SMS-204: Integrative marine sciences.

Lecture 3:

Topic 1: Buoyancy and the Archimedes's principle.

Archimedes's principle: The buoyancy force on an object is equal to the weight of the liquid displaced by the object.

The weight of the displaced fluid is $F_{buoyancy} = m_w \times g = \text{Volume}_{displaced} \times \rho_w \times g$.

Where m_w denotes the mass of the displace fluid, ρ_w denotes density of the displaced fluid and g the acceleration of gravity (9.81m s^{-2} on the surface of the Earth).

Additional force acting at the same time is gravity:

$F_{gravity} = m_{object} \times g = \text{Volume}_{object} \times \rho_{object} \times g$.

Where m_{object} denotes the mass of the object and ρ_{object} denotes the density of the object.

If the object is sinking (it is 'negatively' buoyant), the resulting force on it:

$\Delta F = F_{gravity} - F_{buoyancy} = \text{Volume}_{object} \cdot (\rho_{object} - \rho_w) \times g > 0$.

The weight of the object in the fluid (the net gravitational force pulling it down) has been reduced, compared to its weight in air, by an amount equal to the weight of the water displaced.

Now suppose the object does not sink beyond a certain depth (i.e. it is neutrally buoyant at that depth). At that depth $\Delta F = 0 \rightarrow \rho_{object} = \rho_w$.

Positively buoyant objects float on the water. The buoyancy force on an object that floats equals the weight of the object. Thus,

$\text{Volume}_{displaced} \times \rho_w \times g = m_{object} \times g$.

→ The trick with floating steel (for example an oil tanker) is to displace a large volume (a bowl shaped object full of air)!

Archimedes' principle can be derived from Newton's 2nd law as a result of the difference in forces due the change in hydrostatic pressure below the object and on top of it (which provides a force pointing upward).

Topic 2: Energy, work, power, heat and temperature

(see http://hyperphysics.phy-astr.gsu.edu/hbase/enecon.html for excellent discussion and definitions of concepts associated with energy).

Energy is the capacity to do work, and thus has the same dimensions [M L^2 T^{-2}] (force times distance, ability to push a load from here to there).

Energy comes in many flavors. The energy of a mass in motion is called kinetic energy (it equals the work needed to accelerate it to its current velocity or to stop it).

The electromagnetic energy of photons is called radiative energy and can be captured, for example, by absorbing these photons by a dark material which, in turn, will warm up (thermal energy).

Potential energy, is, as its name convey, the stored energy that can be released under a certain sets of circumstances. For example, the gravitational potential energy is the energy of an object that can fall from height h to the ground ($=mgh$). If we make a hole below the object, we could obtain energy at its point of impact. Other forms of potential energies are elastic (stretched spring), pressure-volume (Champagne bottle shooting its cork), chemical (for example the energy released when two substances interact or when a substance changes phase), nuclear energy (energy released in nuclear chain reaction) and electrical, e.g. due to the attraction of oppositely charged particles.

Heat is another form of energy. Heat is related to temperature though it is not exactly proportional to it (we need to invest heat to change water from ice to liquid without raising the temperature). Temperature is proportional to the mean kinetic energy of the molecules in a system. The faster the molecules move, the higher is the temperature.

Friction is the process through which mechanical energy is converted to heat. The breaks in our car heat when we use them and so do our hands when we rub them against each other. Friction can often be an undesirable conversion of energy to heat. In that process a loss of some kinetic energy (to heat, another form of energy) occurs.

Energy is another conserved physical quantity (in addition to mass and momentum we explored previously). It wears many forms and can change from one form to another (e.g. a steam engine converts chemical potential energy stored in coal to heat to pressure-volume energy that is, in turn, converted to kinetic energy). No energy conversion, however, is 100 % efficient, and some conversions are much less so (the 'left over' energy often leaks as heat, which is an undesirable form of energy in this case).

The amount of work done per unit time and (similarly) the change of energy as function of time is the power.

The SI unit of energy is the joule (Kg m^2 s^{-2}). Often a calorie is used as the unit; 1calorie = 4.18joules is the amount of heat needed to warm one gram of water by one degree C at

14°C and at sea-level pressure. The food Calorie is actually 1000calories, or the amount of heat needed to heat a liter of water by one degree.

Here are some striking examples regarding the energy of food: 1. an 8-min mile burns off energy roughly equivalent to one slice of bread (http://hyperphysics.phy-astr.gsu.edu/hbase/humeng.html#em). 2. The energy of one chocolate kiss (26 Calories) is the energy needed to lift a SUV 2 m up in the air (calculated from $E = mgh$) (http://www.npr.org/templates/story/story.php?storyId=6700905&sc=emaf)!

Topic 3: Energy conservation, Bernoulli's principle and its application.

In fluids we have several forms of energy:

e-internal energy per unit volume, comprised of kinetic and potential *molecular* energy.

PV-pressure-volume energy (P-pressure, V-volume).

mgz-mechanical potential energy associated with the fluid's mass (M-mass, g-gravitational constant, z-height).

$mv^2/2$-mechanical kinetic energy (v-velocity).

When work (*W*) is being done by/on the fluid or when heat is exchanged with the surrounding, some energy can be gained/lost locally, but within the whole system, energy is conserved.

When (a) the flow is steady, (b) no work is performed, (c) the flow is frictionless (no loss of heat), and (d) the fluid is incompressible and of constant density (ρ is constant), then energy is conserved along the flow (from point '1' to point '2') and can be written (per unit mass of the fluid) as:

$$E_1/m = v_1^2/2 + gz_1 + p_1/\rho = v_2^2/2 + gz_2 + p_2/\rho = E_2/m$$

where *E* denotes the total energy (neglecting the thermodynamic terms which are assumed constant). This is known as Bernoulli's principle.

Application of energy conservation: water squirting through a hole in a tube (Fig. 1).

$$\rho gh = \rho u^2/2 \rightarrow u=(2gh)^{1/2}$$

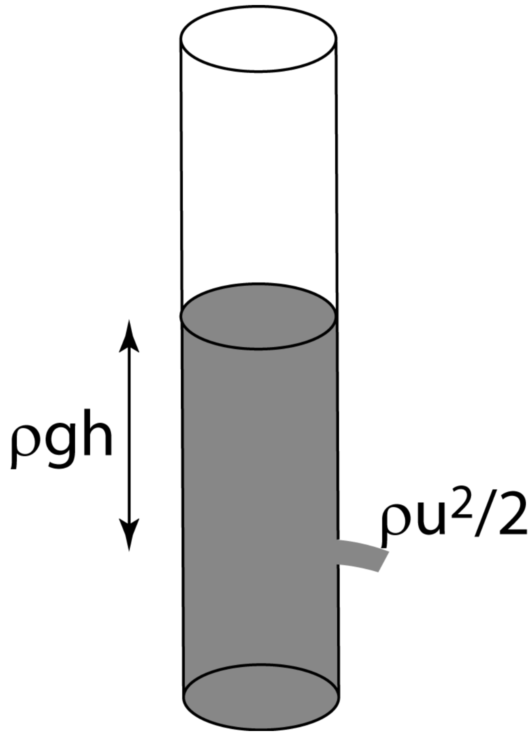

Figure 1. A squirting fountain, where potential energy is converted to kinetic energy.

Note that increasing the size of the hole or the diameter of the tank will not change the speed of the fluid squirting out of the tube (except when the hole is sufficiently small such that friction by its side affect the flow considerably).

One of the interesting and unintuitive results of Bernoulli's principle is that when a fluid approaches a constriction and accelerates (to satisfy continuity) the pressure at the constriction drops (Fig. 2, the pressure must increase ahead of the constriction as we show below). This reduction in pressure can be large enough as to cause the object to be suspended away from the bottom. This principle is what provides a lift to a convex upward wing of an airplane and what provides lift to sand grains on the seabed when they begin to move due to currents associated with waves, tides, river flows etc.

Bernoulli's principle can also be derived directly from Newton's 2nd law ($ma = F$). Neglecting for simplicity the gravitational term:

$mdv/dt = - Vdp/dx$

where v is velocity and V volume.

multiplied by $vdt = dx$:

$\Rightarrow \rho vdv = - dp$.

Integrating both sides:

$\Rightarrow v_1^2/2+p_1/\rho=v_2^2/2+p_2/\rho$

One application is flow over a hill: Flow above a bottom intrusion changes its velocity. This change in momentum is done by the pressure force. Fluid accelerates down a pressure gradient and decelerates when the pressure gradient reverses (Fig. 2).

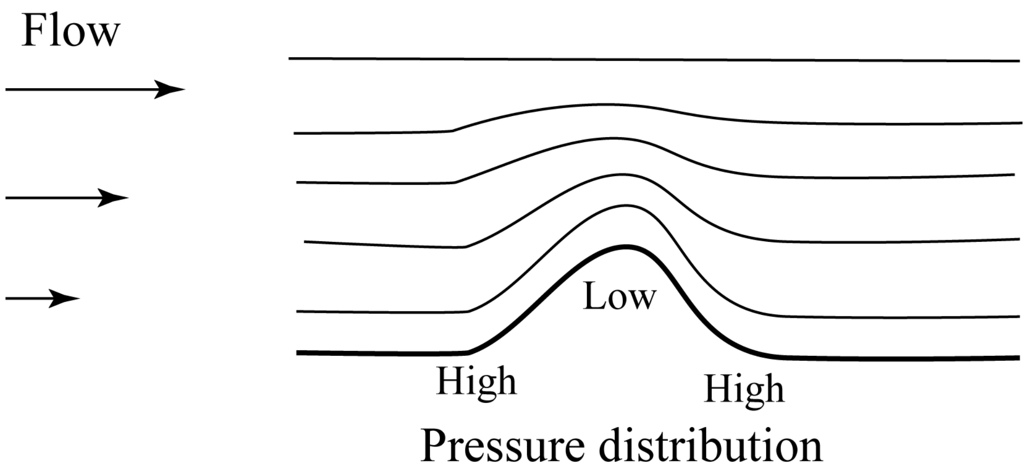

Figure 2. Fluid above a hill (clam, boulder, etc') is accelerated (from high to low pressure) and decelerated (from low to high pressure) with lines representing streamlines (trajectories of neutrally buoyant small objects). The pressure field supplies the force to accommodate that adjustment. More crowded streamlines represent faster flow (as per the continuity principle).

Topic 4: A fluid zoo: streaklines, streamlines, pathlines, and flow tubes.

It is often helpful to be able to visualize the flow using dyes and particles. One way to visualize flows is to release dye continuously from a series of points in the fluid. If the fluid is not turbulent, the dye will form streaks, which are termed streaklines. The paths of individual particles released in a flow that are reconstructed using long exposure photography are called pathlines or particle paths (the trajectory of single particles). Finally, if we use particles in the fluid to map the velocity field (by high-frequency photography) and connect all the instantaneous velocity vectors we draw streamlines (everywhere tangent to the velocity).

In steady flows all three lines coincide and do not change, while in an unsteady flow (where velocity at a point varies in time) they change in time and need not coincide. In steady flows, therefore, these lines are all tangent to the velocity field. If we define an area (A_1) at a point we can map it to area (A_2) where the fluid will be after a time interval Δt. To do so we simply map the points on the boundary of A_1 along the streamlines for an interval of Δt. In this process we define a flow tube. The tube may contract or expand but no fluid can leave the tube from it sides (since by definition its sides are parallel to the flow). By continuity $v_1A_1=v_2A_2$. Thus if the flow tube contracts the fluid must accelerate. Thus if we observe streamlines group together around a body we know that the flow there must be faster than upstream where they were further apart. From Bernoulli's principle (assuming no change in the average vertical position) we know that the pressure must change as well between the streamlines.

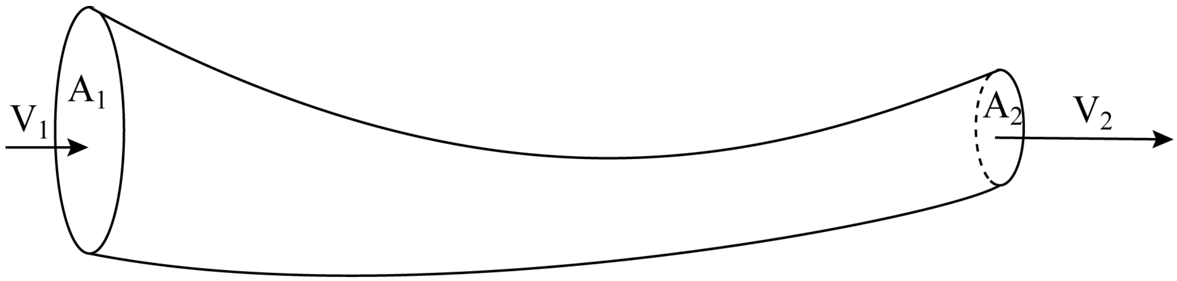

Figure 3. A flow tube, constructed mapping the points on the boundary of A_1 along the streamlines for an interval of Δt.

Topic 5: Profiling floats – changing buoyancy without changing mass.

The Argo global observing system relies on profiling floats as a platform to collect data with. But how can these profiling floats profile with a minimal energy spending (some profile for 5yrs making a profile every 10 days)?

The answer is that the float can change its buoyancy. In order to do that in has to change its volume (it cannot change its mass). Changing its volume it achieved by pumping oil out of a pressurized tank into a balloon to increase its volume (and hence decrease its density). Once at the surface and the data downloaded through a satellite link, the float retract the oil and decrease its volume (increasing its density) and it sinks back to depth. The exact volume it adjusts to is the one at the depth of parking, usually 1000m.

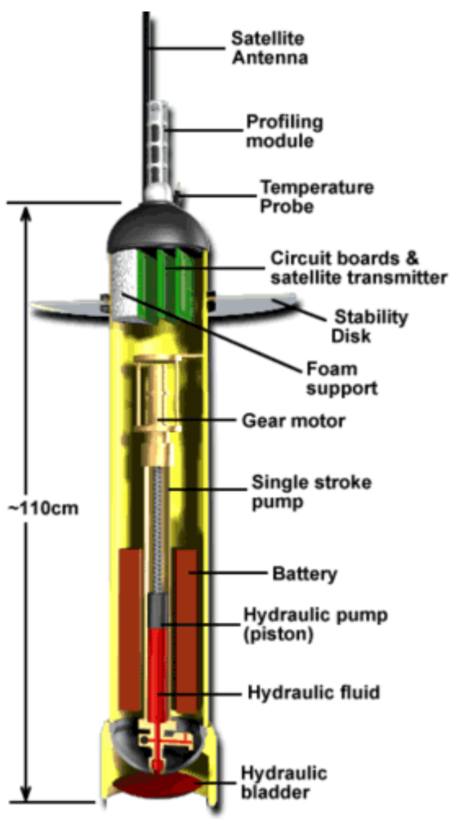

Figure 4. The design of an Argo float. By pumping hydraulic fluid into or out of a pressurized tank and into the hydraulic bladder, the float's volume (and hence density, because mass is constant) changes. Changing the volume result in a change in buoyancy and the float can rise, fall or stay at a fixed depth.

References and additional reading:
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