It is now an easy matter to gradually generalize (4) to the following more general settings: (a) passage through an arbitrary finite number of transparent contiguous media and going on to the limit of continuously varying n; (b) oblique rather than normal incidence of the beam on Y; (c) inclusion of a transmittance factor to allow for scattering and absorption losses from P. Generalizations (a) and (b) result in no change in the form of (4). Generalization (c) results in a multiplicative factor T included on the left side of (4). This factor will be considered at great length in Sec. 3.10 in the discussions of beam transmittance. Specific suggestions for these generalizations are given in Sec. 12 of Ref. [251]. See also [98]. *Henceforth, whenever radiances are related within media of differing indices of refraction, it will be understood that* N/n^2 *rather than* N *will be used, even though "N" only appears in the equations.*

2.7 Scalar Irradiance, Radiant Energy, and Related Concepts

The radiometric concepts studied in this section are those of scalar irradiance, radiant energy, and related radiometric concepts. The first of these concepts is designed to quantitatively describe the volume density of radiant energy in a way which is amenable to operational methods of determination. In addition to the notion of scalar irradiance, we shall develop in this section several closely related notions which together with scalar irradiance comprise a set of useful measures of the volume density of radiant energy. The first of these is radiant density.

Radiant Density

The notion of radiant density is one of several concepts designed to give a measure of the radiant energy per unit volume at a point. Consider a steady beam of radiant flux normally incident on surface S at point x at time t, as shown in Fig. 2.16. Let the field radiance of the beam at this instant be N, its cross sectional area be A, and its solid angle be Ω. The amount of radiant flux incident on S at time t is then $NA\Omega$. An instant t later, the flux of the beam will have moved on a distance $r = vt$, and the flux will have swept out a cylindrical volume of magnitude $V = Avt$. During this time the beam has been steadily pouring an amount of radiant energy into the volume at the rate of $NA\Omega$ watts. Hence the radiant energy content of the beam is $NA\Omega t$, and its average content per unit volume is $NA\Omega t/Avt = N\Omega/v$.

Suppose that point x were simultaneously irradiated at time t by an arbitrary finite number of narrow beams of radiance N_i, $i = 1,\ldots, n$, and corresponding solid angles Ω_i. Then the radiant energy $u(x,t)$ per unit volume at x is given at time t, by means of the D-additivity of Φ (equation (7), Sec. 2.3):

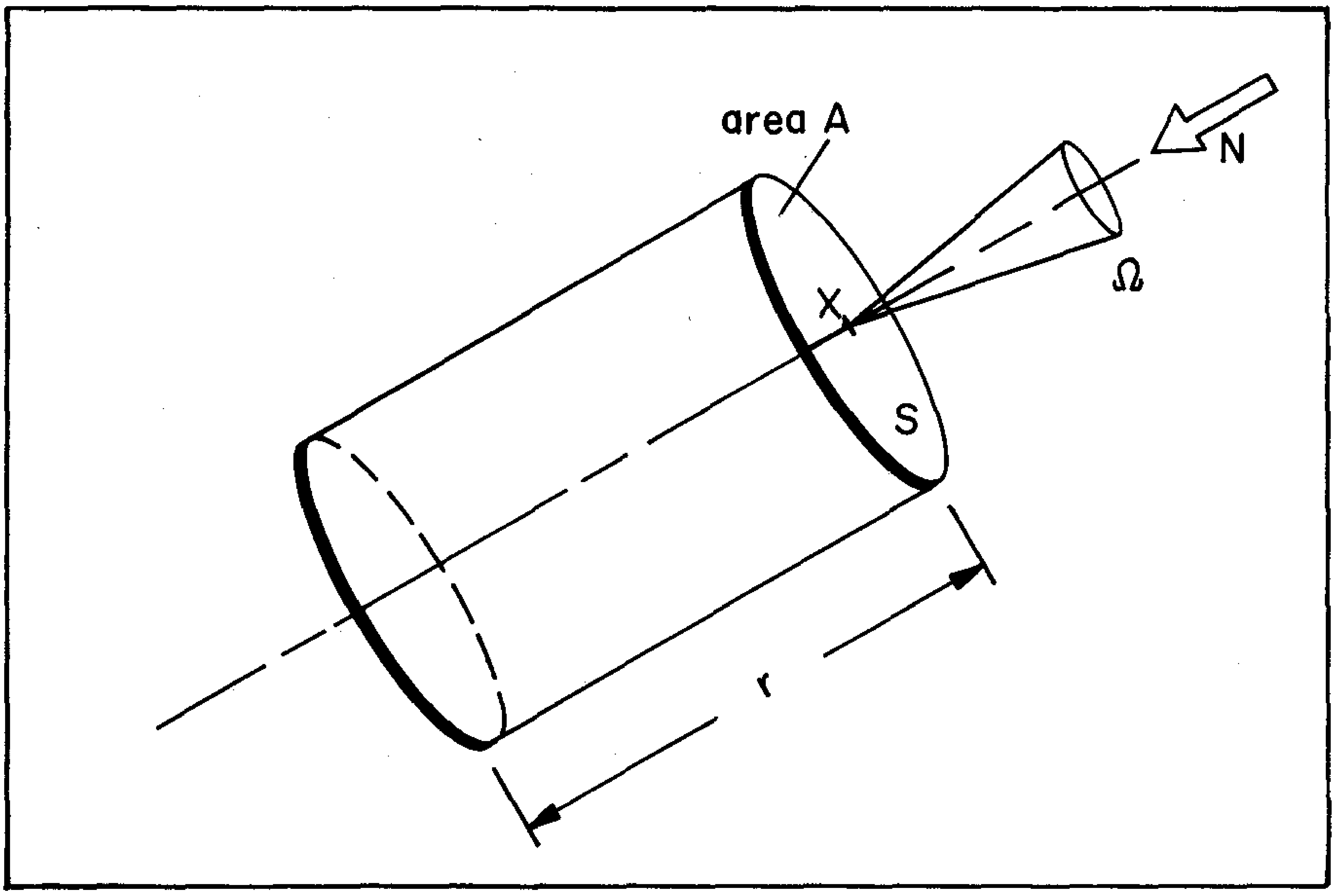

FIG. 2.16 Setting up the connection between radiance and radiant density

$$u(x,t) = \frac{1}{v} \sum_{i=1}^{n} N_i \Omega_i \tag{1}$$

The transition to the continuous case is immediate. Toward this end, let us continue to write "u(x,t)" for the *radiant density*, i.e., we shall also write:

$$\text{"}u(x,t)\text{"} \quad \text{for} \quad \frac{1}{v(x,t)} \int_{\Xi} N(x,\xi,t)\, d\Omega(\xi) \; . \tag{2}$$

The units of u(x,t) are *joules/m*^{3}. We may use either the field or surface interpretation of radiance in this definition.

Scalar Irradiance

Let us go on to write:

$$\text{"}h(x,t)\text{"} \quad \text{for} \quad \int_{\Xi} N(x,\xi,t)\, d\Omega(\xi) \quad . \tag{3}$$

$h(x,t)$ is the *scalar irradiance* at x and time t. The field radiance interpretation of N is most often used in (3), and this interpretation will be in force unless specifically noted otherwise. The reason for singling out $h(x,t)$ for special consideration will be made clear in a moment. For the present it suffices to note that in general:

$$\boxed{u(x,t)v(x,t) = h(x,t)} \tag{4}$$

By virtue of (3) it follows that in this equation the field interpretation of $u(x,t)$ is to be understood, and that while the units of $u(x,t)$ are joules/m³, those of $h(x,t)$ are watts/m². Hence, the term "irradiance" in the name "scalar irradiance" is appropriate. The reason for the modifier "scalar" will also become clear subsequently after vector irradiance has been defined in (2) of Sec. 2.8. A generalization of (3) is obtained by replacing Ξ by a subset D of Ξ. In that case we would write:

$$\text{"}h(x,D,t)\text{"} \quad \text{for} \quad \int_D N(x,\xi,t)\, d\Omega(\xi) \quad .$$

The radiant density associated with $h(x,D,t)$ is $u(x,D,t)$ and (4) holds for these two quantities.

Spherical Irradiance

We shall now show why scalar irradiance is singled out as an alternate (and an actually preferred) description of the radiant density at a point in a radiant flux field. Consider the light field at a point x in a natural optical medium at time t. Let $N(x,\cdot)$ be the radiance distribution at x. Now imagine a small spherical collecting surface S of radius r in the field so that its center is at x. We then ask: *what is the average amount of radiant flux incident per unit area over* S?

To answer this question it is useful to conceptually decompose the great number of radiant flux streams at x into a discrete set of flows. Two such flows are shown in Fig. 2.17. The lines of flux of one of these flows along the direction ξ_i have been fitted with little direction cones of solid angle magnitude Ω_i. Suppose the radiance at x in the direction ξ_i is N_i. Then the irradiance at x on a plane normal to ξ_i is $N_i\Omega_i$. If the sphere is small, say the size of a ping pong ball, then for most natural light fields in the air and sea, N_i will not vary in the region of space taken up by the volume of the sphere. From this we see that we can treat the radiance function N as a constant with respect to location in the vicinity of the sphere and of value N_i for the direction ξ_i. It follows that the amount of radiant flux incident on the sphere contributed by the stream of flux in the direction ξ_i is $(n_i\Omega_i)\pi r^2$. This estimate is based on the

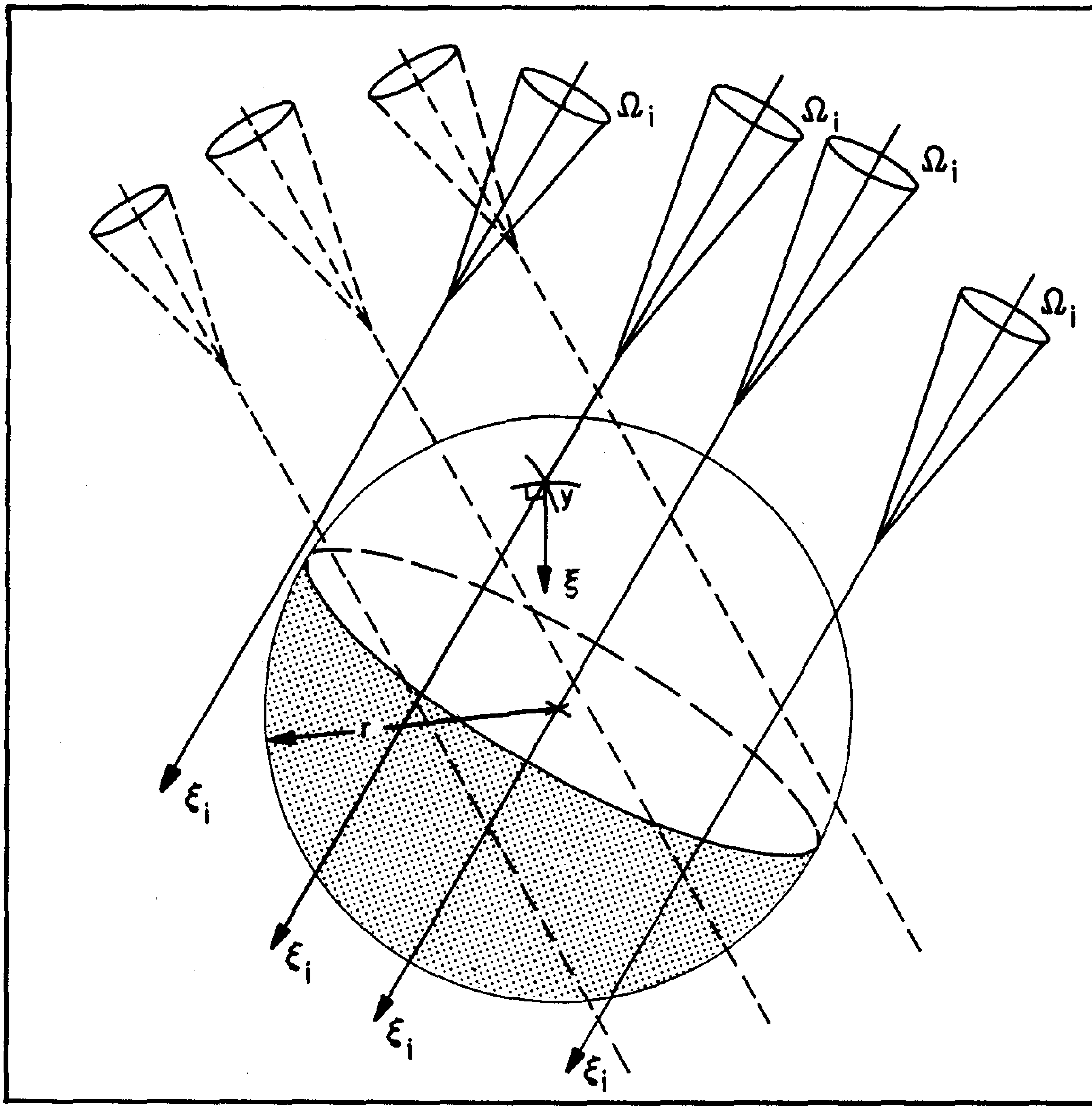

FIG. 2.17 Computing the radiant flux intercepted by a spherical collector in a general light field

assumption that the amount of flux of a narrow beam intercepted by a curved hemispherical surface is the same as the amount intercepted by the great-circle area associated with the hemisphere. The assumption is rigorously defensible for transparent media using the concepts of vector analysis and Stokes Theorem. For the present the reader's intuition will readily allow this assumption to stand even for the case of turbid media as long as r is kept very small. The "line of flux" interpretation will help the intuition considerably in this matter.

The main task in answering the above question has now been dispatched. It remains only to add up all the contributions by the various beams of flux, using as justification Equation (7) of Sec. 2.3. The result is:

$$\pi r^2 \sum_{i=1}^{n} N_i \Omega_i$$

The average radiant flux per unit area of the sphere S is then obtained by dividing this quantity by $4\pi r^2$. Let us designate this average by writing:

$$\text{"}h_{4\pi}(x,t)\text{"} \quad \text{for} \quad \frac{1}{4} \sum_{i=1}^{n} N_i \Omega_i \tag{5}$$

and agree to call it *spherical irradiance*. We shall retain this terminology and notation for the continuous formulation. That is, we shall write:

$$\text{"}h_{4\pi}(x,t)\text{"} \quad \text{for} \quad \frac{1}{4}\, h(x,t) \tag{6}$$

Definition (6) is the basis for an operational determination of scalar irradiance using a spherical collecting surface S. For the average radiant flux per unit area on S is readily measurable and this amount differs multiplicatively from u(x,t) by a fixed numerical factor. Hence, by only slight changes in optical design, the same photoelectric devices used to determine H and N can be directed to obtain scalar irradiance h. Therefore it is spherical irradiance or scalar irradiance which is directly measurable by photoelectric devices. The concept of radiant density u(x,t) is by way of contrast a theoretical concept related to the empirically-based concept h(x,t) by means of (4).

Hemispherical Irradiance

One of the most useful mathematical models of light fields in natural waters is the exact two-flow model to be considered in detail in Chapter 8. A radiometric concept which arises in that theory, and one which also has been found of intrinsic interest to experimenters, is the concept of hemispherical scalar irradiance. We now discuss this concept.

Figure 2.18 (a) depicts a small spherical collecting surface S with center x which is exposed to flux from only one hemisphere of Ξ. Let $N(x,\cdot)$ be the radiance distribution at x. Let us say that light is incident on the sphere in the direction of $\Xi(\xi)$. We ask: what is the average amount of radiant flux incident per unit area over S? Clearly every point of S is in principle exposed to the light field over $\Xi(\xi)$. Fig. 2.18 (b) shows how an obliquely incident beam with a direction in $\Xi(\xi)$ can come close to illuminating the "north pole" of the little spherical surface. If we divide up $\Xi(\xi)$ into pieces analogously to the manner used in deriving the expressions above for spherical and scalar irradiance, then it becomes clear that the integral of $N(x,\cdot)$ over $\Xi(\xi)$ yields the appropriate scalar or spherical irradiance component.

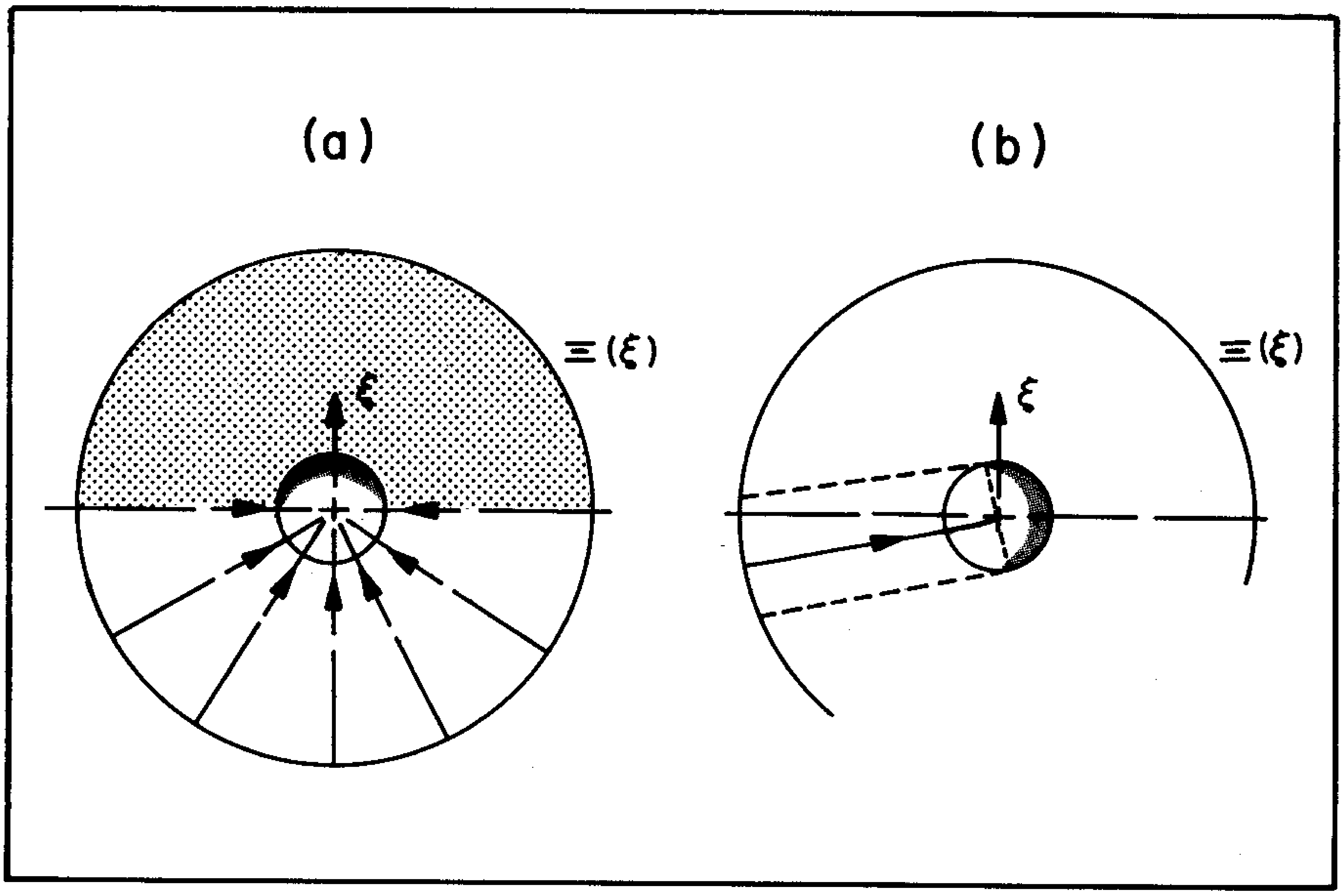

FIG. 2.18 Details for a shielded spherical radiant flux collector

Thus, using field radiance let us write:

$$\text{"}h(x,\xi,t)\text{"} \quad \text{for} \quad \int_{\Xi(\xi)} N(x,\xi',t)\; d\Omega(\xi') \tag{7}$$

and analogously, we write:

$$\text{"}h_{4\pi}(x,\xi,t)\text{"} \quad \text{for} \quad \frac{1}{4}\int_{\Xi(\xi)} N(x,\xi',t)\; d\Omega(\xi') \tag{8}$$

We call $h_{4\pi}(x,\xi,t)$ the *hemispherical irradiance* at x, over the hemisphere $\Xi(\xi)$, at time t. Further, $h(x,\xi,t)$ is the associated *hemispherical scalar irradiance*. It is clearly a special case of $h(x,D,t)$ defined after (3) above. Methods of measuring hemispherical irradiances will be discussed in Chapter 13. It follows immediately from (3) and (7) that:

$$h(x,t) = h(x,\xi,t) + h(x,-\xi,t) \quad . \tag{9}$$

An analogous connection to that displayed in (9) also holds between $h_{4\pi}(x,\pm\xi,t)$ and $h_{4\pi}(x,t)$. The introduction of $h_{4\pi}(x,\xi,t)$ into the family of radiometric concepts is motivated exactly for the empirical reasons that motivated the introduction of its full spherical companion $h_{4\pi}(x,t)$.

When we are working in stratified light fields (Sec. 2.4) then it is possible to drop without loss of generality the "x" and "y" coordinate symbols from the notation and retain only the depth coordinate symbol "z" in the notation. In such contexts we agree to write:

$$\text{"}h(z,\xi,t)\text{"} \quad \text{or} \quad \text{"}h(z,\theta,\phi,t)\text{"} \quad \text{for} \quad h(x,\xi,t) \quad . \tag{10}$$

In particular, if ξ is **k** or -**k**, which occurs in the important case of the two-flow theory (Sec. 8.3), then we agree further to write:

$$\text{"}h(z,\pm,t)\text{"} \quad \text{for} \quad h(z,\pm\mathbf{k},t) \quad , \tag{11}$$

where we read upper signs together and then lower signs together to obtain two separate definitions. As usual, when the light field does not appreciably change in time, or when time is understood, we shall drop "t" from the notation. Applications of these concepts are taken up in Sec. 13.9.

Radiant Energy over Space

The discussion of this section is now continued by officially noting two interpretations of the term "radiant energy". The first interpretation centers on the simple connection that exists between scalar irradiance and radiant energy. Suppose X is a subset of an optical medium over which at time t there is defined a scalar irradiance function h for a given frequency ν. Let "U(X,t)" denote the radiant energy content of X at time t. That is, by the definition of u(x,t), we agree to write:

$$\text{"}U(X,t)\text{"} \quad \text{for} \quad \int_X u(x,t)\ dV(x) \tag{12}$$

and from (4):

$$U(X,t) = \int_X \big(h(x,t)/v(x,t)\big)\ dV(x) \tag{13}$$

where "V" is the volume measure of the optical medium. As a special case, if v(x,t) and u(x,t) are independent of x and t then (13) becomes:

$$U(X) = (h/v)\ V(X) \tag{14}$$

where, for this case, we have written:

$$\text{"}U(X)\text{"} \quad \text{for} \quad U(X,t)$$

$$\text{"}h\text{"} \quad \text{for} \quad h(x,t)$$

$$\text{"}v\text{"} \quad \text{for} \quad v(x,t) \quad .$$

It is clear from (12) that $U(\cdot,t)$ is V-*additive* and V-*continuous*. That is, for every two disjoint parts X_1 and X_2 of an optical medium:

$$U(X_1,t) + U(X_2,t) = U(X_1 \cup X_2,t) \quad , \qquad (15)$$

and for every X and t:

$$\text{If} \quad V(X) = 0, \quad \text{then} \quad U(X,t) = 0 \quad . \qquad (16)$$

Radiant Energy over Time

There is still one more interpretation that can be made of the term "radiant energy". The preceding interpretation of (12) is associated with the energy content of a given region X at time t. There is a complementary interpretation of the total energy incident on or leaving a surface S over an interval T of time. For this interpretation we write,e.g.:

$$\text{"}U^-(S,T)\text{"} \quad \text{for} \quad \int_T \int_S H(x,\Xi(\xi),t)\, dA(x)\, dt \qquad (17)$$

i.e., $U^-(S,T)$ is the radiant energy incident on S over the time interval T. The hemisphere of incident radiant flux at each x is $\Xi(\xi)$, with ξ normal to S at x, in the inward sense. A complementary definition can be made for $U^+(X,T)$ using radiant emittance.

It is worthwhile isolating the important concept, occurring in (17), of radiant flux across a *general* surface S rather than just a collecting surface of the kind encountered in the sections above. Thus we write:

$$\text{"}P^-(S,t)\text{"} \quad \text{for} \quad \int_S H(x,\Xi(\xi),t)\, dA(x) \quad , \qquad (18)$$

where ξ is the unit inward normal to S at x. A similar definition of $P^+(S,t)$ can be phrased. As usual, the signs "+" and "-" can be dropped whenever no confusion results, and also the "t" can be omitted for brevity.

Scalar Radiant Emittance

We conclude this section with the definition of the notion of scalar radiant emittance. This concept is the surface-counterpart to scalar irradiance h defined in (3). Thus, let us write:

$$\text{"}w(x,t)\text{"} \quad \text{for} \quad \int_\Xi N^+(x,\xi',t)\, d\Omega(\xi') \qquad (19)$$

w(x,t) is the scalar radiant emittance at x at time t. This concept is useful in describing certain sources of radiant flux distributed continuously over some region of an optical medium. The emittance counterparts to hemispherical scalar irradiance emittance can now be defined for w(x,t). These definitions would exactly parallel those in (5), (6), (7), (8), (10), (11), and therefore need not be given in detail at this time.

<< Table of Contents << Title Page >> Next Page >>

2.8 Vector Irradiance

The radiometric concept of vector irradiance, which will now be considered, constitutes an interesting and useful complementary concept to that of scalar irradiance. Whereas scalar irradiance in essence measures the volume density of radiant energy at a point and does so without emphasis on the directions of incidence of the component flows but only their magnitudes, vector irradiance in contrast gives a measure of the direction of the preponderant flow of radiant energy at the point without emphasis on the magnitude of the various component flows. Besides serving to complement the geometric properties of scalar irradiance in this way, vector irradiance forms a rigorous tool in deriving the transfer equations for scalar irradiance, and also a powerful means of measuring precisely and directly the absorption properties of real optical media. The basis for the latter means (the divergence relation for **H**) is considered in Chapter 8 and some of its applications are discussed in Chapters 6 and 13. In this section emphasis will be on introducing and explicating the geometric and physical meanings of vector irradiance.

A Mechanical Analogy

The notion of vector irradiance can be introduced by means of an analogy with the vectorial treatment of forces in static mechanics. Figure 2.19 (a) depicts a force diagram familiar to beginning students in static mechanics. A particle at point P is subject simultaneously to two steady forces of magnitude F_1 and F_2 along directions ξ_1 and ξ_2. In order to establish equilibrium of the particle--i.e., to balance out F_1 and F_2 so that the particle is stationary, another force of magnitude F_3' must be applied along direction ξ_3'. The magnitude F_3 and direction ξ_3 of the equivalent force that may replace F_1 and F_2 is found by means of the familiar parallelogram of forces shown in Fig. 2.19 (b). The required balancing magnitude is then $-F_3'$ and its direction is $-\xi_3'$, which follows directly from Newton's Third Law. The central observation to be made here is that, for the purpose of static equilibrium, two forces $\mathbf{F}_1 (= F_1\xi_1)$ and $\mathbf{F}_2 (= F_2\xi_2)$ can be replaced by a single force $\mathbf{F}_3$ $(= F_3\xi_3)$ which serves as a *mechanical* equivalent of the set of forces consisting of F_1 and F_2 together. Thus, $\mathbf{F}_3$ is, for the purposes of an equilibrium computation, equivalent to $\mathbf{F}_1+\mathbf{F}_2$.

Consider now a point P irradiated by two beams of radiant flux which are flowing along directions ξ_1 and ξ_2 with