will be explained in greater detail in the subsequent sections of this chapter.

As a special case of the preceding connection, let $n(x,\xi,t,\nu)$ be constant of magnitude n over S and over a narrow bundle of directions D normally incident on S and let F consist of discrete frequencies. Then, using the photon interpretation of $\Phi(S,D,t,F)$ just described, we can write:

$$\text{"}\Phi(S,D,t,F)\text{"} \quad \text{for} \quad h\nu n A(S)\Omega(D) \sum_{\nu\in F} \nu \tag{2}$$

In summary, we have discussed three possible aids to visualizing the meaning of 'radiant flux'. There is the geometric-optics notion of *lines of flux*, the electromagnetic-theoretic construct of the *Poynting vector*, and the quantum-theoretic construct of the moving *photon*. A composite picture may be made by joining all three of the preceding concepts. Thus, one may visualize the photon not as a particle (i.e., a mathematical point) but rather as a spatially small wave train of electromagnetic waves of predominantly a single frequency and moving along the lines of flux. This concept allows light to have at least intuitively, the properties of both particles and waves.

2.3 Fundamental Geometric Properties of Radiant Flux

In this section we shall assemble the six properties of $\Phi(S,D,t,F)$ on which geometrical radiometry may be based. These six properties summarize precisely and explicitly those macroscopic properties of light which are customarily implicitly assumed in radiometry, and which are based on extended experience with the operational definition of radiant flux. By explicitly recognizing and isolating these six properties we may attain a unified and relatively rigorous development of geometrical radiometry. This fundamental group of six properties falls naturally into three pairs of properties, corresponding to the frequency, surface, and direction parameters occurring in $\Phi(S,D,t,F)$.

We begin with the properties of Φ associated with the frequency parameter F. For every two disjoint sets F_1 and F_2:

$$\Phi(S,D,t,F_1) + \Phi(S,D,t,F_2) = \Phi(S,D,t,F_1 \cup F_2) \tag{1}$$

and

$$\text{if} \quad l(F) = 0, \quad \text{then} \quad \Phi(S,D,t,F) = 0 \ . \tag{2}$$

These properties hold for arbitrary S,D, and t. The first of these is the F-*additivity property* of Φ. The symbol "$\cup$" will be used often below to denote the union of two sets of things. Here "$F_1 \cup F_2$" denotes the set of all frequencies in either F_1 or F_2. By "disjoint sets" we shall mean sets of things which have no elements in common. Thus by "two disjoint sets F_1 and F_2" we mean that F_1 and F_2 have no frequencies in common.

For example the set F_1 of frequencies in the interval from 10^9/sec to 10^{10}/sec inclusive is disjoint from the set F_2 of frequencies from 10^{11}/sec to 10^{12}/sec, inclusive. Thus $F_1 \cup F_2$ is now the set of all frequencies which are either in the interval F_1 or F_2. The second property (2) above is the (absolute) F-*continuity property* of Φ.

The meaning of the F-additivity property is quite simple: imagine radiant flux incident on S through D at time t and consisting of frequencies in $F_1 \cup F_2$. This flux could be irradiated onto S through D simultaneously from two separate sources of frequencies F_1 and F_2, or by means of suitably chosen filters. Alternatively, the flux can be presented first for the frequencies of F_1 and then for the frequencies of F_2. In essence, (1) states that on the macroscopic level there is no interference of two or more wave trains occupying the same region of space. The F-continuity property asserts that, on the macroscopic level and with S,D and t fixed and all other things being equal, the less the number of frequencies in F, the less the radiant flux amount of $\Phi(S,D,t,F)$. In particular, frequency intervals of zero length contain zero radiant flux.

From the F-additivity and F-continuity properties of Φ we derive the concept of monochromatic radiant flux. Thus let us write:

$$\text{"}P(S,D,t,\nu)\text{"} \quad \text{for} \quad \lim_{F \to \{\nu\}} \frac{\Phi(S,D,t,F)}{l(F)} \tag{3}$$

It is precisely the properties (1) and (2) which permit the limit $P(S,D,t,\nu)$ to exist.* When ν is understood, we may drop it from the notation to write:

$$\text{"}P(S,D,t)\text{"} \quad \text{for} \quad P(S,D,t,\nu)$$

and even further, we may write:

$$\text{"}P(S,D)\text{"} \quad \text{for} \quad P(S,D,t,\nu)$$

when both ν and t are understood. We call $P(S,D,t,\nu)$ the *monochromatic* (or *spectral*) *radiant flux* of frequency ν over S within D at time t, per unit frequency interval. The function P which assigns to (S,D,t,ν) the number $P(S,D,t,\nu)$ is

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*The mathematical reader may consult Refs. [216] and [103] for the study of the existence of such limits and their use in radiative transfer theory. For simplicity in exposition we have displayed *finite additivity* of Φ, rather than the full *countable additivity* used in the cited references. Similar remarks pertain to subsequent additivity properties stated in this work.

the *monochromatic* (or *spectral*) *radiant flux function*. It follows from (3) and a theorem of calculus that:

$$\Phi(S,D,t,F) = \int_F P(S,D,t,\nu)\ dl(\nu) \ . \qquad (4)$$

The symbol "l" denotes the length measure along the frequency spectrum. Thus for the interval F consisting of all frequencies from frequency ν_1 to frequency ν_2, where $\nu_1 \le \nu_2$, we have $l(F) = \nu_2 - \nu_1$. Thus in practical computations one can write "$d\nu$" for "$dl(\nu)$" and (4) is then understood to be a Riemann integral. This is the intended interpretation of (4) for use in this work. However, general discussions are occasionally greatly facilitated by the retention of the length measure, as shown in (4). The symbol "l" is also interpretable as the length measure along the wavelength spectrum. Furthermore, since both line spectra and continuous spectra are represented by the set of nonnegative real numbers, l can be used to denote either the Lebesgue or Riemann measure on that set if continuous spectra are envisioned, or the Stieltjes measure, if line spectra are considered. The particular choice of the nature of l will be clear by convention or from the context in each case. Thus, unless specifically stated otherwise, l is to be considered as the usual Riemann type length measure used in ordinary calculus, and we conventionally consider continuous spectra. For integrations over wavelength space, use the transformation (32) of Sec. 2.12.

The second pair of properties of Φ is associated with surfaces. For every two disjoint surfaces S_1 and S_2,

$$\Phi(S_1,D,t,F) + \Phi(S_2,D,t,F) = \Phi(S_1 \cup S_2,D,t,F) \qquad (5)$$

and

$$\text{If} \quad A(S) = 0, \quad \text{then} \quad \Phi(S,D,t,F) = 0 \qquad (6)$$

These properties hold for arbitrary D,t and F. The first of these is the *S-additivity property* of Φ. The second is the *S-continuity property* of Φ.

The S-additivity property is understood as follows. Suppose the radiant flux meter has a variable collecting surface S, so that at one time it can be of extent S_1 and at another time (very soon after) it can be of extent S_2, such that S_1 and S_2 are disjoint. Then (5) states that the sum of the two separate readings associated with S_1 and S_2 equals the reading associated with the union $S_1 \cup S_2$ of these surfaces. This experimental fact is generally valid, provided of course, that D, t, and F are fixed as closely as practicable throughout all three measurements. Statement (5) is the ideal indicated by accumulated empirical findings. Statement (6) is also intuitively clear: positive amounts of flux can only be recorded over surfaces of positive area. This relatively innocuous pair of properties of Φ comprises the logical root of the concepts of irradiance and radiant emittance, to be considered later.

We now turn to the third and final pair of fundamental properties of the radiant flux function Φ. These are associated with the direction set D. For every two disjoint direction sets D_1 and D_2:

$$\Phi(S,D_1,t,F) + \Phi(S,D_2,t,F) = \Phi(S,D_1 \cup D_2,t,F) \tag{7}$$

and

$$\text{If} \quad \Omega(D) = 0, \quad \text{then} \quad \Phi(S,D,t,F) = 0 \tag{8}$$

These properties hold for arbitrary S,t,F. The first of these is the D-*additivity property* of Φ, the second is the D-*continuity property* of Φ. These properties along with the preceding four will lead to the rigorous basis for the discussions of radiance, irradiance and related radiometric concepts.

The meaning of the D-additivity property is perhaps the most interesting of all the additivity properties, for it shows most clearly that on the level of reality within which radiometry conventionally takes place, the interference phenomena of light are not discernable: the light fields are comprised of incoherent electromagnetic fields. Examples are abundant on the microscopic level of light phenomena which illustrate the negation of (7), namely that for some S, t and F, there exist disjoint sets D_1 and D_2 such that:

$$\Phi(S,D_1,t,F) + \Phi(S,D_2,t,F) \neq \Phi(S,D_1 \cup D_2,t,F) \quad .$$

Therefore, the left side can be either > or < the right side. Furthermore, the negation of (8) holds on the microscopic level, too. That is, for some S,D,t,F, we have:

$$\Omega(D) = 0 \quad \text{and} \quad \Phi(S,D,t,F) \neq 0 \quad .$$

The first of these inequalities may be illustrated by means of any diffraction arrangement; the second arises every time Maxwellian electromagnetic theory is applied to a *plane* electromagnetic wave. In such a case D consists of a single direction--the direction of propagation--and $\Phi(S,D,t,F)$ is computed by means of the Poynting vector (cf. Sec. 2.2). It is principally through the properties (5)-(8) that one may discern the differences between the electromagnetic and phenomenological views of light, as far as logical form is concerned. That is, if we assume (5)-(8), then with a few additional physical-process and logical requirements which are common to both the electromagnetic and phenomenological views of light, the fundamental equations of radiative transfer theory are logically deducible. This may be seen, for example, by studying the results of Ref. [251]. Thus the electromagnetic and the phenomenological views of light necessarily part ways in (5)-(8). In a similar manner the phenomenological and quantum views of light differ at the same two points as above and possibly also at property (2). For, in the quantum theory, as in electromagnetic theory, radiant flux of a single frequency as carried by a single photon (or a pure monochromatic wave of light) exists in principle. Therefore, it is possible that $\Phi(S,D,t,\{\nu\})>0$ for some set F consisting of a single frequency ν, and that $l(F) = 0$ at the same time. This follows from use of the usual measure of length on the frequency domain. If one redefines length on the frequency domain by adopting a Stieltjes measure, e.g., so that isolated single frequencies are given nonzero (usually unit) length (instead of the zero length we conventionally assigned them by the usual continuum measure) then (2) would hold on the electromagnetic and quantum levels too, and (5)-(8) remain as the source of the fundamental distinctions between the microscopic and the macroscopic views of light.

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2.4 Irradiance and Radiant Emittance

We now turn to the task of defining the radiometric concepts used in radiative transfer in general and hydrologic optics in particular. The first two of these are the concepts of irradiance and radiant emittance. These concepts describe the flow of radiant energy per unit area across a surface. That is, they describe the area-density of radiant flux. Irradiance describes the flow *onto* a unit area; radiant emittance describes the flow *from* a unit area. From a strictly geometric point of view, this is the only distinction between the two concepts. However, radiant emittance occasionally has an additional physical connotation associated with it, namely that of a flow of radiant flux from a unit area of surface which encloses an emitting source of radiant flux, i.e., a region manufacturing radiant energy. However, within the operational definitions of these concepts, this additional connotation does not exist; the connotation exists only in the mind of the experimenter. We now turn to the detailed definitions of these concepts.

Definition of Irradiance

We begin with the concept of irradiance. Imagine a radiant flux meter transported to a point x in a natural hydrosol, or in the atmosphere. Let the collecting surface S of the meter be placed so that x falls within its small expanse, and orient the set D of directions of the meter as desired. A filter is fitted on the meter so as to pass monochromatic radiant flux of given frequency ν. Hence, the meter can be made to read P(S,D) directly (with ν and t and their units understood). Let "A(S)" denote the area of the collecting surface S. Then we shall write:

$$\text{"}H(S,D)\text{"} \quad \text{for} \quad P(S,D)/A(S) \tag{1}$$

and call H(S,D) the (*empirical*) *irradiance* over S within D. In full notation for the unpolarized context, we would write:

$$\text{"}H(S,D,t,F)\text{"} \quad \text{for} \quad \Phi(S,D,t,F)/A(S) \tag{2}$$