The definition of radiant flux given above is an *operational definition* in the sense that it may be translated into a definite sequence of physical operations with a specific instrument in a given environment and which culminate in a unique nonnegative number $\Phi(S,D,t,F)$. This type of definition can be made to stand out in bold relief from still another type which may also be used as effectively as the operational definition in establishing the theory of radiometry. This alternative definition is known as the *constitutive definition* of radiant flux which uses only the concepts of the mathematical framework within which radiometry is modeled. In a constitutive definition there is no immediate appeal to physical operations with a specific instrument in a given environment. For an example of a constitutive definition of radiant flux and the other radiometric quantities, the reader may consult Secs. 109 and 131 of reference [251].

2.2 The Meaning of 'Radiant Flux'

It will be helpful during the discussions of this and subsequent chapters to have in mind some visualizable construct of radiant flux. By having the reader picture in a relatively concrete manner the meaning of the term 'radiant flux', the various principles and laws of radiative transfer used throughout this work will become more readily understood and applied. We have already given the term 'radiant flux' a relatively concrete meaning by adopting an operational definition of the term. In this section we shall go one step further and suggest three ways in which one may visualize radiant flux directly. What we shall offer, then, are conceptual frameworks within which to view the notion of radiant energy and which, especially during theoretical discussions of radiative transfer, one may use in a heuristic manner.

One manner in which radiant flux may be visualized is by a means similar to that used in geometrical optics. In order to discuss the theory of lenses within geometrical optics one may use the method of ray tracing. The heart of this method resides in the concept of the "light ray" and a few simple rules of construction of a ray of light through a lens. Corresponding to this notion we have in geometrical radiometry the notion of a *line of flux*. One may thus visualize $\Phi(S,D,t,F)$ as proportional to the number of straight or curved lines having directions lying within the set D where they terminate on the surface S. The time t and set F of frequencies are usually fixed or understood during a discussion so that the lines of flux constitute a representation of the geometric construct of $\Phi(S,D,t,F)$. In this representation, the magnitude of $\Phi(S,D,t,F)$ is proportional to the number of such lines of flux, the proportionality factor being some fixed number of lines per unit of radiant flux. One may thus imagine the radiant energy as a fluid travelling along the lines of flux. The closer together the lines are within some region, the greater the radiant flux (i.e., radiant energy flow) through that region. The lines of flux are to be determined using the same formulas of geometrical optics as used in ray tracing. Whenever scattering takes place, however, some lines of flux undergo abrupt changes in direction. Between the points of these abrupt changes in direction the structure of the lines of flux are again governed by the ray tracing formulas of geometrical optics (and the lines are, in most practical instances, merely straight line segments).

Another manner in which radiant flux may be visualized is by means of the Poynting vector of electromagnetic theory. Besides serving to generate an alternative semantic dimension to the term 'radiant flux', such a visualization--if carefully done--serves to establish an analytic link between electromagnetic theory and the concepts of radiometry. Thus, consider the electric and magnetic vector fields **E** and **H**. The vector product **E** × **H** is called the *Poynting vector field* and is usually denoted by "**P**". A dimensional analysis of **P** shows that it has the dimensions of radiant flux per unit area. The direction of this flow is along the direction of **P**, and the magnitude of the flow is that of **P**, and takes place across a unit area normal to the direction of **P**. Generally, **P** varies in both magnitude and direction many times a second at a given point. Thus the quantity $\Phi(S,D,t,F)$ may be viewed as a time average of the magnitude of the Poynting vector **P** confined to the directions within D over the surface S and during some short time period T around time t. The analytic details of the connection between **P** and $\Phi(S,D,t,F)$ beyond those alluded to here will have no application in the present work. The reader wishing to study this connection in more detail may consult Sec. 124 of Ref. [251].

One further manner of visualizing the concept of radiant flux is by means of the notion of photons, that is by means of 'particles' of light. Monochromatic radiant flux, i.e., radiant flux of a single frequency, say, ν may be associated with the flow of a set of photons each of energy $h\nu$, where "h" denotes Planck's quantum of action (per photon). In more detail, let* "$n(x,\xi,t,\nu)$" (the *phase-space density*) denote the number of ν-frequency photons per unit frequency interval and per unit volume at each point x over the surface S, and moving with speed v in a unit solid angle along the direction ξ within the bundle D of directions at time t. Then:

$$h\int_D\int_S\int_F n(x,\xi,t,\nu)v(x,\xi,t)\nu(x,\xi,t)\ \xi\cdot\mathbf{k}(x)\ dA(x)d\Omega(\xi)dl(\nu) \qquad (1)$$

gives the time rate of flow of radiant energy of frequencies in F crossing surface S, within directions D at time t. k(x) is the unit inward normal to S at x. This is then the seat of the meaning of $\Phi(S,D,t,F)$ in terms of the photon concept. In the preceding integral, "A", "Ω" and "l" denoted area, solid angle, and frequency measures, respectively; and these

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*For reference purposes, we observe here that the connection between the density n and the radiance N is: $N(x,\xi,t,\nu) = h\nu v n(x,\xi,t,\nu)$. (See (5a) of Sec. 2.5.)

will be explained in greater detail in the subsequent sections of this chapter.

As a special case of the preceding connection, let $n(x,\xi,t,\nu)$ be constant of magnitude n over S and over a narrow bundle of directions D normally incident on S and let F consist of discrete frequencies. Then, using the photon interpretation of $\Phi(S,D,t,F)$ just described, we can write:

$$\text{"}\Phi(S,D,t,F)\text{"} \quad \text{for} \quad h\nu nA(S)\Omega(D)\sum_{\nu\in F}\nu \qquad (2)$$

In summary, we have discussed three possible aids to visualizing the meaning of 'radiant flux'. There is the geometric-optics notion of *lines of flux*, the electromagnetic-theoretic construct of the *Poynting vector*, and the quantum-theoretic construct of the moving *photon*. A composite picture may be made by joining all three of the preceding concepts. Thus, one may visualize the photon not as a particle (i.e., a mathematical point) but rather as a spatially small wave train of electromagnetic waves of predominantly a single frequency and moving along the lines of flux. This concept allows light to have at least intuitively, the properties of both particles and waves.

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2.3 Fundamental Geometric Properties of Radiant Flux

In this section we shall assemble the six properties of $\Phi(S,D,t,F)$ on which geometrical radiometry may be based. These six properties summarize precisely and explicitly those macroscopic properties of light which are customarily implicitly assumed in radiometry, and which are based on extended experience with the operational definition of radiant flux. By explicitly recognizing and isolating these six properties we may attain a unified and relatively rigorous development of geometrical radiometry. This fundamental group of six properties falls naturally into three pairs of properties, corresponding to the frequency, surface, and direction parameters occurring in $\Phi(S,D,t,F)$.

We begin with the properties of Φ associated with the frequency parameter F. For every two disjoint sets F_1 and F_2:

$$\Phi(S,D,t,F_1) + \Phi(S,D,t,F_2) = \Phi(S,D,t,F_1 \cup F_2) \qquad (1)$$

and

$$\text{if} \quad l(F) = 0, \quad \text{then} \quad \Phi(S,D,t,F) = 0 \quad . \qquad (2)$$

These properties hold for arbitrary S,D, and t. The first of these is the F-*additivity property* of Φ. The symbol "$\cup$" will be used often below to denote the union of two sets of things. Here "$F_1 \cup F_2$" denotes the set of all frequencies in either F_1 or F_2. By "disjoint sets" we shall mean sets of things which have no elements in common. Thus by "two disjoint sets F_1 and F_2" we mean that F_1 and F_2 have no frequencies in common.