SMS-204: Integrative marine sciences.

For homework- remember to provide uncertainties in graphs and display only significant figures for all your results.

HMWK 3 (please pay attention to provide uncertainties and round off numbers to the appropriate significant digits):

1. (25pts) Using results from lab station 1(Archimedes' ball):

Work out mathematically how much air needs to be pulled out of the ball by the syringe in order for the ball to barely start sinking in the surrounding water. How does it compare to your observations (are they within the observation uncertainties)?

Let's assume the dry ball is of mass m (about 125g± 0.5g), volume V (= $4\pi r^3/3$, with r the radius) and density ρ=m/V. The ball diameter is 7± 0.1 cm → $V_{ball} = \pi D^3/6 \sim 180ml = 1.8\times10^{-4} m^3$. $\rho < 1000 Kg/m^3$, as the ball is observed to float.

Let's assume you filled Xml (~52 ± 2.5ml) water in the ball (replacing the same volume of air). The ball is now barely floating, so we can assume all of its volume has displaced water. For that to happen:

Weight of ball = weight of water in the volume it displaced. From Archimedes principles we then find: $mg + Xml \times (\rho_{water}-\rho_{air}) \times g = V_{ball} \times \rho_{water} \times g$

Since $\rho_{water} >> \rho_{air}$, *we can simplify to (after dividing by g):*

$m + Xml*\rho_{water} = V_{ball} \times \rho_{water}$ → $Xml = V_{ball} - m/\rho_{water}$.

If everything was done right, Xml calculated in this way should be similar to the volume of air evacuated into the syringe.

This solution is identical to that which recognize that when the ball barely floats its density=density of water. $\rho_{water} = m/(V_{ball}-Xml)$ → $Xml = V_{ball} - m/\rho_{water} \sim 180ml - 125g/(1\ g\ ml^{-1}) = 55ml$.

The amount calculated is consistent with what was evacuated (52ml) when we take into account the uncertainty of the reading on the syringe (+/-2.5ml) and weight (+/-0.1g), and the fact that the ball was not exactly a ball in shape.

2. (30pts) Using results from lab station 4:

a. Report the cross section area, volume and weight of the empty box.

b. For the four box weights in which the box did not sink quickly create a table showing the weights added, the box weight in air, the box weight in water, and the depth to which the box was immersed in water.

# and type of weights	Weight in air I accepted mass)	Weight in water (I accepted mass)	Immersion depth (cm)
0	25g	0g (floats)	1+/-0.2cm
$2 \times 10g + 1 \times 5g$	50g	0g (floats)	2+/-0.2cm
$4 \times 10g + 2 \times 5g$	75g	0g (floats)	3+/-0.2cm
$6 \times 10g + 3 \times 5g$	100g	0g (bearly floats)	4+/-0.2cm
# and type of weights	Weight in air	Weight in water	Immersion depth

c. Plot the depth to which the box is immersed in water as function of the weight of the box in air (5pts).

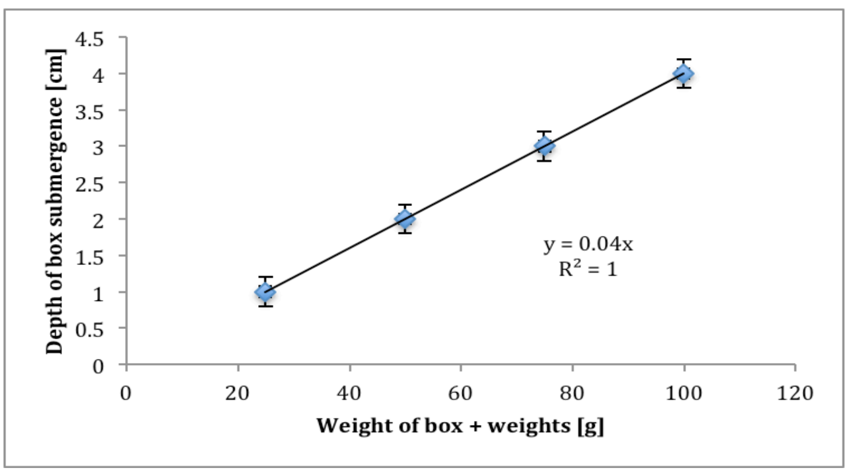

Figure 1. Immersion depth as function of weight of box + weights. The slope is ~0.04cm g^{-1} and the intercept ~0gr as expected (see below).

d. Obtain the slope of the best-fit line. (5pts)

See Fig. 1.

e. What should the slope be based on Archimedes's principle (5pts)?

For the floating box Archimedes tells us that: $Ah\rho_{water}\, g = mg$

→ $Ah = m/(A\rho_{water})$, where h is the depth of submergence, m the mass of the box (with the inside weights) and A the area of the bottom of the box ($25cm^2$).

→ the slope of the graph of h as function of m should be $1/(A\rho_{water}) \sim 1/(25\ cm^2 \times 1gr/cm^3) = 0.04cm\ g^{-1} = 0.4m\ Kg^{-1}$.

f. How does it compare to the slope of your plot? (5pts)

Very well, see Fig. 1.

g. Consider the final trial when you added enough weight so that the box *sinks* in water. In that case, what is the weight of the box when immersed in water and when outside water? What is the difference between them? (5pts)

100gr

h. Is this difference reasonable given what you know about buoyancy? Explain. (5pts)

Let the weight in Air be m_1 and the weight in water be m_2.

By Archimedes principle, the buoyancy force equals to the force due to the weight of the water displaced (Volumeρ_{water}*g), and equals the difference between weight in air (m_1*g) and in water (m_2*g). Dividing all sides by the gravitational acceleration g we get:*

→m_1-m_2 = Volume$\boldsymbol{\times}_{water}$.*

The Volume of the box=$100cm^3$ → m_1-$m_2 \sim 100cm^3 * 1gr/cm^3 = 100gr = 0.1Kg$.

If we measured this difference in weight than theory and measurements are consistent!

3. (25pts) In 2nd station of the 2nd lab you were studying water squirting out from a hole in a cylinder filled with water into a tub. Just as a falling ball converts potential energy to kinetic energy, water pressure pushed water out of the hole by converting potential energy per unit volume (ρgh) to kinetic energy per unit volume ($\rho v^2/2$). Assume you have a 30cm head of water above the hole and that the hole is 30cm above ground.

1. What is the horizontal speed at which the water leaves the hole?

Potential energy per unit volue: ρgh_1 -> $\rho v^2/2$: kinetic energy per unit volume.

→ $v^2=2gh_1$ → v=(2 x 9.81 x 0.3)^0.5=2.43ms^{-1}

h_1 denotes the distance from top of water to hole.

2. How long will it take it to reach the ground (think mechanics)?

You learned in mechanics that an object starting from rest (there is no vertical velocity to the water) obeys: $gt^2/2=h_2 \rightarrow t^2=2h_2/g \rightarrow t=(2x0.3/9.81)$^$0.5=0.25s$

h_2 *denotes the distance from hole to ground.*

3. How far will the water reach by the time it hits the ground?

L=vt~0.6m or 60cm, rounding to the closest cm.

4. How does the place where the water reaches change with the height of *the water surface* above the hole (provide an equation or a relationship)?

5. How does the place where the water reaches change with the height of *the hole* above ground (again, provide an equation or a relationship)?

Answer for 4 & 5:

$L=vt=sqrt(2gh_1)$ x $sqrt(2h_2/g) = 2sqrt(h_1h_2)$ *- where sqrt denotes square root.*

Hence the place where the water splash is proportional to the square-root of the height of the water above the hole times the square-root of the height of the hole above ground (it is their geometric mean and it increases with increase in any of them).

4. (20pts) Working with profiling float data:

a. Go to: http://www.mbari.org/science/upper-ocean-systems/chemical-sensor-group/floatviz/ Select a float in the 'Select Float' window. Choose density anomaly in the 'select one X Variable' window. Choose depth in the 'Select Y variable'. On the left most part, click on 'plot' to generate a plot of all the density anomalies (=density-1000kg/m^3) and click on 'Send'. Copy the image you generate to your homework.

I chose float 7619SoOcn, a float that was deployed in the SO.

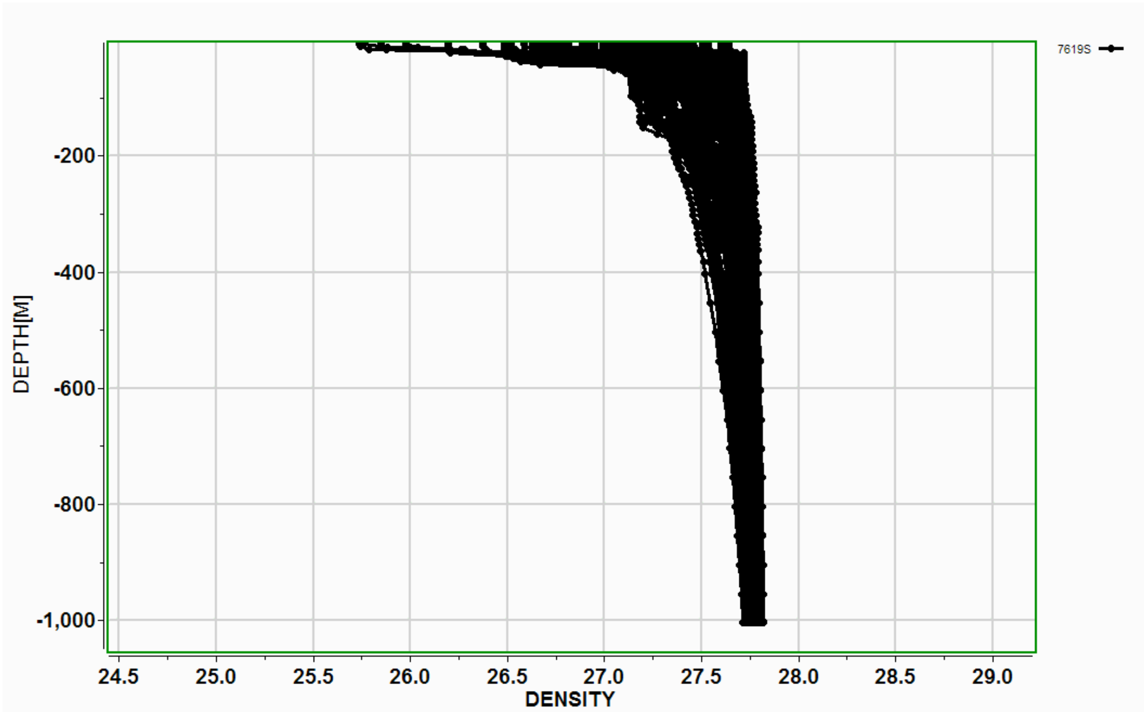

Figure 2. density anomaly profiles of float 7619SoOcn.

c. (5pts) On average, what is the difference in water density between the surface and depth? Where does it vary more?

Maximal difference is about 2.2 kg m^{-3}, average about 1kg m^{-3}. The largest variability in density is near the surface.

d. (5pts) Given that a float is parked at the deepest depth (~10 days between profiles) as well as the surface (to send data), in the least, what is the range of densities it should be able to accommodate?

Float density should span the range from 1025.5 to 1028 kg m^{-3}

e. (5pts) The float is a perfect cylinder with a 30cm diameter and 1.5m length. What should be its mass, such that its density without inflating a bladder, matches the density at depth?

*Volume = π 0.3m^2 /4 * 1.5m = 0.1060m^3.*

Mass=Volume x density = 108.936 Kg (density=1027.7);

f. (5pts) Given the above mass and volume, how much should the bladder be inflated (in cm^3), to allow the float to reach the surface for all the conditions it encountered?

To reach the surface in all condition: density = 1025.6 kg m^{-3}=108.9362 /Volume
→ Volume = 0.1062m^3 → an inflation of 0.0002 m^3 = 200cm^3 of the bladder is necessary to change the float buoyancy to make it reach the surface.
