	See the previous example. You can not factor out a constant if there is a power on the parethesis!
$\sqrt{-x^2 + a^2} \neq -\sqrt{x^2 + a^2}$	$\sqrt{-x^2 + a^2} = (-x^2 + a^2)^{\frac{1}{2}}$ Now see the previous error.
$\frac{a}{\left(\frac{b}{c}\right)} \neq \frac{ab}{c}$	$\frac{a}{\left(\frac{b}{c}\right)} = \frac{\left(\frac{a}{1}\right)}{\left(\frac{b}{c}\right)} = \left(\frac{a}{1}\right)\left(\frac{c}{b}\right) = \frac{ac}{b}$
$\frac{\left(\frac{a}{b}\right)}{c} \neq \frac{ac}{b}$	$\frac{\left(\frac{a}{b}\right)}{c} = \frac{\left(\frac{a}{b}\right)}{\left(\frac{c}{1}\right)} = \left(\frac{a}{b}\right)\left(\frac{1}{c}\right) = \frac{a}{bc}$

For a complete set of online Algebra notes visit http://tutorial.math.lamar.edu.