radiative transfer. The results will embody powerful extensions which appear to be capable of solving--in principle and in practice--every known current problem of applied radiative transfer theory in the domains of the air and the sea.

As an aid in studying the present work Fig. 1.2 indicates the *logical interdependence* of the various volumes and chapters. Actually every chapter is connected in some way with every other; however, some connections are stronger than others, and these are shown in the diagram. Thus the prerequisite most essential to understanding a given chapter is the chapter (or chapters) which stand immediately above it via the horizontal and vertical lines in the diagram. For example Chapter 11 depends directly on 4,5, 7 and 10, while 6 depends directly only on 3. Furthermore, the chapters whose contexts are developed on the level of general radiative transfer theory (Fig. 1.1) are outlined in heavy boxes; those that are more directly concerned specifically with hydrologic optics (or the theory of stratified plane parallel media) are outlined in the dashed boxes.

1.1 A Primer of Geometrical Radiometry and Photometry

After the solar radiant energy incident on the upper levels of the atmosphere has rapidly percolated down through the atmosphere and redistributed itself via scattering processes throughout the lower reaches and in the upper layers of the seas and lakes, its flow within these media assumes an intricate, and relatively steady geometric pattern. A particularly useful mode of representation of this flow of scattered radiant energy is possible by means of the concepts of geometrical radiometry, whose definitions and interrelations we shall now briefly study. A relatively complete and detailed study of geometrical radiometry and photometric concepts is reserved for Chapter 2.

The Nature of Radiant Flux

The radiant energy streaming in from the sun is understood to be electromagnetic energy. The atomic radiative processes of the sun generate a wide range of frequencies (or wavelengths) of electromagnetic energy, only a small part of which is visible to the human eye, or detectable by human skin, or usable by the plants and animals of the earth. The part of the electromagnetic spectrum visible to normal human eyes lies essentially in the range from 400 to 700 millimicrons wavelength, the 400 mμ light being deep blue-violet, the 700 mμ light being deep red, with all the colors of the rainbow ranging continuously between these extremes. The wavelength of electromagnetic energy evoking the greatest sensation of brightness is the yellow-green at 555 mμ under normal daylight conditions. If radiant energy of wavelengths much less than 400 or much greater than 700 mμ fall on normal retinas, there is relatively no conscious awareness of such an event by the associated brain, though--in some extraordinary cases, some ultra violet (380 mμ) and some infra red (780 mμ) phenomena are still within the range of detectability by the human visual organs. By and large, however, the human visual sensor system effectively samples and reacts to only the minute portion of the whole outpouring of radiant energy by the sun between 400 and 700 mμ--much in the way that a taut wire of given length and diameter resonates most sharply to a single acoustic frequency and less sharply to the frequencies in a small interval surrounding the central frequency, outside of which the wire is essentially insensitive to the vibrations. Figure 1.3 depicts the place of the visible portion of the spectrum within the electromagnetic spectrum, along with schematic diagrams of those portions of which we are aware by means of various devices used to detect and measure radiant energy. (Current manufacturer's catalogs should be consulted for precise details of individual devices.) Any observable part of the electromagnetic spectrum, observable not only as visible light but also by suitable technical means, falls under the aegis of geometrical radiometry.

The central construct of geometrical radiometry is *radiant flux* which we define generally as the time rate of flow of radiant energy of given wavelength (or frequency) across a given surface. (It has dimensions of (radiant) energy per unit time per unit frequency.) Thus radiant flux is a time density* of radiant energy. For our present purposes and in the exposition of radiative transfer theory, we may imagine the flow of radiant energy to be in the form of mutually non-interfering swarms of tiny colored particles--which we call *photons*. While this may not correspond in all aspects to physical reality, it nevertheless is a helpful construct in practical work. Each photon contains a well defined amount $h\nu$--a *quantum*--of radiant energy associated with its color, or frequency ν. This means of picturing radiant energy *for the purposes of geometrical radiometry* is quite useful and correct within the modern framework of physics. It will make the exposition of the notions of geometrical radiometry a relatively simple task, and the visualizations of the various concepts an almost trivial matter. In the terminology of electromagnetic theory, we shall work with electromagnetic fields produced by mutually incoherent sources and which are studied on a macroscopic level, i.e., where the dimensions of the detectors are very large compared to the observed wavelengths.

The Unpolarized-Flux Convention

The radiant flux always will be assumed unpolarized, unless specifically noted otherwise. This will result in simplified working formulas of relatively great practical value and of adequate accuracy in the pursuit of most applications of hydrologic optics. Whenever it is necessary to indicate how the theory may be elevated to the polarized level,

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*Because most of our discussions center on an arbitrary frequency (or wavelength) of radiant flux, the reference to the "per unit frequency" part of the dimension of radiant flux will be omitted, unless specifically noted otherwise.

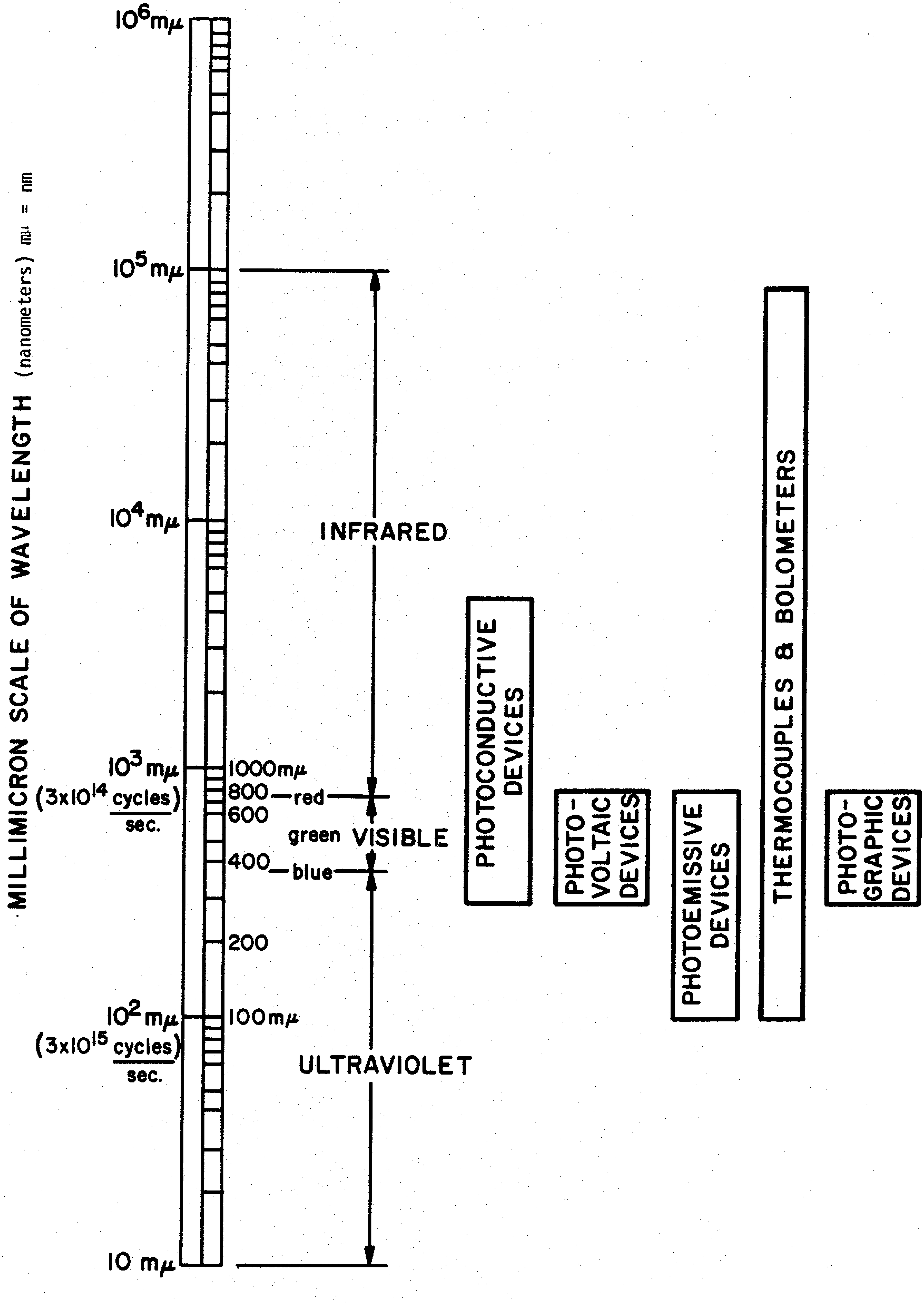

FIG. 1.3 The electromagnetic spectrum and the ranges of some typical radiant energy detector domains.

notes will be made to that effect. The general theory of polarized radiative transfer is outlined in Sec. 114 of Ref. [251], and the problem of the relative consistency of the polarized and unpolarized theories is examined in Sec. 13.11, below.

Geometrical Channeling of Radiant Flux

Once the nature of radiant flux is clarified, as above, the descriptions of the remaining concepts, theorems and procedures of geometrical radiometry are essentially geometric in nature. There are only two distinct, ideal modes of describing a flow of particles past a point in three dimensional space, and these are shown in Fig. 1.4. In part (a) of the figure a parallel flow of photons is described in terms of the passage of particles through a small region S on a plane normal to the flow around a point p on the plane. A complementary mode of the flow is in terms of the passage of particles through a small set D of directions around a given direction ξ and through the point p. Considering these two modes in a given flow of photons, let P(S) and P(Ω) be the radiant fluxes in each of these cases, with A(S) the area of S and Ω(D) the solid angle content of the bundle D of directions. Further, let the central direction ξ of the bundle D be normal to S at p. Then we write:

"P(S)/A(S)" for the area density of radiant flux

"P(D)/Ω(D)" for the solid angle density of radiant flux

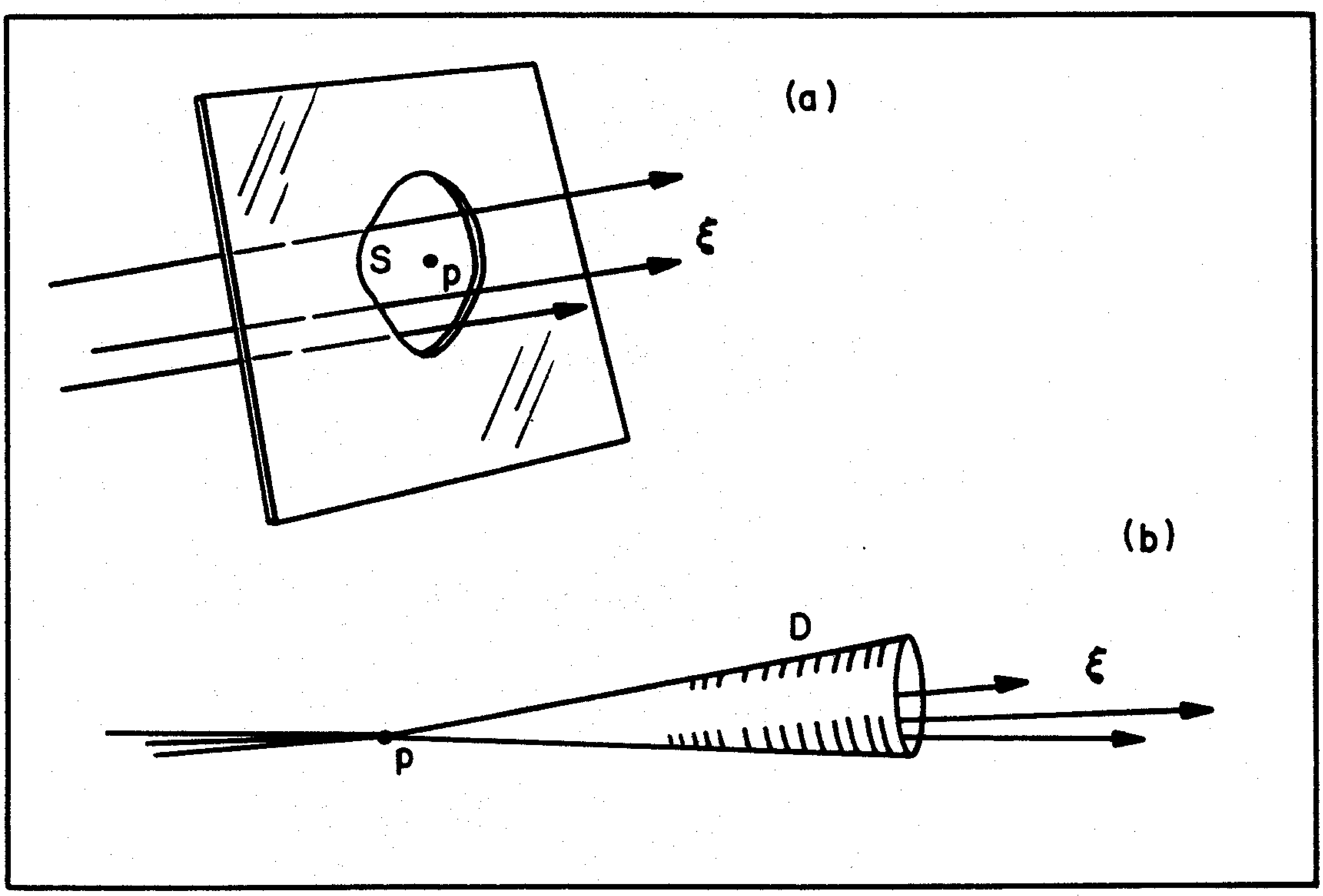

FIG. 1.4 Two geometric modes of describing radiant flux.

It is convenient in geometrical radiometry to call $P(S)/A(S)$ simply a (radiant) *flux density* and $P(D)/\Omega(D)$ a (radiant) *intensity*.

These are the two basic modes of conceptually channeling the flow of photons in space or matter. There is an important third mode which is the result of the direct union of these two modes. If we reconsider the setting of Fig. 1.4 and imagine a narrow bundle of directions D around a central direction ξ normal to S *at each point* p of S, then there would be an associated flow P(S,D) of radiant energy across the combined set $S \times D$ of the surface set S and the direction set D. We write:

"$P(S,D)/A(S)\Omega(D)$" for the *phase density* of radiant flux

The term "phase density" is simply a convenient descriptive term for the combined areal and directional densities, and it can be related to the phase space concept of classical statistical mechanics, though there is no need to do so here. The conventional term for phase density of radiant flux, the one we adopt for use in this work is *radiance*; it is radiance which is used to describe the monochromatic brightness of radiant flux.

Operational Definitions of the Densities

An operational definition of radiance and its companion densities is effected by means of a radiant flux meter, depicted schematically in (a) of Fig. 1.5. A radiant flux meter forms the heart of the radiance meter, as shown in (b) of Fig. 1.5, and may embody any one of several means of measurement of radiant flux, such as photoconductive, photoemissive, or photovoltaic devices (see Sec. 2.1). Before the radiant flux reaches the collecting surface S of the radiance meter, it is filtered to the desired wavelength and is also confined to flow onto S about point x through a narrow circular conical bundle D of directions whose central direction ξ is normal to S. A good radiance meter will have D so that $\Omega(D)$ is as small as practicable. A magnitude of $\Omega(D) \le 1/30$ steradians serves well for most geophysical optics tasks. If the reading of the radiant flux meter is P(S,D) when it is located at x and oriented by ξ (see Fig. 1.5), then the associated radiance is $P(S,D)/A(S)\Omega(D)$, which we can denote by "$N(x,\xi)$". Here "x" denotes where the flow is, and "ξ" denotes its direction. The associated radiant intensity is $P(S,D)/\Omega(D)$ and the radiant flux density is $P(S,D)/A(S)$. These operational definitions reduce to a practical level the ideal situations pictured in Fig. 1.4. They are ideal because in (a) of Fig. 1.4 the flow was assumed to be along a single direction and in (b) the flow was assumed to be through a single point. The operational definitions give workable approximations to these ideals and form the basis for a rigorous transition to the ideal limit, which will be made in Chapter 2.

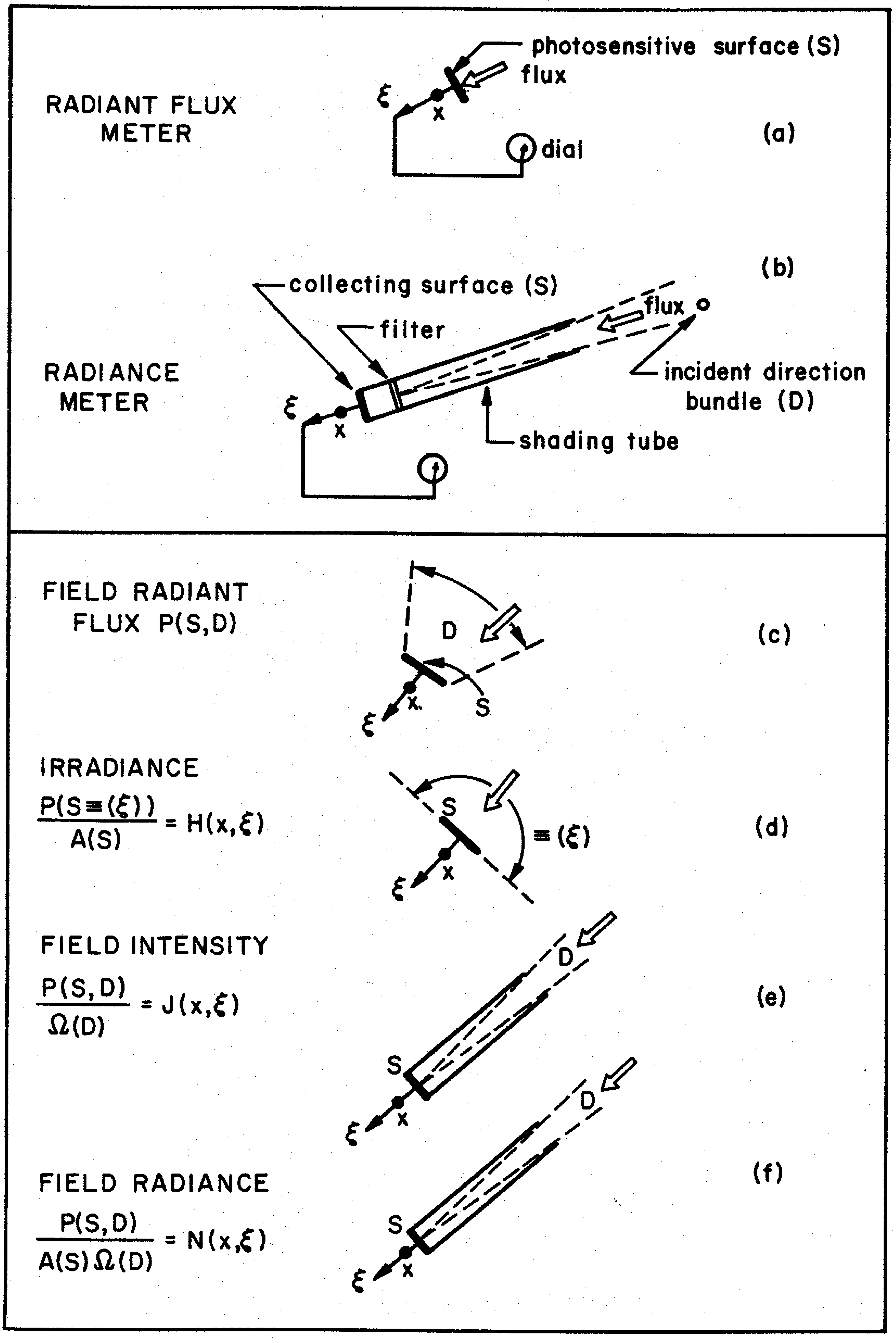

FIG. 1.5 Operational definitions of the radiometric concepts. Errata

Field and Surface Interpretations of Radiant Flux and its Densities

In Fig. 1.4 one important fact about the radiant flux was omitted, namely its *sense* of flow. In practice we often find it useful to distinguish between the flow of radiant energy *onto* a surface S and *from* the surface S. When we do so, the three central densities introduced above each have either one of the two possible interpretations, according as the radiant flux comprising the density is viewed as flowing onto or from a surface. When radiant flux comes from the radiometric field and falls onto the collecting surface S of the radiance meter we call the associated radiance the *field radiance*. When the radiant flux is seen to leave a surface (either real or imaginary) for the surrounding radiometric field we use the term *surface radiance*. Similarly for radiant flux density: when radiant flux falls onto a surface we speak of the radiant flux density as the *irradiance* of the flux at a point, and when the radiant flux density leaves S, we speak of the *radiant emittance* of the radiant flux at a point. Similarly also for (radiant) intensity: we have *surface* (radiant) *intensity* and *field* (radiant) *intensity*. The parenthesized "radiant" indicates that this adjective can be omitted when *radiant* flux is understood to be the flux of interest.

Operational Definitions of Field and Surface Quantities

We may summarize the preceding definitions in parts (c)-(f) of Fig. 1.5. These diagrams emphasize the operational procedures used to measure the various quantities in actual radiometric environments.

Thus field radiant flux can be defined over the surface S of the radiant flux meter for an incoming bundle D of directions. The heavy arrows give the general sense of the flow. When the meter is oriented so that at point x the inward unit normal to its collecting surface is ξ, and D is opened up to be the hemisphere $\Xi(\xi)$ of all directions ξ' such that $\xi \cdot \xi' = \cos\theta \geq 0$ then by definition we measure the irradiance at x for the orientation ξ of the collector. The field (radiant) intensity $J(x,\xi)$ and the field radiance $N(x,\xi)$ are defined analogously. It is important to emphasize that the $\Omega(D)$ in the latter two cases should be on the order of 1/30 of a steradian or smaller for best results. The 'surface' counterparts to the preceding 'field' quantities may be pictured by reversing the flux arrows in parts (c) to (f) of Fig. 1.5.

Figure 1.6 shows the details of how a surface radiance may generally be assigned to a real or imaginary surface. We use the radiance invariance law (Sec. 2.6) to assign to the direction ξ at point p on S the radiance $N(x,\xi)$ when p is viewed by a radiance meter oriented as shown. This is a consistent assignation since the radiance-invariance law states that for a fixed ξ, $N(x,\xi)$ is independent of y along a

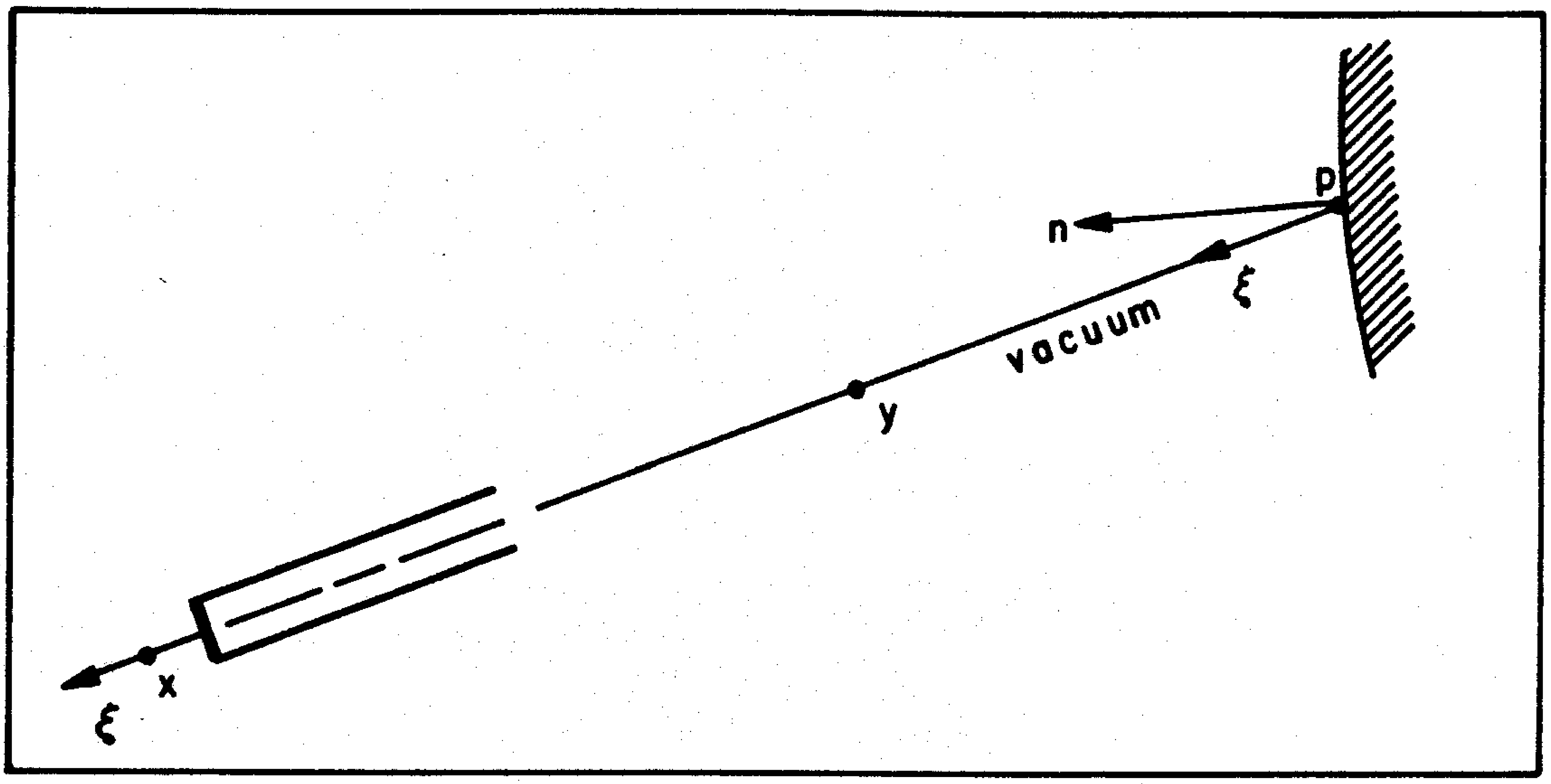

FIG. 1.6 The method of assigning radiances to real or imaginary surfaces.

vacuous path between x and p. In this way each ξ at p in the outward hemisphere Ξ(n) of directions at p can be assigned a radiance.

A useful property of irradiance is the *cosine law*, which follows directly from the present operational considerations. Fig. 1.7 shows a thin collimated steady stream of photons incident normally on a small hypothetical plane surface S. If P(S,D) is the radiant flux produced on S by this stream, then this same flow P(S',D) exists across the surface S' whose unit normal is tilted θ' from the direction of the stream. The connection between the two irradiated areas is:

$$A(S') \cos \theta' = A(S)$$

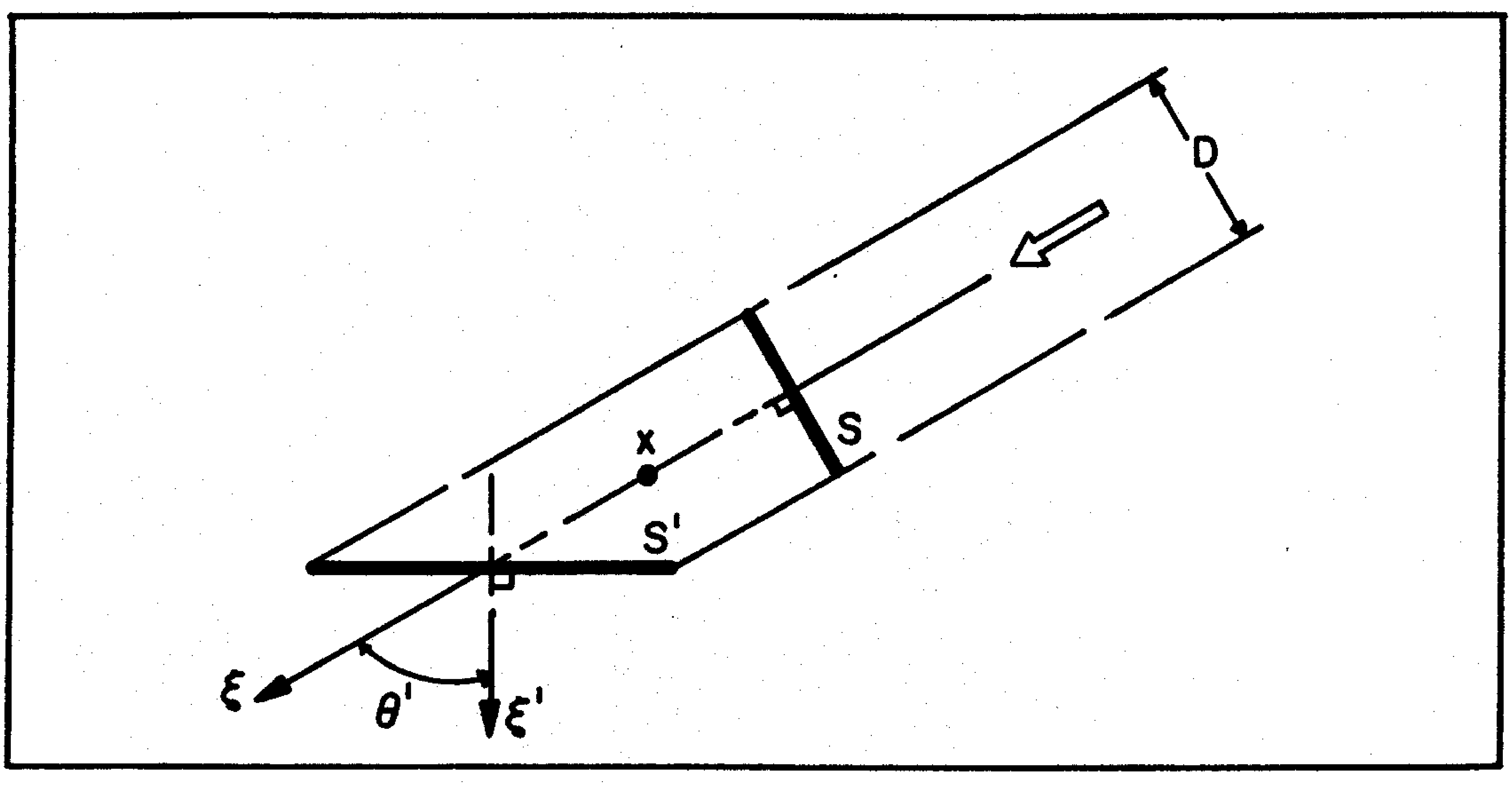

FIG. 1.7 Deriving the cosine law for irradiance.

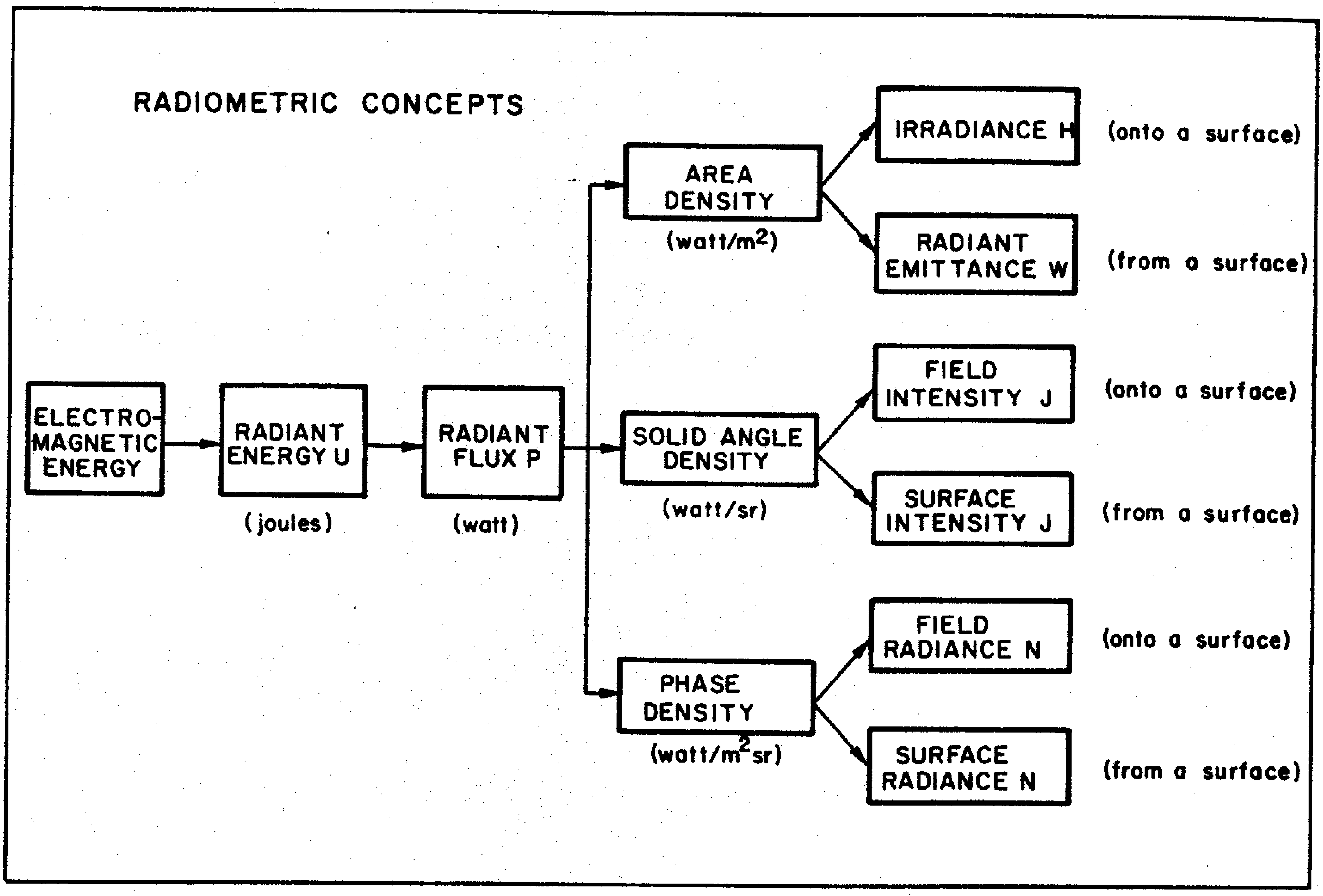

FIG. 1.8 Logical lineage of the radiometric concepts.

Hence the connection between the *irradiances* on S' and S produced by the stream is:

$$H(x,\xi') = \frac{P(S',D)}{A(S')} = \frac{P(S,D)}{A(S)} \cos\theta' = H(x,\xi) \cos\theta'$$

That is,

$$H(x,\xi') = H(x,\xi) \cos\theta'$$

which is a form of the cosine law for irradiance (the general law is given in Sec. 2.8). The companion law to this for the radiant emittance of S' is:

$$W(x,\xi') = W(x,\xi) \cos\theta'$$

Summary of Concepts and Some Principal Formulas of Geometrical Radiometry

A schematic diagram of radiometric concepts, developed in the manner described above, which summarizes the geometric derivatives of radiant energy, along with their mks units, and current standard symbols, is given in Fig. 1.8. The names of the six concepts above, and their designating symbols may come and go with the years, but the logical lineage of the concepts depicted above, with their tap root in the concept of radiant energy and indicated branching structures, will withstand the rigors of time. For while the names in the boxes are transient conventions, the arrangement of the boxes, and the underlying concepts for which the boxes stand are simply manifestations of the way we naturally view radiant energy and the flow of radiant energy in space and time. In this sense the indicated conceptual scheme in Fig. 1.8 is immutable. The full developments of the analytical connections among the radiometric concepts are not needed in this introductory chapter, and are reserved for Chapter 2. However, a brief survey of some of the main formulas of geometrical radiometry is given here for convenient reference during the remainder of this chapter's discussions.

The primary concept of geometrical radiometry in practice is the phase density concept, namely radiance. We find it possible to describe all other concepts in terms of this density. Thus for example in the case of the flux density concept:

$$H(x,\xi) = \int_{\Xi(\xi)} N(x,\xi')\xi'\cdot\xi \, d\Omega(\xi') \quad \text{(with field radiance)} \tag{1}$$

$$W(x,\xi) = \int_{\Xi(-\xi)} N(x,\xi')\xi'\cdot\xi \, d\Omega(\xi') \quad \text{(with surface radiance)} \tag{2}$$

$H(x,\xi)$ is the irradiance at x on a surface whose inward normal is the direction ξ. The basis for (1), (2) rests in the cosine law for irradiance and the possibility of the linear superposition of radiant fluxes. The symbol "$\Xi(\xi)$" stands for the hemisphere of all directions ξ' such that $\xi'\cdot\xi > 0$, (hence $\Xi(-\xi)$ is the hemisphere of all directions ξ' such that $\xi'\cdot(-\xi) > 0$, i.e., $\xi'\cdot\xi < 0$). Here "$d\Omega(\xi')$" is short for "$\sin\theta' \, d\theta' \, d\phi'$", where (θ',ϕ') define ξ' in some reference frame. Of course $\xi'\cdot\xi$ is the scalar or dot product of the directions ξ' and ξ. The representations of the solid angle density in terms of radiance are not needed at present and may be found, along with many related concepts, in Sec. 2.9. We shall also find it convenient to introduce at this time two cousins of the flux density concept, namely *scalar* and *vector* irradiance, defined, respectively, by writing:

$$\text{"}h(x)\text{" for } \int_{\Xi} N(x,\xi') \, d\Omega(\xi') \quad (\text{watt/m}^2) \tag{3}$$

and:

$$\text{"}\mathbf{H}(x)\text{" for } \int_{\Xi} N(x,\xi')\xi' \, d\Omega(\xi') \quad (\text{watt/m}^2) \tag{4}$$

Here Ξ is the set of all unit vectors (directions) in euclidean three space. The scalar irradiance $h(x)$ is the total radiant flux per square meter coursing through point x in all directions. It is related to radiant energy per cubic meter u(x) (the *radiant density*: Joules/m^3) by means of the formula:

$$v(x)\ u(x) = h(x) \tag{5}$$

where v(x) is the speed of light at x (in m/sec). The quantity **H**(x) is a vector; the indicated equation is really three equations: one for each of the x, y, z components of **H**(x), as given by the corresponding components of ξ'. The vector **H**(x) also has units of watts per square meter: its magnitude is the maximum net irradiance attainable as one samples all possible directions ξ of flow about x. The direction of **H**(x) defines this direction of maximum net irradiance. The *net irradiance* $\vec{H}(x,\xi)$ at x in the direction ξ is defined as $H(x,\xi)-H(x,-\xi)$; see Sec. 2.8 for complete details.

It will be necessary in this introductory chapter to also consider *hemispherical scalar irradiance*, defined by writing:

$$\text{"}h(x,\xi)\text{"} \quad \text{for} \quad \int_{\Xi(\xi)} N(x,\xi')\, d\Omega(\xi') \quad (\text{watt/m}^2) \tag{6}$$

$$\text{"}h(x,-\xi)\text{"} \quad \text{for} \quad \int_{\Xi(-\xi)} N(x,\xi')\, d\Omega(\xi') \quad (\text{watt/m}^2) \tag{7}$$

where, by (3),

$$h(x) = h(x,\xi) + h(x,-\xi) \tag{8}$$

for every ξ in Ξ. A convenient terrestrial reference frame in hydrologic optics is that depicted in Fig. 1.9. We will often use the special case of (6), (7) where $\xi = \mathbf{k}$, and we shall write

$$\text{"}h(z,\pm)\text{"} \quad \text{for} \quad h(p,\pm\mathbf{k}) \tag{9}$$

where we retain only the depth variable z of the usual (x,y,z)-coordinates of the point p. Corresponding to $h(z,\pm)$ we have the companions from (1) in which $\xi = \pm\mathbf{k}$; we write

$$\text{"}H(z,\pm)\text{"} \quad \text{for} \quad H(p,\pm\mathbf{k}) \tag{10}$$

Irradiances associated with plus signs are *upwelling* (or upward) irradiances; those with minus signs are *downwelling* (or downward) irradiances. All these irradiances have units of watt/m^2. In natural hydrosols $H(z,\pm)$ can be measured by horizontal flat plate collectors, while $h(z,\pm)$ can be measured by spherical collectors, suitably shielded (see Sec. 2.7). Some useful special cases of the preceding formulas are the following.

Let $N(x,\xi)$ be uniform, i.e., independent of ξ at some x and of magnitude N; then by (1)

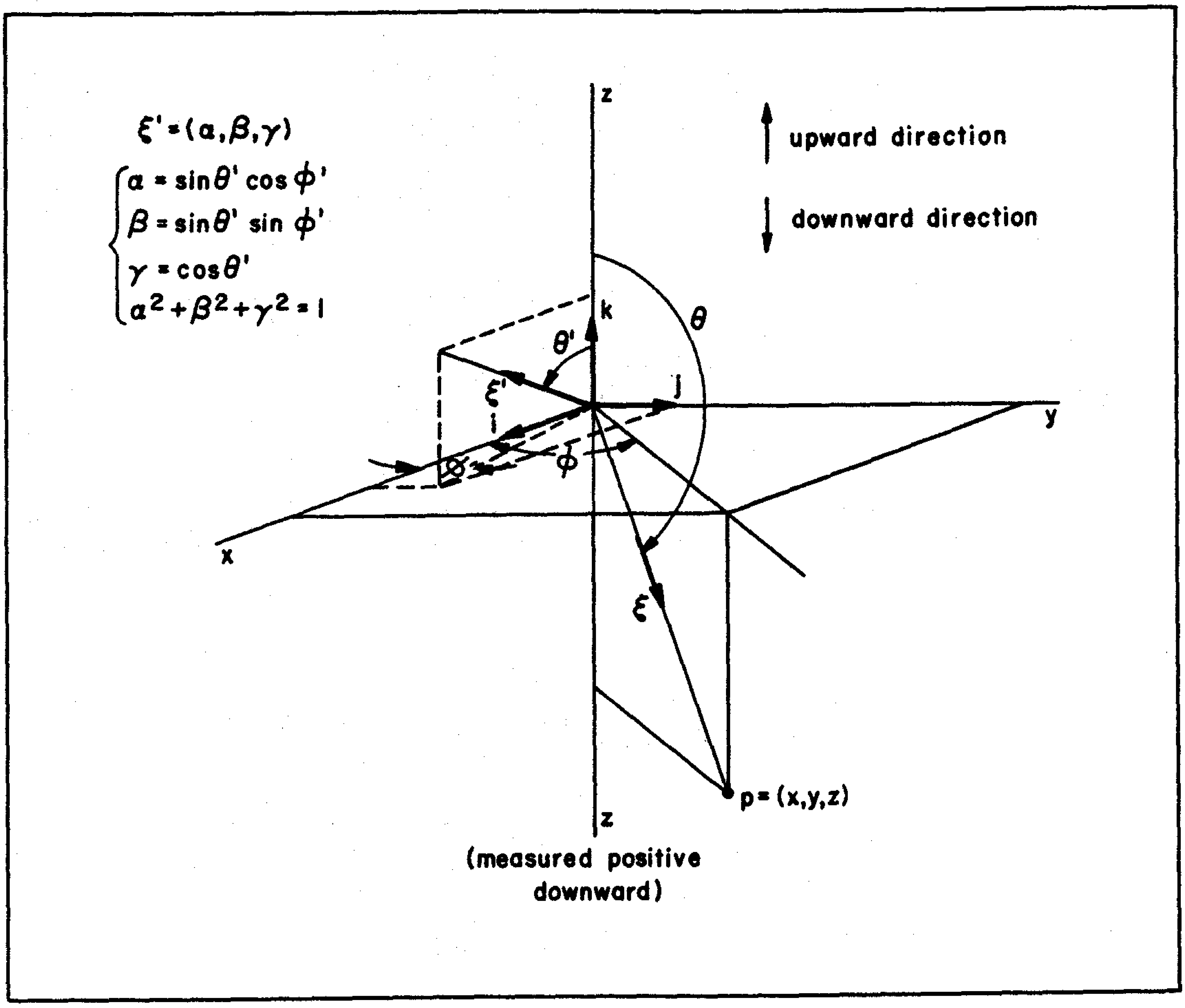

FIG. 1.9 The standard terrestrially-based coordinate system in hydrologic optics.

$$H(x,\xi) = N\int_{\Xi(\xi)} \xi'\cdot\xi \; d\Omega(\xi') = N\int_{\phi'=0}^{2\pi}\int_{\theta'=0}^{\pi/2} \cos\theta' \sin\theta' \; d\theta' d\phi'$$

$$= \pi N \tag{11}$$

which holds for all ξ at x. The computation was made with the **k** axis momentarily shifted parallel to ξ. Further, from (2), in the same way:

$$W(x,\xi) = \pi N \tag{12}$$

for all ξ at x. Next, by (3):

$$h(x) = N\int_{\Xi} d\Omega(\xi') = N\int_{\phi'=0}^{2\pi}\int_{\theta'=0}^{\pi} \sin\theta' \; d\theta' d\phi' = 4\pi N \tag{13}$$

By (4)

$$H(x) = N\int_{\Xi} \xi' \, d\Omega(\xi') = 0 \tag{14}$$

By (6)

$$h(x,\xi) = 2\pi N \tag{15}$$

Observe the effect of the cosine in the integrand: for a uniform radiance distribution at x, $h(x,\xi)=2H(x,\xi)$, for every ξ. Further examples are given in Sec. 2.11.

n^2-Law for Radiance

We mention in passing an important law of geometrical radiometry concerning radiance: *If $\mathcal{P}$ is an arbitrary photon path through a transparent optical medium within which the index of refraction* n *varies continuously with location, then photon flux along the path $\mathcal{P}$ having radiance* N *moves such that* N/n^2 *is invariant along the path* (cf. Sec. 2.6). This is the n^2-law for radiance.

The Bridge to Geometrical Photometry

The conceptual bridge from geometrical radiometry to geometrical photometry is built on the empirical fact that not all wavelengths of radiant flux invoke the same sensation of brightness in the human eye. The green-yellow wavelength 555 mμ is the brightest. In fact one would require, e.g., about 2 watts of blue-green light of 510 mμ or 2 watts of orange light of 610 mμ to produce the same sensation of brightness as one watt of green-yellow light of 555 mμ. The *photopic luminosity* curve depicted in Fig. 1.10 summarizes a quantitative measure $\bar{y}(\lambda)$ of the brightness-sensation producing capabilities of a wavelength λ in the electromagnetic spectrum. Observe that for wavelengths λ below 400 mμ and above 700 mμ, electromagnetic radiation no longer is seen by normal human eyes. A fuller discussion of this curve is given in Sec. 2.12. See also Sec. 1.8.

The conversion rule from a radiometric concept to its photometric counterpart is based on the photopic luminosity curve and is given as follows:

Let $\mathcal{R}$ be any radiometric concept (e.g., U, P, H, W, J, *or* N*) which is defined over the electromagnetic spectrum. Then the photometric concept $\mathcal{L}$ (namely* Q, F, E, L, I, *or* B, *respectively) associated with $\mathcal{R}$ is given by*

$$\boxed{\mathcal{L} = 680\int_0^\infty \mathcal{R}(\lambda)\, \bar{y}(\lambda)\, d\lambda}$$

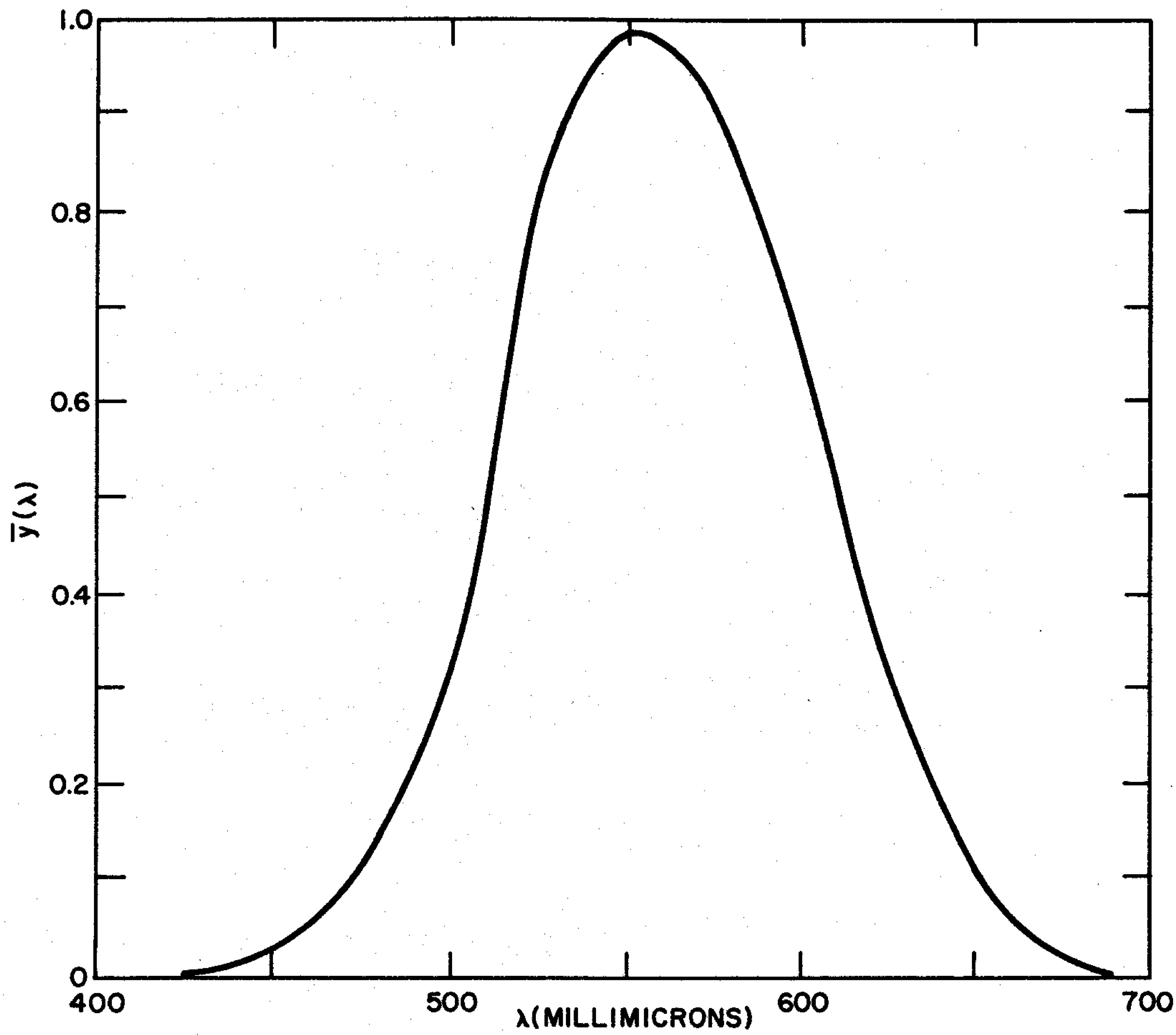

FIG. 1.10 The photopic luminosity function.

If $\mathcal{R}$ has units *watt*/(*), then $\mathcal{L}$ has units *lumen*/(*), where "(*)" stands for (meter) or (steradian) or various permissible combinations of these geometrical units. For example,

$$B(x,\xi) = 680\int_0^\infty N(x,\xi,\lambda)\ \bar{y}(\lambda)\ d\lambda \quad , \text{ lumens/m}^2\text{ sr}$$

This gives the *luminance* (loosely, the "brightness") produced by a given sample of radiance. This is what, in essence, we can see as a result of the radiant flux of photons at x in the direction ξ. Again, for example, *illuminance* is:

$$E(x,\xi) = 680\int_0^\infty H(x,\xi,\lambda)\ \bar{y}(\lambda)\ d\lambda \quad , \text{ lumens/m}^2$$

The logical interrelations among the photometric concepts precisely parallel those of radiometry. Thus, starting with *luminous energy* Q, which, according to the rule above, we define as:

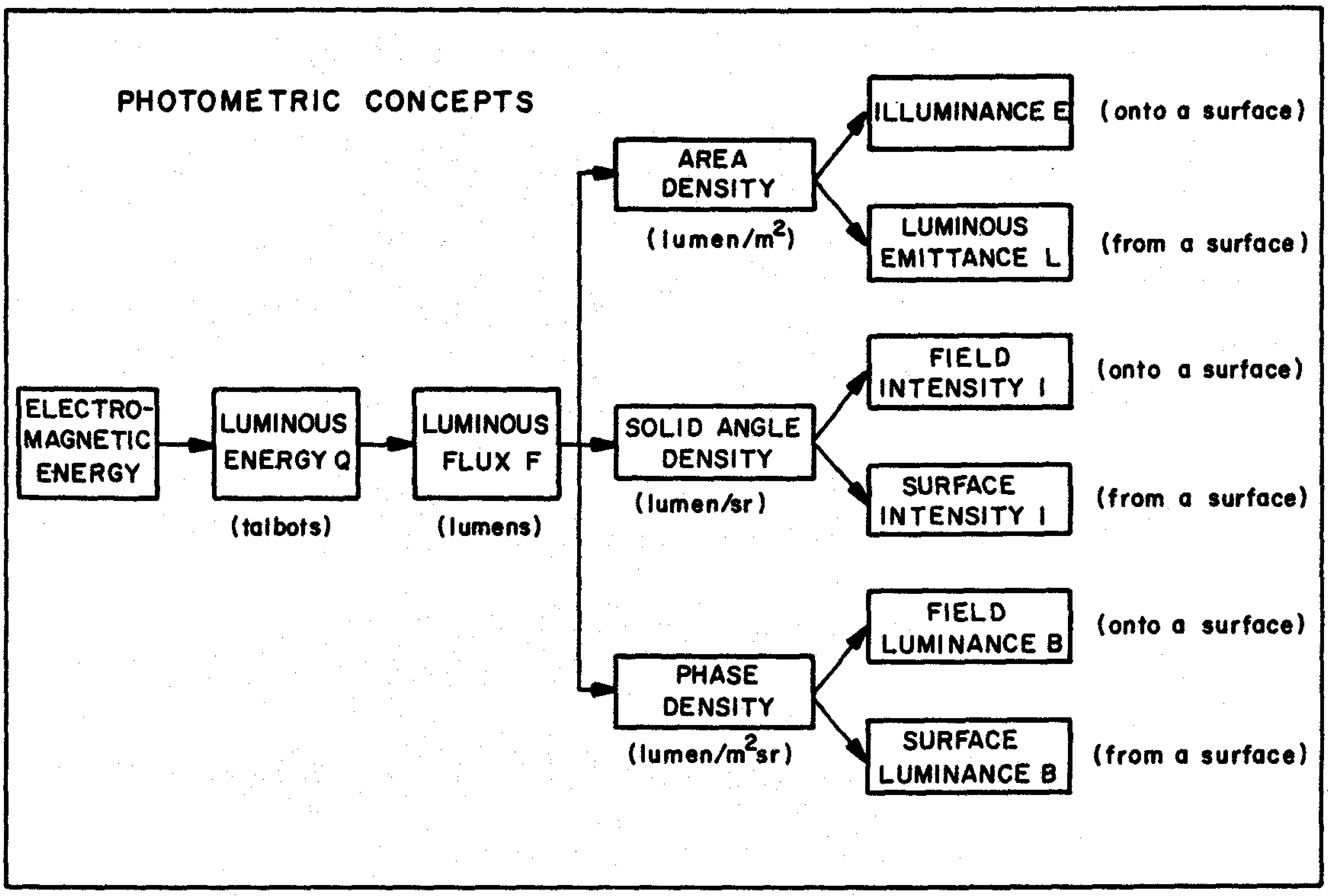

FIG. 1.11 Logical lineage of the photometric concepts.

$$Q = 680\int_0^\infty U(\lambda)\ \bar{y}(\lambda)\ d\lambda \quad ,$$

we then can construct a diagram similar to that in Fig. 1.8. This is shown in Fig. 1.11. Consequently, everything we can say about the *geometrical* properties of the radiometric concepts, we can also say about the corresponding properties of photometric concepts.

We mention in passing some classical alternate sets of photometric units:

1 foot candle = 1 lumen/ft² (area density of flux) (16)

1 candela = 1 lumen/sr (solid angle density of flux)(17)

$$\left.\begin{aligned} &1\ \text{(centimeter) lambert} = \tfrac{1}{\pi}\ \text{lumen/cm}^2\ \text{sr} \\ &1\ \text{(meter) lambert} = \tfrac{1}{\pi}\ \text{lumen/m}^2\ \text{sr} \\ &1\ \text{(foot) lambert} = \tfrac{1}{\pi}\ \text{lumen/ft}^2\ \text{sr} \end{aligned}\right\}\ \text{(phase density of flux)} \qquad (18)$$

From (17) we can compactly express luminance generally in terms of candelas/m^2 when using the mks system (the preferred system). The lambert unit arises as follows: let a surface, which has both unit reflectance with respect to irradiance for each wavelength and also a directionally uniform reflected radiance for each wavelength, be called *a perfectly diffusing* surface, for short. *By definition*, a perfectly diffusing surface irradiated by one lumen has a luminance of *one lambert*. (Use Eq. (12).) However, the conversion rules above in (18) are by convention now used under arbitrary directional and reflectance conditions.

Thus we have the general rule: *To convert* $B(x,\xi)$ *lumens/m^2sr to meter lamberts, multiply* $B(x,\xi)$ *by* π. (This follows from the fact that as defined above the meter lambert is about 1/3 of a lumen/m^2sr; so it takes about 3 meter lamberts to every lumen/m^2sr to describe the same scene.)

With due respect to the historical origins of the preceding terms, it is felt that the continued employment of "foot candle" and "lamberts" will serve no logical purpose. Their mention here simply serves to keep open the passageway to the classical literature of photometry and radiative transfer theory to which we must refer now and then during this work. New students are advised to use the lumen, meter, steradian system of units in photometry, along with the watt, meter, steradian system in radiometry in their future studies. A convenient abbreviated mks unit of radiance is the (unrationalized)* herschel:

$$1 \text{ herschel} = 1 \text{ watt/m}^2\text{sr} \tag{19}$$

and an mks unit of luminance is the (unrationalized) blondel:

$$1 \text{ blondel} = 1 \text{ lumen/m}^2\text{sr} \tag{20}$$

These abbreviations should be used only when the sheer frequency of mention of "watt/m^2sr" or "lumen/m^2sr" becomes so great in a given discussion that facile communication is impaired; otherwise they simply should be spelled out in full using watts, meters and steradians. Further discussion of the foundations of photometry is given in Sec. 2.12.

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*An unrationalized radiance (or luminance) unit is one for which a uniform radiance distribution of magnitude N produces an irradiance of πN. A rationalized unit would associate to a uniform N the irradiance N. An unrationalized radiance unit is thus logically simpler than a rationalized unit. The term "rationalized" here means "removed π-factor". It is irrational to rationalize radiance units just because it is too tiresome to carry around a π-factor which arises in calculations with radiance distributions which in fact *do not occur in practice in real environments* in the first place! (namely directionally uniform distributions).