Appendix:

Uncertainties of a measurement:

In measuring lengths of rods or their weights, an uncertainty due to finite digitization of the measurement device is associated with the measurement. This uncertainty is ± 0.5 times the smallest unit that we can resolve. This uncertainty provides the precision of the measurement. We assume that the measurements are unbiased, that is, that our measuring devices are accurate. Our only assurance that this assumption is true is that the scale and meter that we use are regularly calibrated. Often uncertainties are termed "errors," and the two terms are used interchangeably.

Uncertainties in multiple *independent* measurements of the same quantity:

Each measurement (*e.g.,* of settling velocity) has an uncertainty (*e.g.,* due to the precision of the stopwatch and length-measuring device) that is most often smaller than the uncertainty of the group of measurements (due to variability in measurement techniques between different individuals and between groups). These sources of variability are separated in the uncertainty analysis into uncertainties within and among groups and can be nested to any arbitrary degree (*e.g.,* multiple observations by a single individual within a group are pooled to produce the group's measurements, which in turn are pooled to produce the class' measurements). In a normal distribution, spread is quantified by the standard deviation; 68 % of data lie within one standard deviation from the mean; 95.5 % of the lie within 2 standard deviations. Uncertainty of the estimate of the mean (the standard error of the mean) is reduced by a factor of $1/\sqrt{n}$ when making n *independent* measurements. This outcome reflects the way that n independent measurements increase confidence in the outcome. Notice that when n = 2 a few more replicates decrease uncertainty a lot, whereas once you reach about 20 or so replicates, it takes a good many more replicates to improve matters much further.

While the standard deviation of a measurement does not necessarily change as we add more measurements, the standard error of the mean, that is the uncertainty in the mean value, equals the standard deviation divided by $\sqrt{n}$, and decreases the more measurements we have.

Uncertainty in a derived quantity (propagation of errors):

The absolute (*dz*) or relative (*dz/z*) uncertainties of a sum, difference, product or the ratio (*z*) of two independent variables (*x, y*) with uncertainties (*dx*, *dy*) are computed as follows. (You can derive it yourself from the chain rule in differentiation or see it in Taylor J., 1997, *An Introduction to Error Analysis,* 2nd, Ed., University Science Books, Herndon, VA). Here *dz, dx* and *dy* denote the uncertainties in *z, x* and *y,* respectively.

$$z = x + y \rightarrow dz = \sqrt{(dx)^2 + (dy)^2}$$

$$z = x - y \rightarrow dz = \sqrt{(dx)^2 + (dy)^2}$$

$$z = x \cdot y \rightarrow \frac{dz}{z} = \sqrt{\left(\frac{dx}{x}\right)^2 + \left(\frac{dy}{y}\right)^2}$$

$$z = \frac{x}{y} \rightarrow \frac{dz}{z} = \sqrt{\left(\frac{dx}{x}\right)^2 + \left(\frac{dy}{y}\right)^2}$$

For example from measurements of three sides of a cube (say, x+/-dx, y+/-dy, z+/-dz) we would like to compute the volume of a cube and the uncertainty. The volume is computed from:

$$Volume = x \cdot y \cdot z \rightarrow \frac{dVolume}{Volume} = \sqrt{\left(\frac{dx}{x}\right)^2 + \left(\frac{dy}{y}\right)^2 + \left(\frac{dz}{z}\right)^2}$$

$$\rightarrow dVolume = Volume\sqrt{\left(\frac{dx}{x}\right)^2 + \left(\frac{dy}{y}\right)^2 + \left(\frac{dz}{z}\right)^2}$$

$$density = \frac{Mass}{Volume} \rightarrow \frac{d\_density}{density} = \sqrt{\left(\frac{dMass}{Mass}\right)^2 + \left(\frac{dVolume}{Volume}\right)^2}$$

Significant figures

In everyday life, writing the number 1.3 means just that. In science, it means that you are reasonably sure that the true value (precise value, assuming that your measurement tool is working accurately) lies between 1.25 and 1.349999... When you publish any number, be sure that you can support the precision state; don't simply copy all the digits from your calculator or instrument. Do not add extra zeros after the decimal point: 4.60 implies that the true value lies between 4.595 and 4.604999… Think about what the number of significant figures implies; Reporting one significant figure implies a fractional uncertainty (the ratio between your uncertainty, δx, and your best estimate, x_b, $|\delta x|/x_b$) of 10-100% (roughly 50%), reporting two significant figures implies a fractional uncertainty of roughly 5%, and three, roughly 0.5%. Avoid arbitrary rounding by quoting rational number, for example 45/87 eggs proved infertile [instead of 0.52 of the individuals (n=87) proved infertile].

Both measurement and error should have the same dimensions AND units. Always include the leading zero (±0.4 and not ±.4). The following are all acceptable: 55.9±0.4; 56±4; 50±40. If you get disparate precision in calculations of the best estimate and its imprecision, round the more precise one to be compatible with the less precise one, after you have made sure that you can support even that number of significant figures. In scientific notation, place the uncertainty before the power of 10, e.g., $(3.43\pm0.02)x10^{-7}$ not $3.43x10^{-7} \pm 2x10^{-9}$. The latter expression is ungainly. Even though you would end up without any ambiguity if you followed the rules (multiplication before addition or subtraction), the meaning is much less quickly apparent than with the uncertainty out front, and it is much harder to see whether the precision of the measurement and the error match. Some situations warrant mild exceptions. For example, if your best estimate of the mean length of adult shrimp in a population is 38.6 mm, and you measured to the nearest whole millimeter, it is arguably less misleading to write 38.6 ± 1 mm than to write 39 ± 1 mm. Similarly, if you are doing a string of calculations, you should carry extra digits to the extent that rounding before calculation could introduce unnecessary errors, but at the end you should report the final result with the appropriate number of significant figures.