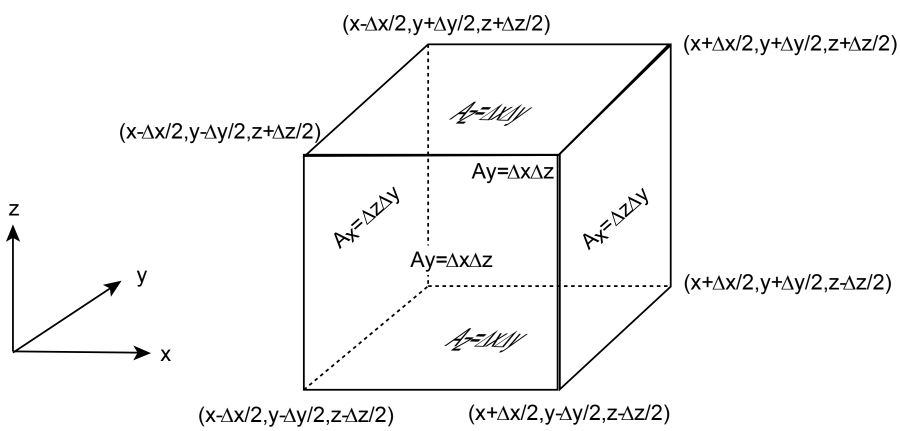
SMS-618, Particle Dynamics, Fall 2003 (E. Boss, last updated: 10/15/2003)
Bottom boundary layer

Review

The Navier-Stokes equation in one dimension (say x):

$$\frac{Du}{Dt} = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{\mu}{\rho}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) \qquad (1)$$

is a statement of Newton's 2nd law (when multiplied by density and integrated on a volume).
The forces acting on the fluid can be understood as follows:



1. Pressure: the force due to pressure in the x direction is due to the difference in pressure between both sides of the control volume (remember, pressure is a force per unit area, and fluid accelerates from high to low pressure):
$F_p = A_x * p(@x-\Delta x) - A_x * p(@x+\Delta x) = -V\partial p/\partial x$.
Dividing by the mass ($V*\rho$) we get the first term on the RHS of (1).

2. Stress: the force due to stress in the x direction is due to the difference in shear ($\partial u/\partial x$, $\partial u/\partial y$, $\partial u/\partial z$) between two sides of the control volume (remember, stress is a force per unit area, and momentum 'diffuses' down gradient):
$F_\tau = A_x * \{\tau_x(x+\Delta x) - \tau_x(x-\Delta x)\} + A_z\{\tau_z(z+\Delta x) - \tau_z(z-\Delta x)\} + A_y\{\tau_y(z+\Delta x) - \tau_y(z-\Delta x)\}$, where $\tau_x = \mu\partial u/\partial x$, $\tau_y = \mu\partial u/\partial y$, and $\tau_z = \mu\partial u/\partial z$.
$\rightarrow F_\tau = V[\mu\{\partial^2 u/\partial x^2 + \partial^2 u/\partial y^2 + \partial^2 u/\partial z^2\}]$. Dividing by mass ($V*\rho$) we get the 2nd term on the RHS of (1).

Let's decompose the velocity, density, and pressure into a time average and fluctuating components.

$$\frac{\partial(\overline{u}+u')}{\partial t}+(\overline{u}+u')\frac{\partial(\overline{u}+u')}{\partial x}+(\overline{v}+v')\frac{\partial(\overline{u}+u')}{\partial y}+(\overline{w}+w')\frac{\partial(\overline{u}+u')}{\partial z}=$$
$$-\frac{1}{(\overline{\rho}+\rho')}\frac{\partial(\overline{p}+p')}{\partial x}+\frac{\mu}{(\overline{\rho}+\rho')}\left(\frac{\partial^2(\overline{u}+u')}{\partial x^2}+\frac{\partial^2(\overline{u}+u')}{\partial y^2}+\frac{\partial^2(\overline{u}+u')}{\partial z^2}\right) \quad (2)$$

where over bars imply time average. By definition $\partial\overline{u}/\partial t\equiv 0$ and $\overline{u'},\overline{v'},\overline{w'},\overline{p'}=0$. It is also a fact that $\rho'<<\overline{\rho}$. Using these, and taking the time average of equation (2) we have:

$$\overline{u}\frac{\partial\overline{u}}{\partial x}+\overline{u'\frac{\partial u'}{\partial x}}+\overline{v}\frac{\partial\overline{u}}{\partial y}+\overline{v'\frac{\partial u'}{\partial y}}+\overline{w}\frac{\partial\overline{u}}{\partial z}+\overline{w'\frac{\partial u'}{\partial z}}=-\frac{1}{\overline{\rho}}\frac{\partial\overline{p}}{\partial x}+\frac{\mu}{\overline{\rho}}\left(\frac{\partial^2\overline{u}}{\partial x^2}+\frac{\partial^2\overline{u}}{\partial y^2}+\frac{\partial^2\overline{u}}{\partial z^2}\right) \quad (3)$$

The continuity equation applies to both mean and perturbation quantities and thus:

$$\overline{u'\frac{\partial u'}{\partial x}}+\overline{v'\frac{\partial u'}{\partial y}}+\overline{w'\frac{\partial u'}{\partial z}}=\overline{u'\frac{\partial u'}{\partial x}}+\overline{v'\frac{\partial u'}{\partial y}}+\overline{w'\frac{\partial u'}{\partial z}}+\overline{u'\left(\frac{\partial u'}{\partial x}+\frac{\partial v'}{\partial y}+\frac{\partial w'}{\partial z}\right)}= \quad (4)$$
$$\frac{\partial\overline{u'u'}}{\partial x}+\frac{\partial\overline{v'u'}}{\partial y}+\frac{\partial\overline{w'u'}}{\partial z}$$

Combining (4) with (3) and moving the Reynolds stress to the RHS:

$$\overline{u}\frac{\partial\overline{u}}{\partial x}+\overline{v}\frac{\partial\overline{u}}{\partial y}+\overline{w}\frac{\partial\overline{u}}{\partial z}=-\frac{1}{\overline{\rho}}\frac{\partial\overline{p}}{\partial x}+\frac{\mu}{\overline{\rho}}\left(\frac{\partial^2\overline{u}}{\partial x^2}+\frac{\partial^2\overline{u}}{\partial y^2}+\frac{\partial^2\overline{u}}{\partial z^2}\right)-\frac{\partial\overline{u'u'}}{\partial x}-\frac{\partial\overline{v'u'}}{\partial y}-\frac{\partial\overline{w'u'}}{\partial z} \quad (5)$$

Correlations between different perturbation velocity components (the Reynolds stresses) can thus force the mean flow. We call them correlation because by definition of the correlation coefficient is given by:

$R(A,B)=corrcoeff(A,B)=\frac{\overline{A'B'}}{\overline{A}\,\overline{B}}$.

If u' and v' are uncorrelated than R(u',v')=0.

Bottom boundary layer dynamics (steady, neglecting waves):

Flows in pipe have been studied for nearly a century. One of the basic findings was that the resistance to the flow depends on the Reynolds number (Re=UD/ν, where U is the velocity at the center of the pipe, D the pipe diameter and ν the fluid viscosity). In both laminar (low Re number flow) and turbulent flows a boundary layer is present near the pipe walls where the no-slip condition requires the velocity to be zero.

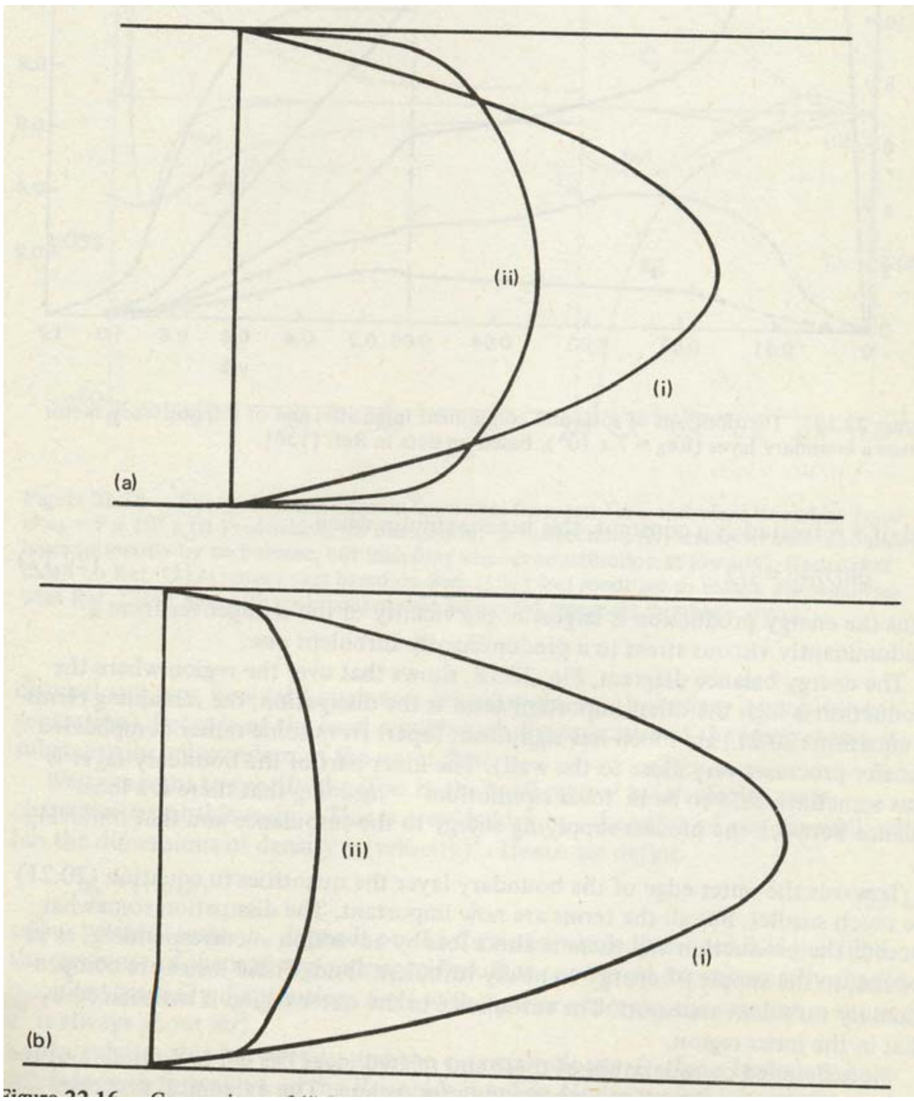


Figure 1. Comparison of laminar (i) and turbulent (ii) velocity profiles in a pipe for (a) the same mean velocity and (b) the same driving force (pressure difference). Figure 22.16 from Tritton, D.J. 1977. Physical Fluid Dynamics. Van Nostrand Reinhold, NY. p. 277.

Turbulent boundary layers in pipes are more dissipative and the vertical shear (du/dz) is decreased compared to the laminar flow.

Observations of flow in turbulent bottom boundary layers reveal similar structures as flows in pipes (Fig. 2). A boundary layer exists where the flow varies from the free flow (which is not affected by the presence of the boundary) to zero at the bottom.

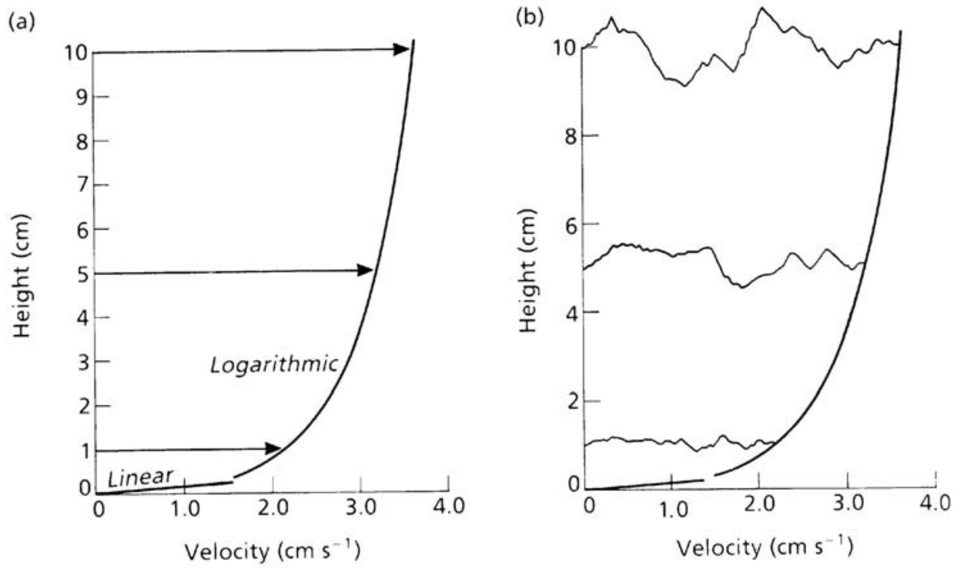


Fig. 2.04 (a) A vertical profile of mean water velocity through the boundary layer above a smooth surface showing the linear sublayer where viscous stresses dominate the stress between the water and the surface and the logarithmic layer where turbulent or Reynolds stresses dominate. (b) The same profile as in (a) including time series measurements of velocity at three levels to illustrate the increase in the size of the turbulent fluctuations with height above the boundary.

Figure 2. Structure of a bottom boundary layer very close to the bed. From Mann and Lazier, 1996. Notice the change in u' as function of depth.

The stress in the BBL varies from being dominated by the Reynolds stress away from the wall to being dominated by viscous stress right next the bed. The BBL layer (δ_{BBL}) is usually defined as the region where the mean fluid flow along the bottom varies from 0 to 0.99 of the 'free' velocity. Despite the BBL being turbulent several dynamical regions have been defined for it where certain momentum balances hold (Fig. 3).

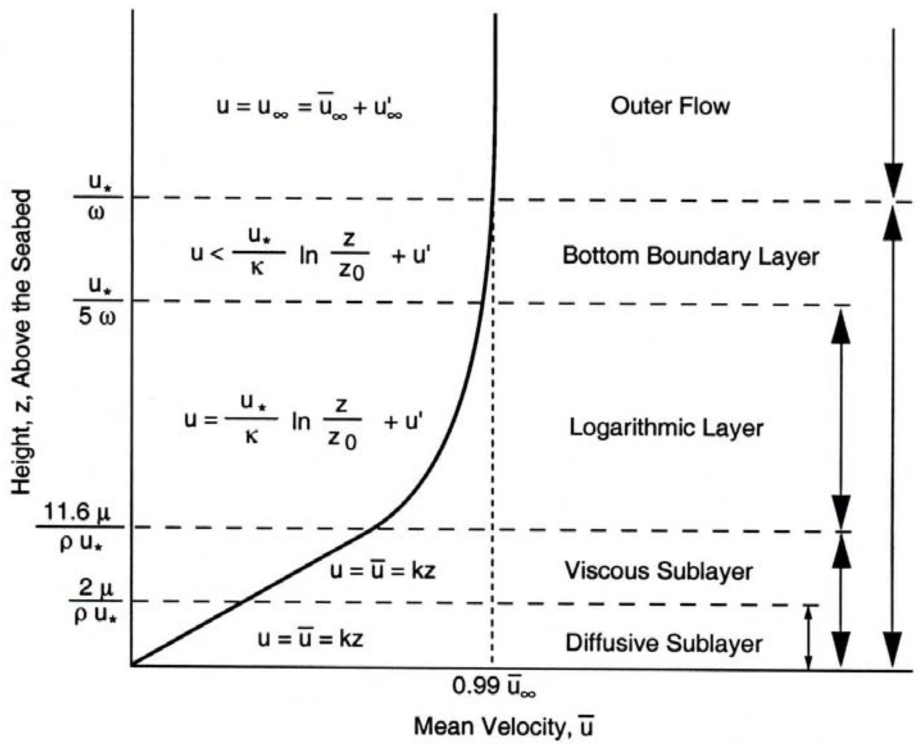


Figure 3. Division of the BBL into different regions based on dynamical/kinematical balances. Oceanic bottom boundary layers rarely grow to equilibrium thickness, but are constrained by either the frequency, ω, of forcing (e.g., of wave oscillations or tides) or water depth, whichever imposes the smaller limit. Viscous (momentum) and diffusive (mass) sublayers exist only when conditions are not fully rough turbulent. The layers are not drawn to scale. Where turbulent fluctuations are important, the fluctuating component of velocity is denoted by u'. As per the text, u is shear velocity, whereas κ is Von Karman's constant (0.41) and ρ and μ are fluid density and dynamic viscosity, respectively from Jumars (1993).*

1. Viscous sub-layer (δ_v): In this region viscous stresses dominate and shear stress is constant.

$$\bar{\rho} u^{*2} = \tau_0 = \mu \frac{d\bar{u}}{dz} = const. \text{ and } \bar{u}(z=0) = 0 \Rightarrow \bar{u} = \frac{\tau_0}{\nu} z = \frac{u^{*2}}{\nu} z$$

The layer thickness is scales with $\delta_v = \nu/u^*$ (Fig. 4).

2. Logarithmic layer (up to (20%) of BBL): In this region Reynolds stresses dominate. Assuming a constant stress, dimensional analysis suggests the shear of the mean velocity should be proportional to the shear velocity and distance from the bottom:

$$\frac{d\bar{u}}{dz} \propto \frac{u^*}{z} \rightarrow \frac{d\bar{u}}{dz} = \frac{u^*}{\kappa z} \rightarrow \frac{u}{u^*} = \frac{1}{\kappa}\ln(z) + Const \rightarrow \frac{\bar{u}}{u^*} = \frac{1}{\kappa}\ln\left(\frac{z}{z_0}\right)$$

z_0 is called the 'roughness' length and denotes the height where $\langle u \rangle=0$. $\kappa=0.41$ is the von-Karman constant. When the viscous sub-layer is present (smooth bottoms) z_0 is proportional to $\delta_v=\nu/u^*$. This is equivalent to assuming that the eddy viscosity be linear with depth. Indeed, for this profile $K_{eddy}=\kappa u^* z$.
In addition transition layers connect the viscous sub-layer with the logarithmic layer and between the logarithmic layer to the outer layer (where empirically $\langle u \rangle/u_{max} \propto (z/\delta_{BBL})^{1/n}$, $5<n<7$).

Additional considerations for BBL:
The current approach ignored presence of waves and stratification. Waves induce a small wave boundary layer confined closer to the bed than that due to tides, winds or gravity (in a river). Stratification dampens turbulence by reducing w'. Both are treated by a recent paper by Styles and Glenn (2000) and references therein.

References:

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