SMS 303: Answer to homework 3. Coriolis, Inertial Oscillations calculus and Baseball

From Newton's 2nd law we know that:

Mass×acceleration=Force.

It is often convenient when dealing with fluids to divide both sides by mass:

Acceleration=Force/Mass.

Now, consider an object on a rotating platform that rotates at angular velocity Ω (= 2 π radians / period, where the period is the time it takes for one full rotation). In order to account for the rotation of the platform an apparent force known as the Coriolis force is added to the equations. In two dimensions those are:

$du/dt=F_x/\text{Mass}+2\Omega\times v$

$dv/dt=F_y\,/\text{Mass}-2\Omega\times u$

where (u,v)=(dx/dt, dy/dt), are the velocities in the x and y directions respectively. Here we assume x is eastward and y is northward.

Let's assume that we give a kick to the object in the direction y at time zero and observe how it moves without applying any extra force. Initial condition $v(t=0)=V_0$. Let us also denote $f=2\Omega$ (On the Earth and latitude ϕ, f is the Coriolis parameter = $4\pi\sin\phi$ / 24 hr^{-1}).

$du/dt= f\,v$

$dv/dt=-\,f\,u$

Homework (be careful regarding units).

1. Check that the following is a solution the equations above (5 pts):

$u= V_0\cos(f\,t)$, $v= -V_0\sin(f\,t)$

Plugging it into the equations above:

$du/dt= -f\,V_0\sin(f\,t)$ ***is equal*** *fv*

$dv/dt=-f\,V_0\cos(f\,t)$ ***is equal*** *$-f\,u$*

2. Solve for the position (x, y) as function of time, assuming x(t=0)=y(t=0)=0 (remember, u=dx/dt, v=dy/dt, so x(t)=∫u(t')dt' and y(t)=∫v(t')dt', with the boundaries of the integral being from 0 to t). (15 pt)

$X(t)= V_0\sin(f\,t)/f$ +constant_1, $y= V_0\cos(f\,t)/f$ +constant_2

substitute, x(t=0)=0 → constant_1=0

substitute, y(t=0)=0 → constant_2= V_0/f.

→$[x(t),y(t)]= V_0/f \times [\sin(f\,t), \cos(f\,t)-1]$.

This describes a circle with radius V_0/f (which could be derived from dimensional analysis). Period is $T=2\pi/f$.

3. Plot the position of the object as function of time for 24 hours (every 1hr, using $f = 4\pi \sin\phi$ / 24 hr^{-1}) assuming a starting latitude of 30°N.

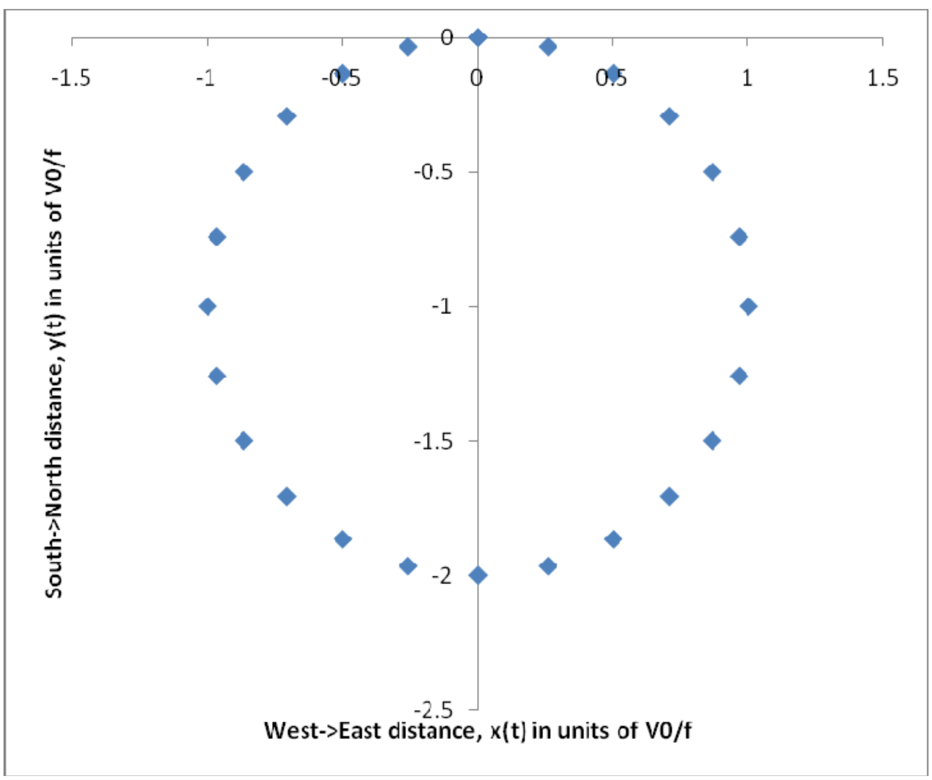

Figure 1. Trajectory of the object (position every hour) as function of time. Trajectory is clockwise.

a. What is the shape of the trajectory (15 pts)?

A circle with a radius of V_0/f.

B. How does the trajectory depends on V_0 and f (15 pts)?

The radius of the trajectory is V_0/f.

C. How long does it take for the object to come back to its initial position in terms of f and/or V_0 (15 pts)?

The time it takes is the period $T=2\pi/f$ *which in our case is 24hr.*

4. Now, assume that we are dealing with baseball and Fenway park (~42°N). The speed of the ball leaving the bat is 40m/s. Neglecting friction, what would be the position of the ball after 2 seconds (15pts)?

Using our results from the previous sections, and assuming the ball is thrown East, we have $[x(t),y(t)] = V_0/f \times [\sin(f\,t), \cos(f\,t)-1]$.

Using: $V_0=40\text{m s}^{-1}$. $f= 9.73211\text{E-05 s}^{-1}$ *and* $t=2\text{s}$ *we get:*

The position is $[x(t=2s), y(t=2s)]=[79.99999949, -0.007785688]$ *meters. The ball deflection is 8mm.*

Plot the trajectory of the ball in the x-y plane (10pts).

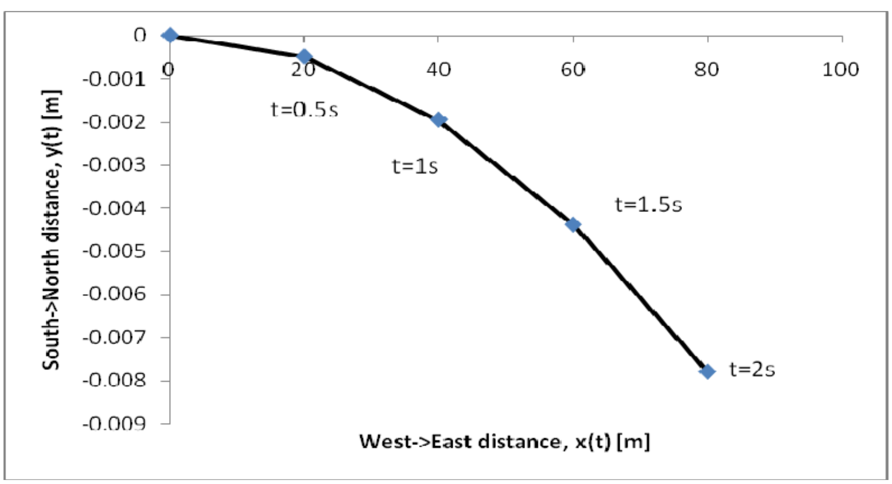

Figure 2. trajectory of a baseball thrown East at 40m/s during its first 2seconds at 42°N.

Further reading:

http://en.wikipedia.org/wiki/Coriolis_effect

Durran, D. R., 1993: *Is the Coriolis force really responsible for the inertial oscillation?*, Bull. Amer. Meteor. Soc., 74, 2179–2184; Corrigenda. Bulletin of the American Meteorological Society, 75, 261 (http://www.atmos.washington.edu/~durrand/pdfs/Coriolis_BAMS.pdf)

This page was last edited on 11/4/2010