SMS-618, Particle Dynamics, Fall 2003 (E. Boss)

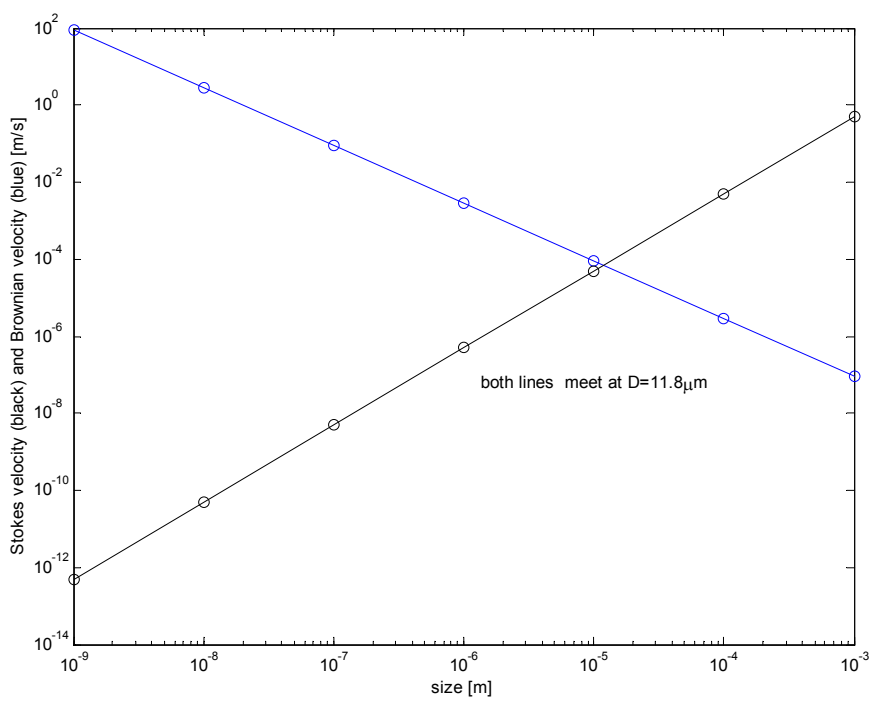
Solution of assignment 1

Q3:

Matlab routine:

```
%solution for problem 3 in assignment 1.

D=[10^-9,10^-8,10^-7,10^-6,10^-5,10^-4,10^-3]; %diameter in meters
%Stokes settling:
g=9.81;%gravitational constant
mu=1.79*10^-3;%dynamic viscosity
ro_water=1000;%density of fresh water
ro=2650;%denisty of particle
ws=g*(ro-ro_water)*D.^2/18/mu;

%Brownian motion:
Kb=1.38*10^-23;%Bolzman's constant
m=pi/6*D.^3*ro;%mass of particle
T=273;%Temperature in K
v_Brown=sqrt(3*Kb*T./m);

loglog(D,v_Brown,'b-o',D,ws,'k-o')

%find at what D both are equal
D=(3*Kb*T/((g*(ro-ro_water)/18/mu)^2*(ro*pi/6)))^(1/7)
```

Output figure:



Doubling the particle density changes the diameter where both velocities are equal to 8μm and halving it changes it to 20μm.

Brownian motion and settling, a different approach:

In the 1st assignment you were requested to find the size for which the settling velocity (assuming Stokes settling in water) equals the Brownian motion. The size found was near 10μm, suggesting Brownian motion is a significant source of motion for particles. While the magnitude of velocity is comparable, the actual distance a particle move by Brownian motion is very small, thus not affecting much its position. A better way to evaluate the importance of Brownian motion is to ask: how will the profile of the particles be in water if the water is quiescent? It turns out (see Berg, 1993, Ch. 5) that the concentration of the substance will be distributed according to:

$C(z)=C(0)\exp(-z/x_s)$, $x_s=k_BT/mg$,

Where C is concentration, z distance from bottom, k_B Boltzman's constant, T the temperature in Kelvin, and mg the particle's weight. x_s us called the scale height. For particles in a fluid, mg should be modified to $(m_{particle}-m_{fluid\text{-}displaced})g$.

At what size of particle is the scale height equivalent to the particle size?

$\rightarrow D=k_BT/\{\pi g D^3(\rho_{particle}-\rho_{fluid\text{-}displaced})/6\} \rightarrow D=[6k_BT/\{\pi g(\rho_{particle}-\rho_{fluid\text{-}displaced})\}]^{1/4}$

For an inorganic particle ($\rho_{particle}-\rho_{fluid\text{-}displaced}\sim 1500 m^3/Kg$) this occurs at $D\sim 1\mu m$. Smaller particles will have a boundary-layer-like distribution with substantial concentration above bottom while bigger particles will have no measurable concentration in the water.

One can look at the scale height as the height where the potential energy (mgh) equals the kinetic energy in the z direction (k_BT).

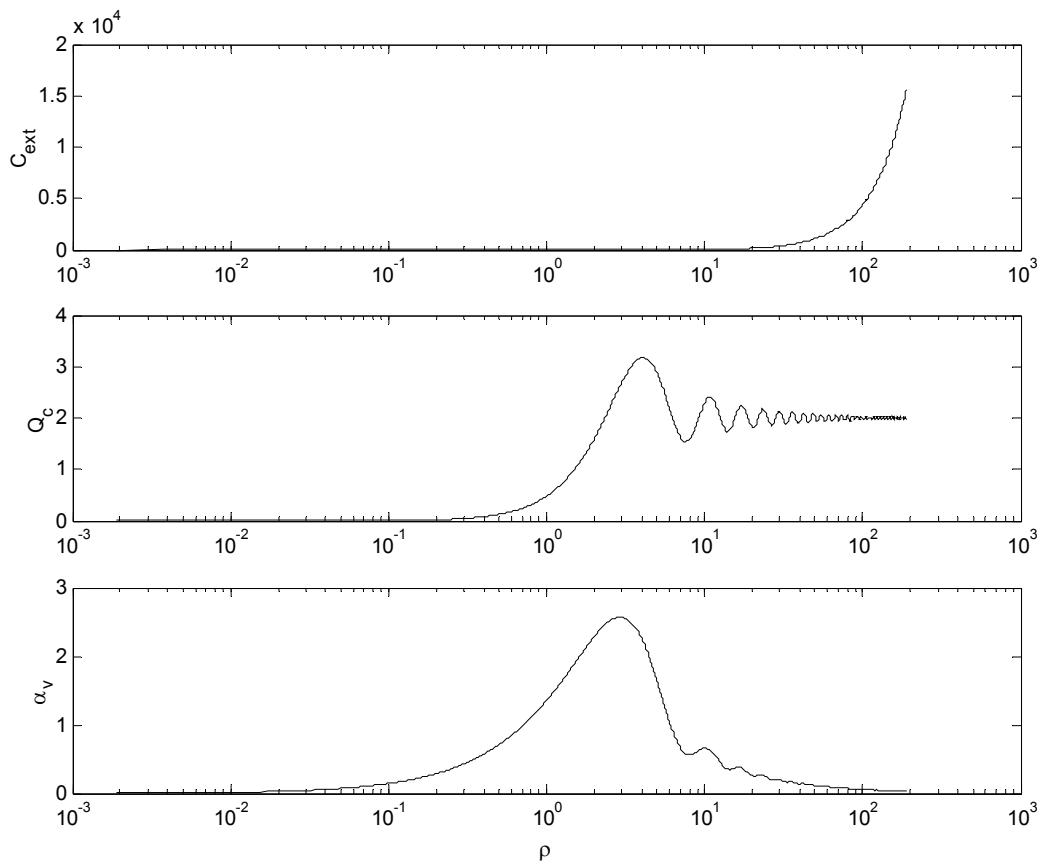
Reference:

Berg, H., 1993. *Random walks in biology*. Princeton University Press. 152pp.

Q4:

Matlab routine:

```
%solution for problem 4 in assignment 1:

n=1.15;
lambda=0.66*0.75; %wavelength in micron
D=[10^-3:10^-3:100]; %particle size in micron
ro=2*pi*D*(n-1)/lambda;


Qc=2-4*sin(ro)./ro +4*[1-cos(ro)]./ro.^2;
C_ext=Qc*pi.*D.^2/4;
alpha_v=C_ext./(pi.*D.^3/6);

subplot(3,1,1)
semilogx(ro,C_ext,'k-')
ylabel('C_{ext}')
subplot(3,1,2)
semilogx(ro,Qc,'k-')
ylabel('Q_c')
subplot(3,1,3)
semilogx(ro,alpha_v,'k-')
ylabel('\alpha_v')
xlabel('\rho')
```



C_{ext}, the attenuation cross-section, represent the attenuation of a single particle. The bigger the particle is the more it attenuates.

Q_c, the attenuation efficiency, represent the attenuation per particle normalized by its cross sectional area. Large particles have Qc of 2, attenuating twice the light that impinges on their cross section.

α_v, represent the attenuation of a particle normalized by its volume. Thus, for n=1.15 and λ=660nm, if we packaged the same amount of mass in different size particles, the particles of 3μm will be the most efficient attenuators.