## SMS-618, Particle Dynamics, Fall 2003 (E. Boss, last updated: 10/2/2003) Particle settling in a fluid

## Low Re \# settling:

The process of settling is governed by a balance between three forces; gravity, buoyancy, and drag.

Newton's 2nd law: $\mathrm{mdv} / \mathrm{dt}=\mathrm{F}_{\text {gravity }}-\mathrm{F}_{\text {buoyancy }}-\mathrm{F}_{\text {drag }}$
A settling particle will accelerate under gravity until it reaches a constant speed. At that point in time, $\mathrm{dv} / \mathrm{dt}=0$ and $\mathrm{F}_{\text {gravity }}=\mathrm{F}_{\text {buoyacy }}+\mathrm{F}_{\text {drag }}$.

What is likely to affect the settling velocity of a particle?
a. The particle's excess density to the fluid $\left(\rho_{\text {particle }}-\rho_{\text {fluid }}\right) \mathrm{g}\left[\mathrm{Kg} / \mathrm{m}^{3} \mathrm{~m} / \mathrm{sec}^{2}\right]$
b. The fluid's viscosity, $\mu[\mathrm{kg} / \mathrm{m} / \mathrm{sec}]$
c. The particle's cross sectional area, $\mathrm{A}=\pi \mathrm{D}^{2} / 4 \sim \mathrm{D}^{2}\left[\mathrm{~m}^{2}\right]$
to a lesser degree:
d. The particles shape
e. proximity to other particles

From dimensional analysis alone we find that:
$V_{\text {settling }} \alpha\left(\rho_{\text {particle }}-\rho_{\text {fluid }}\right) g \mathrm{DD}^{2} / \mu[\mathrm{m} / \mathrm{sec}]$.
Additionally, a function of a non-dimensional variable may also be represented in the equation. The only non-dimensional parameter in this problem is the Reynolds number, $\operatorname{Re}=\rho_{\text {fluid }} \mathrm{DV}_{\text {settling }} / \mu$.

So: $\mathrm{V}_{\text {settling }} \alpha$ func $(\operatorname{Re})\left(\rho_{\text {particle }}-\rho_{\text {fluid }}\right) \mathrm{gD}^{2} / \mu$.
It turns out that for spheres with $\operatorname{Re}<1$ : $\mathrm{V}_{\text {settling }}=\left(\rho_{\text {particle }}-\rho_{\text {fluiid }}\right) \mathrm{gD}^{2} / 18 \mu$.
This equation was derived by Stokes in the middle of the 19th century. It implies that the drag force on a sphere at low $R e \#$ is: $F_{D}=3 \pi \mu \mathrm{DV}$, linearly proportional to velocity (V and D ), unlike the $\mathrm{V}^{2}$ dependence at large $\mathrm{Re} \#$.

The agreement with experimental evidence is testimony that the theory of fluid flows at low $\operatorname{Re} \#$ is exact.

High Re \# settling:


Figure 4.3 Flow patterns around circular cylinders normal to the flow, for Reynolds numbers from less than six to
about $10^{6}$.
Figure 1. The flow pattern around a circular cylinder at different Re \#. From Middleton and Southard, 1984, Mechanics of sediment movement, SEPM

At low $\operatorname{Re} \#(\operatorname{Re}<6)$ the flow is steady and symmetric with respect to the cylinder. The flow accelerate at the middle point (streamlines are closer together) and thus the pressure there is maximal.
As we increase the Re number ( $\sim 40$ ) fore-aft symmetry is broken and flow separation occurs. Fluid from the boundary layer separates away from the body. There are two counter-rotating attached vortices on the lee side of the cylinder.
At higher Re number ( $\sim 100$ ) these vortices periodically break away from the cylinder and are shed down stream (known as 'Von Karman trail').
At higher Re numbers ( $\sim 1000$ ) a turbulent (non-coherent, non-periodic, disorganized)
wake exists at the back of the body with a laminar wake (where fluid is trapped) closer to the body.
At higher Re numbers (~100000) the turbulent wake occupies a large portion of the back of the body.
Increasing the Re number ( $\sim 1000000$ ) decreases the area of the turbulent wake

The Re \# when the transition to turbulence occurs for a given body depends on the shape of the body and how stable is the flow.
Flow separation can occur in both laminar and turbulent flows. When the fluid in the boundary layer (the layer affected by viscosity) has exhausted its kinetic energy along the boundary (due to dissipation) it detaches and continues along with the free flow.

## Drag force on a body:

Two forces act on bodies in flow. Skin friction, the stress parallel to the body, where the no-slip condition applies and a form drag, the force due to the pressure (or normal stresses) on the body. The sum of both is the drag, the force that needs to be applied to keep the body moving at a constant speed.

It is convenient to define a drag coefficient, $\mathrm{C}_{\mathrm{D}}$, which is defined the ratio of the drag force to half the inertial force: $C_{D}=F_{D} /\left\{0.5 \rho U^{2} A\right\}$. A is the cross-sectional area of the body.

For spheres the following regressions were found (see Fig. 2):
For low $\operatorname{Re}<0.5$ : $C_{D}=24 / \operatorname{Re}$.
For $0.5<\operatorname{Re}<1000$ : $C_{D}=24 / \operatorname{Re}\left\{1+0.15 \operatorname{Re}^{0.687}\right\}$
For $1000<\operatorname{Re}<\operatorname{Re}$ crictical: $\mathrm{C}_{\mathrm{D}}=0.44$
At Re_critical $\left(\sim 2.5^{*} 10^{5}\right)$ a sharp drop in CD is observed (termed drag 'crisis') and from there on $C_{D}$ increases.


Figure 2. The drag coefficient of different shaped particles as function of the Re \#. From www.wm.edu/geology/geo304/ Lecture20/sId002.htm.

Hindered settling (based on Allen, 2001):
When an ensemble of particles settles, the concentration of particles affects the settling of the individual grains. This can be explained by invoking the continuity equation: the particles are falling down relative to the fluid but the fluid needs to replace the volume occupied by the particles. Denoting by C the relative volume of particles within the fluid,
$\mathrm{V}_{\text {rel }}(1-\mathrm{C})=\mathrm{V}$
Where $\mathrm{V}_{\text {rel }}$ is the flow relative to the fluid and V the velocity relative to the ground.
V has been compared with the solitary particle settling velocity Vo and it has been found that $\mathrm{V} / \mathrm{Vo}=(1-\mathrm{C})^{\mathrm{n}}$ with $2.3<\mathrm{n}<4.65$ (decreasing monotonically with $\mathrm{Re} \#$ ), and that the drag coefficient is higher for a suspension with: $C_{D} / C_{D, 0}=(1-C)^{2-2 n}$.

Another approach to treat hindered settling is through the change of the apparent viscosity of the fluid:

$$
\mu_{\text {suspension }}=\mu *(\mathrm{C}),
$$

at low concentration, for hard spheres, $\mathrm{f}(\mathrm{C})=1+2.5^{*} \mathrm{C}+\mathrm{O}\left(\mathrm{C}^{2}\right)$ (based on Einstein's work).

Dietrich (1982):
(modified from:
http://www.ocean.washington.edu/people/faculty/parsons/OCEAN542/settle-
lect.htm)
See:http://woodshole.er.usgs.gov/staffpages/csherwood/sedx equations/RunSedCalc s.html for a web applet.

Dietrich (1982) was interested in an empirical expression that would express the settling velocity as an explicit function of particle characteristics.
Used dimensionless quantities -
$R=\left(\rho_{\text {particle }}-\rho_{\text {fluid }}\right) / \rho_{\text {fluid }}$
$W_{*}=\frac{w_{s}^{3}}{R g v}$
$D_{*}=\frac{R g D_{n}{ }^{3}}{v^{2}}$

Where $w_{s}$ is the settling velocity, $v$ is the kinematic viscosity and $D_{n}$ is the nominal diameter of the largest projected area.

He broke up the effects of shape and its production of a turbulent wake into three coefficients in the equation -
$W_{*}=R_{3} 10^{R_{1}+R_{2}}$
where $R_{1}$ represents the effects of 'density', $R_{2}$ represents the effects of shape, and $R_{3}$ encompasses angularity (roundness).

$$
\begin{align*}
R_{1}=- & 3.76715+1.92944\left(\log D_{*}\right)-0.09815\left(\log D_{*}\right)^{2} \\
& -0.00575\left(\log D_{*}\right)^{3}+0.00056\left(\log D_{*}\right)^{4} \tag{5a}
\end{align*}
$$

$$
\begin{align*}
& R_{2}=\left(\log \left(1-\frac{1-C S F}{0.85}\right)\right)-(1-C S F)^{2.3} \tanh \left(\log D_{*}-4.6\right)+  \tag{5b}\\
& 0.3(0.5-C S F)(1-C S F)^{2.0}\left(\log D_{*}-4.6\right) \\
& R_{3}=\left[0.65-\left(\frac{C S F}{2.83} \tanh \left(\log D_{*}-4.6\right)\right)\right]^{(1+(3.5-P) / 2.5)} \tag{5c}
\end{align*}
$$

where CSF is the Corey shape factor defined by

$$
C S F=\frac{c}{\sqrt{a b}}
$$

where $a$ is the largest length scale associated with the particle, $b$ is an intermediate length and $c$ is the minimum length. $P$ is Powers value of roundness, which is a qualitative measure of roundness described by Powers (1953). Basically P is smaller for more angular material. Perfectly round material has $P=6$ (for which $R_{3}$ becomes equal to one). Highly angular material (crushed silica, for instance) generally has $P \sim 2-3$.

Dietrich (1982) is particularly good for fluvial and aeolian sands. Oceanic flocs require a different treatment.

Not only concentration hinders motion. Particles in the vicinity of a side walls or a slow compared to freely settling particles.

## Settling of aggregates:

Aggregates are particles composed of primary particles or molecules bound to each other with interstitial waters in between them. Their settling is influenced by the amount of water within them which reduces the aggregate's density (thus decreasing fall speed) and increases their size (thus increasing their fall speed). Passage of water within the pores of the particle reduces drag compares to an impermeable particle (by reducing the need for the water to flow around the particle, and thus the resistance on the falling particle). Since these effects are contradictory it is not obvious, a priori, whether 1 . Does aggregation increases settling speed of material? and 2. Does an aggregate sinks faster/slower the more loosely packaged it is?

Johnson et al., 1996, derive the settling speed of an aggregate under the assumption that $\mathrm{Re} \ll 1$ :

$$
U_{\text {settling }}=\frac{g\left(\rho_{\text {primary_praticles }}-\rho_{\text {fluid }}\right)(1-P)}{18 \mu} D_{\text {aggregate }}^{2}=\frac{g\left(\rho_{\text {primary_praticles }}-\rho_{\text {fluid }}\right)}{18 \mu} \frac{V_{\text {primary_particles }}}{V_{\text {aggregate }}} D_{\text {aggregate }}^{2}
$$

Where P is the particles porosity (the relative ratio of fluid volume to aggregate volume). Assuming this formula to hold, we are now in a position to answer the second question: $\mathrm{U}_{\text {settling }} \propto 1 / \mathrm{D}_{\text {aggregate }}$ : collapsing an aggregate results in increasing settling velocity.

While the above settling velocity assumes an impermeable particle, a similar result can be derived from data of Johnson et al., 1996, who found (their equation 22 and table 3) that: $\mathrm{C}_{\mathrm{D}}=\mathrm{aRe}{ }^{-\mathrm{b}}$, where a and b are empirical constants, with $\mathrm{b} \sim 1$. Assuming that this drag balances the particles buoyancy we get:

$$
C_{D}=\frac{2 g\left(\rho_{\text {primary }}\right. \text { praticles }}{}-\rho_{\text {fuid })} V_{\text {aggregate }(1-P)}^{A \rho_{\text {fluid }} U^{2}}
$$

Assuming $\mathrm{b}=1$, and equating the last to expression for the drag coefficient we get again that $U_{\text {settling }} \propto 1 / D_{\text {aggregate }}$.
Note: the above discussion is for $\mathrm{Re}<1$. When $\mathrm{Re}>1$ the drag coefficient actually increases compared to that predicted from Stokes and thus the above conclusion holds as well.

For small Re \#, sinking speed is proportional to $D^{2}$. In the process of aggregation density is reduced: $\Delta \rho_{\text {aggregate-fluid }}=\Delta \rho_{\text {primary-particle-fluid }}(1-\mathrm{p})$. For fractal aggregate (1-P) scales like $\mathrm{D}^{\delta-3}$, where $1<\delta<3$ is the fractal dimension. Thus $U$ scales with $\mathrm{D}^{\delta-1}$ which increases with size for $\delta>1$. Indeed there is a lot of empirical and scaling evidence showing that aggregates sink faster than their primary particles (Johnson et al., 1996).
The relationship between U and $\delta, \mathrm{U} \propto \mathrm{D}^{\delta-1}$, has been used to quantify the fractal dimension of aggregates.

