SMS-204: Integrative marine sciences II.

Lecture 2:

Topic 1: Mass, mass conservation, volume and density.

Mass is a fundamental physical property of matter. It is conserved, which makes it a very useful quantity. Conservation does not mean that the amount of water in a reservoir is always constant. It means that the total mass changes only if we remove some of it (for example by evaporation, by pumping it out or by using some for a chemical reaction such as photosynthesis) or add some (for example by rain). Mass is related to volume and density:

ρ (density) = M/V (mass/volume). $[M\ L^{-3}]$

When the density of a fluid changes (for example, due to changes in temperature), its volume changes (since mass is conserved). In solids, the changes in density with temperature are smaller than in fluids. When comparing fluids (for example to determine which will flow on top of which) mass itself is not useful; we may have lots of one fluid and little of the other, yet it is the one that is denser that will sink below the less dense fluid. We care about the mass per unit volume, *i.e.*, the density.

Topic 2: Continuity, flux.

Continuity is a principle which incorporates motion (dynamics) into the conservation of mass. Assume a flow in a pipe with diameter D_1. Assume downstream the pipe is attached to a narrower pipe of diameter D_2 (Figure 1). Conservation of *mass* for an *incompressible* fluid implies that the same *volume* of fluid passes per unit of time through any cross-section of the pipe. If the velocity at B_1 is v_1 and at B_2 is, in average, v_2 then:

$$v_1 \times \pi D_1^2/4 = v_2 \times \pi D_2^2/4 \Rightarrow v_2 = v_1 D_1^2/D_2^2.$$

More generally,

$$v_2 = v_1 A_1/A_2,$$

where A_1 and A_2 are the cross-sectional areas, *perpendicular* to the flow, at B_1 and B_2.
It follows that when a pipe narrows the fluids much accelerate and when it widens the fluid decelerates. Notice that velocity times cross-section has dimensions of volume per unit time and, indeed, the volume flux is computed as the velocity times the cross sectional area perpendicular to the direction of the flow, $\Phi_\varpi = vA$, $[L^3\ T^{-1}]$

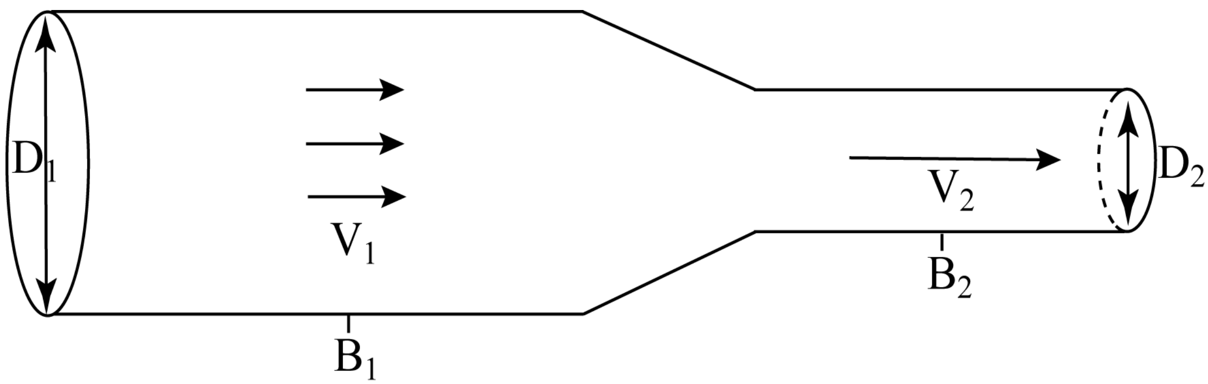


Figure 1. Conservation of mass for a non-compressible fluid (which to a large degree is correct for water and even for air in natural flows, but not in pumps and car tires) implies that the same volumetric amount of fluid passes across the pipe at any given time at both B_1 and B_2. Since $D_2<D_1$ the fluid must accelerate and flow faster at B_2. Fluid cannot 'concentrate' within the system if it is not compressible.

The mass of fluid passing through a pipe (or river, or a vein) per unit of time is called the mass flux. Mass flux has dimensions of M/T. It is related to other quantities we encountered by:

$$\Phi_M = v\rho A, \; [\mathrm{M\ T^{-1}}]$$

where Φ_M denotes the mass flux, A the cross sectional area, ρ the fluid's density, and v the velocity *perpendicular* to the cross sectional area.

What makes water flow through a pipe in the first place? There must be force acting on it (to overcome friction imparted by the pipe's walls). For a pipe it is the pressure difference between the two ends of a pipe that causes water to go through (more about that pressure difference later); water flows from high to low pressure.

Topic 3: Momentum, forces, and Newton's law of dynamics.

Momentum is the product of mass and velocity. Newton's 1st law states that in the absence of any force the momentum of a body stays constant. This is another conservation principle. The only way momentum can change is by application of a force (Newton's 2nd law):

$$d(mv)/dt=F.$$

This statement is more general than, $ma=mdv/dt=F$, because it allows for changes in mass (as might happen in a rocket as the fuel burns). It illustrates well why even a slow moving SUV can impact more force than a fast driving Honda civic when it comes to a sudden stop during a collision (the change of momentum is greater).

The amount of momentum passing through a pipe (or river, or a vein) per unit of time is called the momentum flux. Momentum flux has dimensions of M L/T^2. It is related to other quantities we encountered by:

$$\Phi_{mom}=v\Phi_{M}=v^2\rho A. \; [M\ L\ T^{-2}]$$

Within fluids we experience many forces; some forces act on the boundaries of fluid (*e.g.* friction with the bottom, wind stress at the top) and propagate into the fluid through viscosity and pressure. Some forces act everywhere within the fluid, such as gravity (a 'body' force). Objects at rest within a fluid experience a gravitational force (oriented towards the Earth's center) and a buoyancy force, pointing in the opposite direction. Moving objects within the fluid experience, in addition, drag due to pressure forces and shear forces caused by the motion of the object in a the fluid. Viscosity resists motion relative to the fluid and causes a force in opposite direction to the motion of the object. On a rotating planet another force is needed to explain motion, the Coriolis force. Earth's rotation has little effect on fluids on short time and space scales as those associated with the experiments we conduct in SMS204. However, it does have a major impact on ocean currents.

Topic 4: Pressure and stress.

It is often the force per unit of area, rather than the force itself, that matters. A force per unit of area is called a stress. Case in point (pun intended), a blunt object cannot penetrate the skin at a given force, while the same object, when sharpened, can (think of a knife). The skin can withstand a certain pressure (force per unit area perpendicular to it) that is exceeded when the same force is applied to a smaller area.

Force is a vector and we can always decompose a vector into its three spatial components. Similarly, the force per unit area is a vector. The component of the force per unit area perpendicular to the surface is the pressure whereas the two components parallel to the surface are called shear stresses. Thus the wind applies a stress on the ocean surface (and some pressure when waves are present) while the weight of the atmosphere applies pressure.

Within a fluid *at rest* pressure is the same in all direction (isotropic) at any given depth within the fluid and is perpendicular to the boundaries encompassing the fluid. Hydrostatic pressure refers to the pressure due to the weight of the fluid above. We feel the pressure of the atmosphere above us (in particular, athletes perform better in elevated places where the pressure is lower as in the Mexico City Olympics games). Organisms in the ocean feel in addition to the weight of the atmosphere the pressure due to the weight of the water on top of them. Pressure does change in the vertical within a fluid at rest; however that pressure change is balanced exactly by the weight of the fluid and therefore creates no motion.

Let's assume a cylinder filled with fluid and compute the pressure on the bottom due to the fluid. This pressure is:

p=Perpendicular force/Area= weight/Area =$(m_{air}+m_{water})g/A=p_{air}+Ah\rho_w g/A=p_{air}+h\rho_w g$,

where we assumed a uniform density for water (ρ_w, water density changes by only a few percents throughout the world's ocean) and a height of the fluid equal to h (thus the volume is Ah, where A is the cross-sectional area of the cylinder). The atmospheric pressure was assumed constant (p_{air}~$1.01\cdot10^5$Pa ±10%). The pressure varies with depth and is given at depth (h) by the same formulae. Within the cylinder at each level the pressure applied by the fluid on the walls is applied back by the walls on the fluid (a consequence of Newton's 3rd law of motion).

Problem: What is the side force applied on a vertical Dam of width W and height H by a fluid?

Solution: At any depth h the net sideway pressure felt by the dam is: $h\rho_w g + p_{air}$, where p_{air} is the air pressure at the surface of the water. The average pressure over its full height, H, is $H\rho_w g/2 + p_{air}$. The pressure is applied over the whole area of the dam ($A=W \times H$). Thus the force applied by the fluid on the dam (in addition to the air pressure which is equal on both sides) is: $F = pA = WH^2\rho_w g/2 + WH\, p_{air}$ (average pressure x area of dam).

Question: what if instead of a vertical container we had a container shaped like an inverted cone? It turns out that the pressure is exactly the same as for a cylinder. Can you explain why?

Divers (and animals who have air or gas cavities), need to equalize pressure with their surrounding as they dive. This is because air and gases are compressible (unlike liquids that have very little compressibility). Similarly, a diver needs to exhale into the mask to prevent it from collapsing inward as (s)he dives deeper.

Problem: What force will a diver feel on his/her face if he/she dives to 10m without equalizing his/her mask?

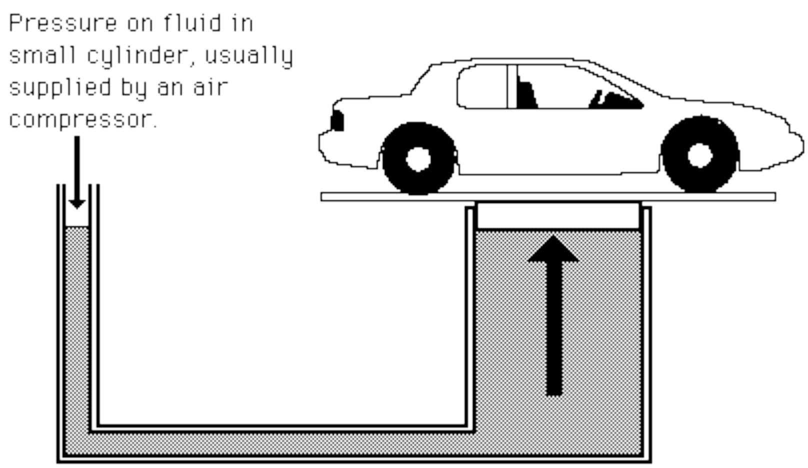
Solution: At the surface the pressure inside the mask is similar to that outside and equals p_{air}. At depth h the pressure outside is $h\rho_w g + p_{air}$ while the pressure within the mask is the same, p_{air} (assuming the mask doesn't compress). The net difference at 10m is 10m × 1030kg m^{-3} ×9.81m s^{-2} or about 10^5 Pa (about the same as the atmospheric pressure on its own). A typical mask has a surface area 0.1m × 0.15m=0.015m^2. Thus the net force on the mask is $F=pA$=1500N, equal to the weight of 150Kg (~330 pounds) person sitting on ones face!

Pressure exerts another (dynamic) force on fluids: fluids flow from high to low pressure. Presence of a pressure difference (called: a gradient) forces fluid motion (Remember, $mdv/dt=F$ and pressure is a force per unit area. Gradients in pressure indicate that there is a net force in a given direction). Similarly, water will not flow through a pipe unless there is a pressure change along it and will flow from high to low pressure. This is what allows us to drink through a straw (dropping the pressure in our mouths relative to the atmospheric pressure)! In a similar way a flowing fluid applies pressure; think of water coming out a hose towards you.

The hydraulic press (Fig. 2) illustrates how we can use fluids to raise a heavy object. Pressure is transmitted rapidly (in fact at the speed of sound) to all part of the fluid. The work required (work = force × distance) to raise a heavy object is equal to the work performed in pushing the fluid. Pushing a small area requires less force ($F_1=pA_1$), than a large area. The pressure is transmitted in the fluid and raises the car ($F_2= pA_2$). The excess pressure (above the hydrostatic) in the fluid is the same everywhere. The work (force × distance) done at both ends and along the fluid flow path of the press is the same:

Work=$F_1d_1=F_2d_2 \rightarrow d_2=d_1(F_1/F_2)=A_1/A_2$

Note that continuity implies that the volume of fluid displaced downward (d_1A_1) equals the volume of fluid displaced upward (d_2A_2). This is the same result as above! Note that the car is lifted a smaller distance than we pushed the fluid downward, yet with less force applied over a greater distance we were able to move an object we couldn't elevate directly at all.



Figrue 2. The Pascal press. Figure from: http://hyperphysics.phy-astr.gsu.edu/hbase/pasc.html#pp

Question: could this mechanism work with air?

Topic 5: Compressibility and the equation of state.

Compressibility of a fluid is its tendency to change its density under pressure. Water has very little compressibility and for our purposes can be treated as incompressible (sound, however, depends on compressibility to propagate so we need to add compression effects to have sound waves). Air is compressible, yet its ambient motion through wind is not sufficient to cause substantial compression. The equation of state of an ideal gas, which is applicable to air in normal conditions, is:

$$\text{Pressure} \times \text{Volume} = n \times R \times T = k_B \times N \times T,$$

where T is absolute temperature (K), n the numbers of moles of the gas present (1 mole=$6.022 \cdot 10^{23}$ molecules) and R (8.3145 J mole^{-1} K^{-1}) the gas constant. In the last expression, k_B is Bolzmann's constant = 1.3806504 J K^{-1}, and N is the number of molecules present. Note that unlike an incompressible fluid, for an ideal gas at a fixed temperature, increase in Pressure result in decrease in volume (also called Boyle's law). Thus, if we sink a balloon (or our ears) in water, the volume of contained gas will shrink due to the increase in pressure. Similarly, releasing a helium balloon to the atmosphere will cause it to expand as it rises and experiences a decreasing ambient pressure. How much expansion or contraction occurs depends on the elastic forces associated with the balloon's (or ear drum's) skin. For this reason divers need to equalize their ears and mask. If ears were filled with an incompressible liquid (as we can approximate for our body outside the lungs), no equalization would be needed.

By definition: $\rho = \text{Mass/Volume} = n\, M_w/\text{Volume}$

where M_w is the molecular weight (a constant for a given gas). It follows that for an ideal gas $\rho = M_w p/(RT)$.

Thus the density of an ideal gas increases with pressure at a constant temperature. In the Earth's atmosphere density decreases with distance from the Earth (z) approximately exponentially:

$$\rho(z)/\rho(z=0) = p(z)/p(z=0) = \exp(-z/8400\text{m}).$$

For water the equation of state is a complicated function of temperature, salinity, and pressure (Fig. 3 and 4).

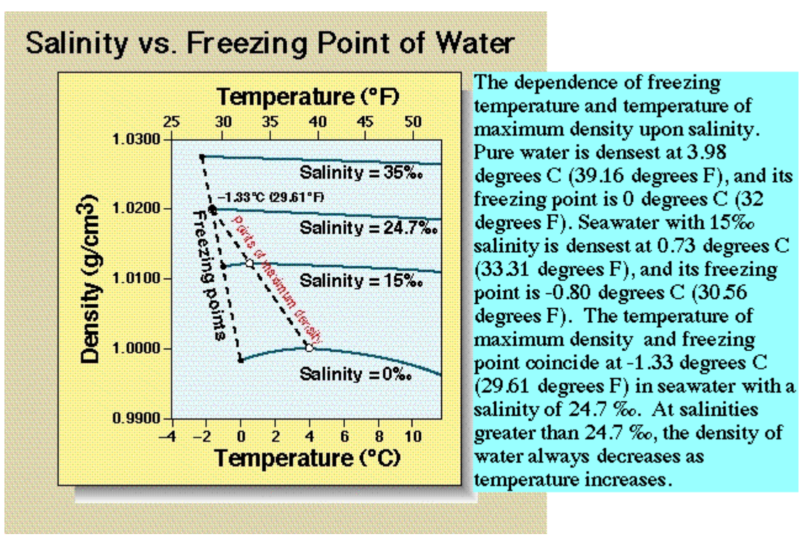


Figure 3: density as function of salinity and temperature at sea level. from: *http://geoserv.geology.wmich.edu/dave/otln7.htm*

When water freezes it is less dense than when liquid (that is often referred to as the anomaly of water).

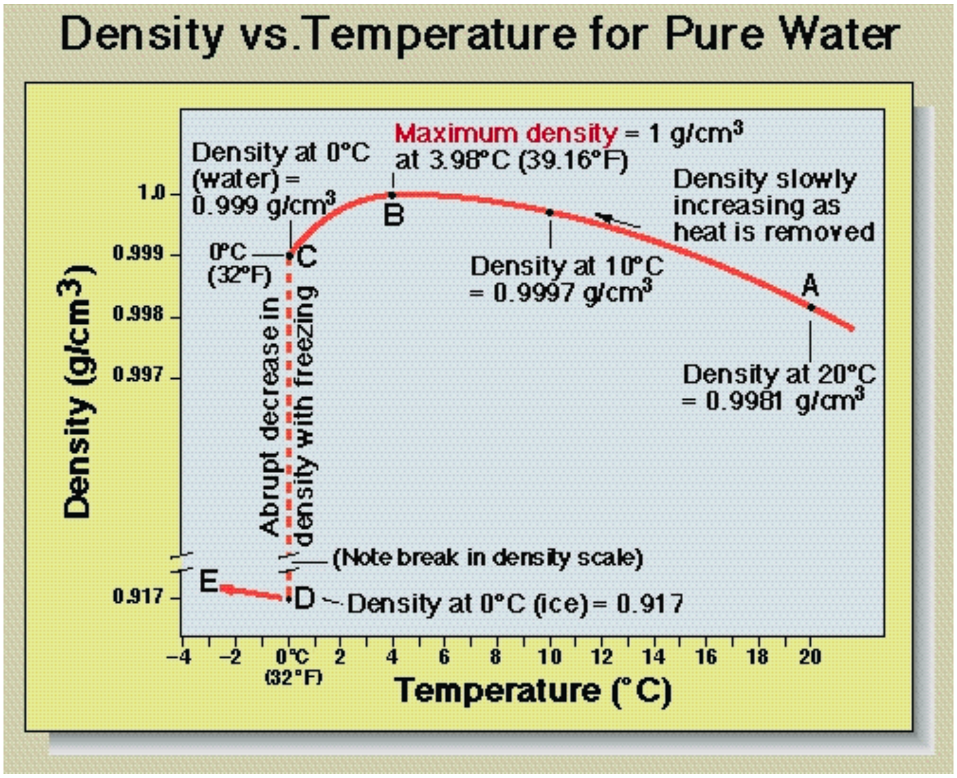


Figure 4: density as function of temperature at sea level. from: http://geoserv.geology.wmich.edu/dave/otln7.htm. Note that the change in density in liquid water form 0 to 20℃ is less than 0.2%.

For many purposes (for example modeling winds and currents near the coast) we can assume that air and water are incompressible. However, compressibility of these substances is essential to the transfer of sound waves within them, as well as explaining why the local temperature in the water increases at deeper depth (over large depths on the order of 100-1000 of meters).

The effect of pressure on density (or compressibility) is illustrated in Fig. 5. A 200 fold change in pressure changes density by less than 1%.

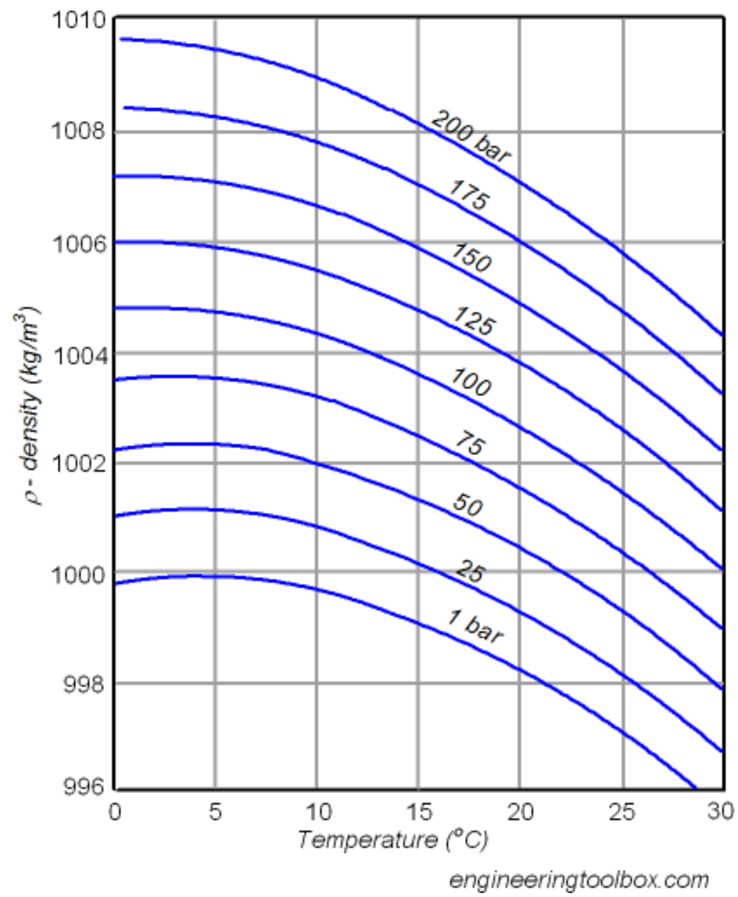


Figure 5. Changes of density of fresh water as function of temperature and pressure. From: http://www.engineeringtoolbox.com/fluid-density-temperature-pressure-d_309.html

Topic 6: Pressure and density distributions.

A stratified fluid at rest will have density and pressure levels parallel. If they are not a hydrostatic pressure gradient exist (higher pressure where the integrated density above is highest). In the world's ocean (as well as in estuaries) we do observe density gradients in the horizontal (see Fig. 6 for an East-West transect across the Equatorial Pacific and Fig. 7 for a North-South transect in the Pacific). In the oceans, wind stress, mixing, air-sea exchange of heat and the Earth rotation all help form and maintain horizontal density gradients. Such density gradients cause pressure gradients that provide the energy for motions that will act to erase those density differences in the horizontal (as these gradient are loaded with 'potential' energy, energy that can be converted to kinetic energy, that is motion). If all forcing came to a halt (for example with no sun and wind) these lateral gradients will be erased and the ocean would, eventually, be equally stratified everywhere (densest waters at depth and least dense water at the top).

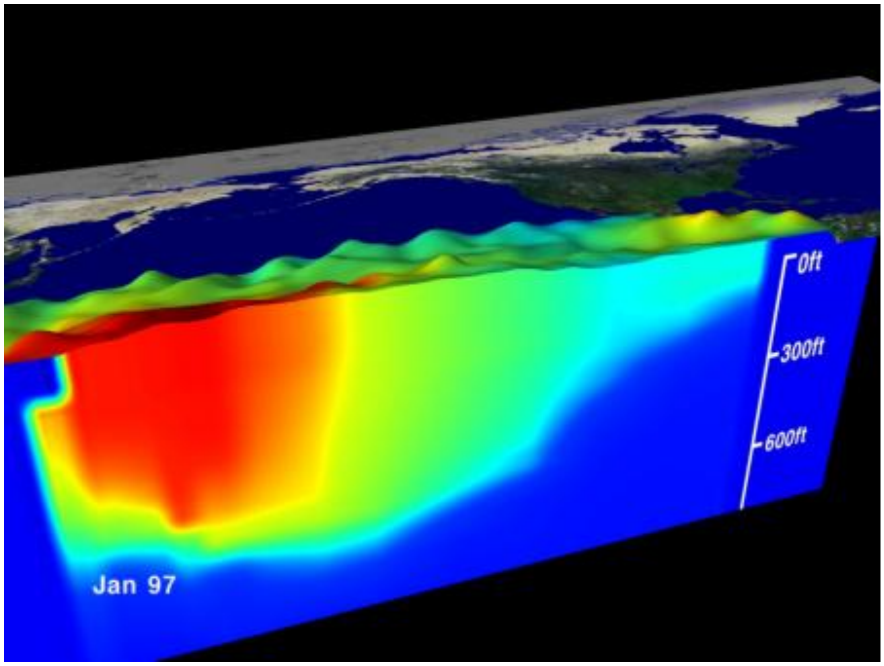


Figure. 6. Temperature (comparable to density) distribution across the equatorial Pacific during La nina period. Notice the warm water pool in the western part.

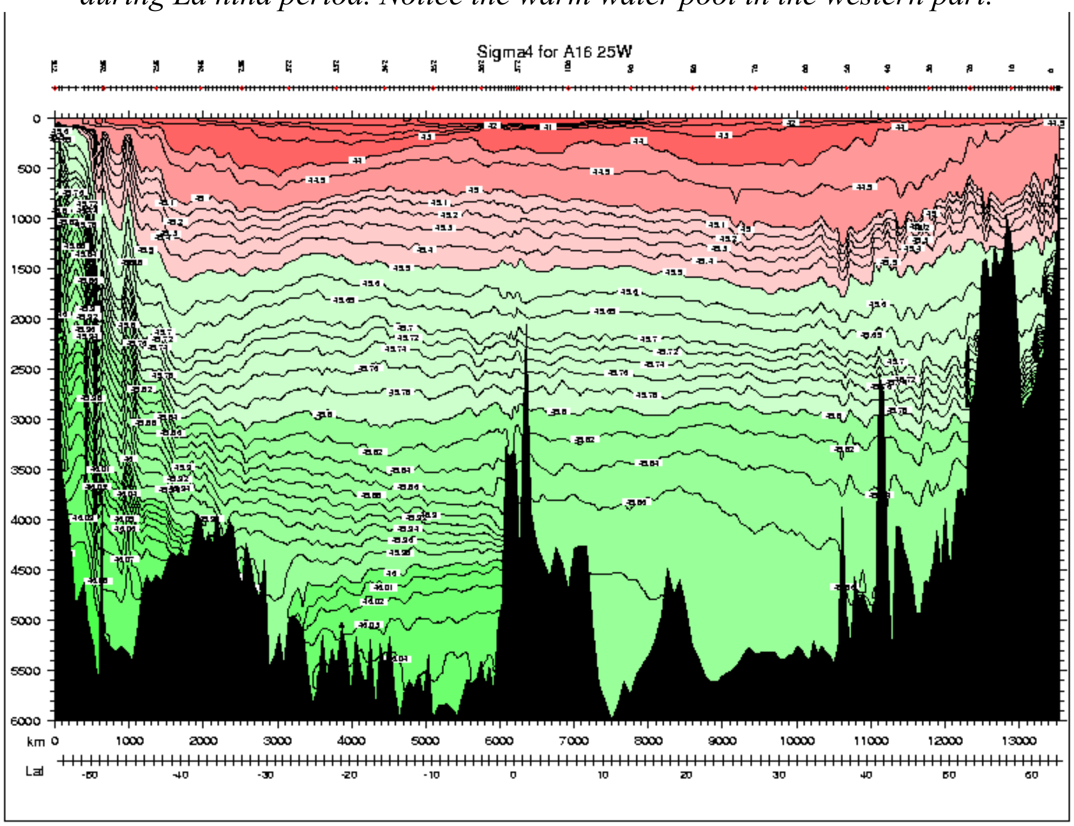


Figure 7. A density transect across the Atlantic Ocean (along 25W) Notice the horizontal changes in density gradients that exists, in particular as the Poles are approached (left) where deep water are formed.

Topic 7: Pressure and depth.

Because, to a large degree of accuracy, the pressure in the ocean is hydrostatic, that is:

$$p(z) = p_{air} + g\int_{-z}^{0} \rho(z)dz$$

and because density changes by only a few percent in the ocean, often pressure is used as an alternative measure of depth (and depth is measured in dbar, which are approximately 1m).

This is, off course, not exact. The correct way to measure depth is to measure temperature and salinity (itself a function of conductivity, temperature and pressure) as function of pressure, use temperature, salinity and pressure to obtain the water density profile, and then covert it to depth. The programs to do that (for example with data collected with profiling floats, or ship's CTD rosettes) are available on-line (e.g. http://www.teos-10.org/software.htm, http://pordlabs.ucsd.edu/ltalley/sio210/propseawater/ppsw.html).

References and additional reading:
Denny, M. W., 1993, Air and Water, Princeton U. Press, Chapter 3-4.
Vogel S., 1996, Life in moving fluids, Princeton U. Press, Chapter 1-2.
Wilkes, J. O., 1999, Fluid Mechanics for Chemical Engineers, Prentice Hall PTR, Chapter 1.
First 4 lectures of introduction to physical oceanography: http://www-pord.ucsd.edu/~ltalley/sio210/index.html
Great web site about properties of water: http://www1.lsbu.ac.uk/water/index2.html

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