2.9 Radiant Intensity

The concept of radiant intensity, the last of the set of basic radiometric concepts to be introduced in this chapter, is designed to give a measure of the solid angle density of radiant flux. Thus radiant intensity is a dual concept to irradiance in the sense that the latter gives a measure of the *area density* of radiant flux while the former gives a measure of the *solid angle density* of radiant flux. At one time the concept of radiant intensity enjoyed the place now occupied by radiant flux. In the days of the candle and gas lamp there were very few artificial extended light sources. The controlled artificial point source, such as a candle's flame, was the sole basis for radiometric standards and its radiant output was conveniently measured in terms of intensity. However, with the passing of years the numbers of extended artificial light sources increased and the classical mode of use of radiant intensity has become correspondingly less frequent than that of radiant flux. Eventually radiance for the most part usurped the once centrally disposed radiant intensity concept. Despite this displacement of radiant intensity's status, it appears that there will always exist times when its use arises naturally. For example when emitting 'point sources' are considered, the use of radiant intensity seems automatically indicated. This useful aspect of radiant intensity will be discussed during its systematic development, to which we now turn.

Operational Definition of Empirical Radiant Intensity

In presenting the concept of radiant intensity we shall be guided by operational considerations so as to give the concept a secure footing relative to the other radiometric concepts already defined. Thus our first encounter with the notion of radiant intensity is in the following context: in the operational definition of P(S,D) (Sec. 2.3), a radiant flux meter with collecting surface S and collecting directions D and monochromatic filter passing a single frequency ν records an associated amount P(S,D) of radiant flux incident on S through the set of directions D. Once the datum P(S,D) is obtained, then one conceptual path leads, as in Sec. 2.4, to irradiance H(S,D), i.e., the area density of P(S,D) over S; another path leads to J(S,D), i.e., the solid angle density of P(S,D) over D, where we have written:

$$\text{"}J(S,D)\text{"} \quad \text{for} \quad P(S,D)/\Omega(D) \quad . \tag{1}$$

We call J(S,D) the (empirical) *radiant intensity* of P(S,D) over D on S. The dimensions of radiant intensity are *radiant flux per solid angle (per unit frequency interval)*, and convenient units are *watts/steradian (per unit frequency interval)*. In full notation for the unpolarized context, we would write:

$$\text{"}J(S,D,t,F)\text{"} \quad \text{for} \quad \Phi(S,D,t,F)/\Omega(D)$$

or:

$$\text{"}J(S,D,t,\nu)\text{"} \quad \text{for} \quad P(S,D,t,\nu)/\Omega(D) \quad . \tag{2}$$

However, we shall need only to employ the briefer notation in most of our discussions.

An examination of the operational definition of empirical radiant intensity, summarized in (1), will show that there is no restriction on the set of directions D. That is, D may be an arbitrary fixed set of directions along which the radiant flux funnels down onto the points of the collecting surface S. In practice, however, a radiance meter is the device used to estimate the radiant intensity of the light field at a point in an optical medium. In such an instrument, the set D is a relatively narrow conical bundle of directions whose axis is perpendicular to the collecting surface S of the meter. The connection between field radiance N(S,D), and radiant intensity in such a context, follows from (1) and is readily stated:

$$J(S,D) = N(S,D)A(S) \tag{3}$$

The connection between J(S,D) and N(S,D) can be generalized to take into account radiant flux which crosses S obliquely within the narrow set of directions D. The geometric setting is essentially that depicted in Fig. 2.6, the setting for the cosine law for irradiance.

To establish the generalized version of (3), we return to (1) and within the setting of Fig. 2.6, compute P(S',D) i.e., the radiant flux over S':

$$P(S',D) = P(S,D) = N(S,D)A(S)\Omega(D) \quad .$$

The reason for the equality of P(S',D) and P(S,D) stems from the hypothesized setting of Fig. 2.6, and the arguments presented earlier. Therefore, from (1):

$$\begin{aligned} J(S',D) &= P(S',D)/\Omega(D) \\ &= N(S,D)A(S) \quad . \end{aligned}$$

But:

$$A(S) = A(S') \cos \vartheta \quad .$$

Hence:

$$J(S',D) = N(S,D)A(S') \cos \vartheta \quad .$$

By the radiance invariance law:

$$N(S',D) = N(S,D) \quad .$$

Hence:

$$\boxed{J(S',D) = N(S',D)A(S') \cos \vartheta \ ,} \tag{4}$$

whenever the inward unit normal ξ' to S' makes an angle ϑ with the central direction ξ of D, as in Fig. 2.6, Eq. (4) should be compared with the special case (6) of Sec. 2.5. It is worthwhile re-emphasizing that relations (3) and (4) are relations among *empirical* radiometric quantities, i.e., radiometric quantities obtained with the use of a radiance meter of finite solid angle opening $\Omega(D)$, and finite collecting surface area $A(S)$. The more finely-honed *theoretical* radiometric concepts come later with the help of the various additive and continuity properties of Φ postulated in Sec. 2.3. The empirical concepts serve to establish the bridge between theoretical constructs and the immediately given physical realities. The empirical concepts serve also to block out in rough form the incipient analytical structures of the theoretical relations.

Field Intensity vs. Surface Intensity

There is a distinction that can be made in practice between two types of radiant intensity, a distinction that is exactly analogous to the distinction made in Sec. 2.5 between field and surface radiance. Indeed, by referring to Fig. 2.12 wherein is depicted the two types of radiance, $N^+(S,D)$ and $N^-(S,D)$, which in turn are defined as in (30) and (31) of Sec. 2.5, we are led to write:

$$\text{"}J^+(S,D)\text{"} \quad \text{for} \quad P^+(S,D)/\Omega(D) \tag{5}$$

and

$$\text{"}J^-(S,D)\text{"} \quad \text{for} \quad P^-(S,D)/\Omega(D) \quad , \tag{6}$$

in complete analogy to the definitions of $W(S,D)$ and $H(S,D)$ in (17) and (18) of Sec. 2.4. We call $J^-(S,D)$ the *field intensity* and $J^+(S,D)$ the *surface intensity* over D within S. The utility of this distinction and the basis for the names of these concepts rest once again on the remarks for $N^+(S,D)$ and $N^-(S,D)$ in Sec. 2.5. In actual practice in natural optical media it is the surface intensity which is used with greatest frequency. However, in these settings it is the field intensity (or rather radiance) which, in the final analysis, must be measured before the surface intensity is obtained. The basic quantitative connection between the two types of radiant intensity is analogous to that between surface and field radiance in (22) of Sec. 2.5:

$$\boxed{J^+(S,D) = J^-(S,D) \quad .} \tag{7}$$

It follows from (30) of Sec. 2.5 and (5) above that:

$$N^+(S,D) = W(S,D)/\Omega(D) = J^+(S,D)/A(S) \tag{8}$$

and from (31) of Sec. 2.5 and (6) above that:

$$N^-(S,D) = H(S,D)/\Omega(D) = J^-(S,D)/A(S) \quad . \tag{9}$$

Henceforth we shall drop the "+" and "-" superscripts from the symbol "J" when it is clear from the context (or immaterial) which interpretation of radiant intensity is to be used in reading a statement using the concept of radiant intensity. Occasionally, however, especially for the purpose of emphasizing a delicate point in a discussion, the plus and minus appendages will be reattached to "J". In general, the following rule may be observed in regard to the base symbols "J" and "N": whenever "+" and "-" are omitted from "J" and "N", then the associated statement or term in which "J" and "N" appear is valid under both surface and field interpretations.

Theoretical Radiant Intensity

Suppose now that in the operational definition (1) the set of directions D becomes smaller and smaller, such that it always contains the direction ξ and such that the flow of radiant energy is onto S. Then write:

$$\text{"}J(S,\xi)\text{"} \quad \text{for} \quad \lim_{D\to\{\xi\}} J(S,D) \tag{10}$$

The existence of this limit is guaranteed by the D-additive and D-continuity properties of Φ postulated in Sec. 2.3. The radiant intensity $J(S,\xi)$ is called the (theoretical) *radiant intensity* in the direction ξ on S. It is important to note that non-zero values of $J(S,\xi)$ are necessarily associated with surfaces S which have non-zero area A(S). This fact is based on the S-continuity property of Φ recorded in Sec. 2.3. Thus, by S-continuity and S-additivity of Φ we have:

$$\lim_{S\to\{x\}} J(S,\xi) = 0 \quad , \tag{11}$$

for every x in S. However, once again by S-continuity and S-additivity of Φ, we have from (1), (4), (10) and the definition of $N(x,\xi)$:

$$N(x,\xi) = \lim_{S'\to\{x\}} J(S',\xi)/\big(A(S')\xi\cdot\xi'(x)\big) \tag{12}$$

where $\xi'(x)$ is the unit inward (or outward)* normal to S' at x. See Fig. 2.6.

From (10) and the fundamental theorem of calculus we obtain:

$$P(S,D) = \int_D J(S,\xi)\, d\Omega(\xi) \quad . \tag{13}$$

---

*Recall our convention on field intensity and surface intensity stated above.

From (4) and (12) we have for similar reasons:

$$\boxed{J(S,\xi) = \int_S N(x,\xi)\,\xi\cdot\xi'(x)\ dA(x)} \tag{14}$$

where $\xi'(x)$ is the unit inward (or outward) normal to S at x and is in $\Xi(\xi)$.

At this point it would be instructive to view (12) in terms of (5) of Sec. 2.5. Furthermore, one can compare (14) with its 'dual' in (8) of Sec. 2.5. This 'duality' stems from a comparison of what is held constant and what is varied in (8) of Sec. 2.5 and (14) above. In (8) of Sec. 2.5, x in S is held fixed while ξ varies over all directions in $\Xi(\xi)$. In (14), ξ in $\Xi(\xi)$ is held fixed while x varies over all points in S. Furthermore, while the integration in (8) of Sec. 2.5 was limited for physical reasons to a hemisphere $\Xi(\xi)$ at x, the integration in (14) is limited, for similar reasons, to an S over which $\xi'(x)$ also stays within $\Xi(\xi)$. Hence the duality between $H(x,\Xi(\xi))$ and $J(S,\xi)$ is quite deep and complete.

Radiant Intensity and Point Sources

As noted in the introductory statements of this section, radiant intensity first arose as a measure of the directional radiant flux output of spatially very small emitters of flux. We shall now show that this feature of radiant intensity can still be employed within the operational point of view adopted in the present development of geometrical radiometry. The net result of this observation will be the recovery of the original conceptual feature of radiant flux but in a manner which will, it is hoped, now be operationally meaningful.

We begin by defining the notion of a point source of radiant flux. A part Y of an optical medium X is a (radiometric) *point source* with respect to point x in X if the set D(Y,x) of directions subtended at x by the points of Y is such that $\Omega(D(Y,x)) \le 1/30$. The basis for this definition rests in two facts, one empirical, and the other theoretical.

The empirical fact is that radiance meters with solid angle openings such that $\Omega(D) \le 1/30$ have been found to be adequate for the practical purposes of geophysical optics to distinguish the radiance variations occurring in natural optical media. Hence any part Y of an optical medium X which can be encompassed by the field of view of a radiance meter located at point x in X is radiometrically a 'point source' of flux. It might be that Y is a ship or an extensive wheat field, or a large patch of ocean surface, or a great cumulus cloud. As long as these objects (they can be either opaque solids, surfaces, or certain well-defined nearly transparent volumes of water or air) fall within the field of view of a standard radiance meter, they are considered 'point sources' with respect to that meter.

The second fact on which the definition of 'point source' is based is that a part Y of X, such that $\Omega(D(Y,x)) \leq 1/30$, has the property that the irradiance from Y on a surface about point x will vary, to within one percent, inversely as the square of the distance from x to Y whenever Y is some definitely localizable object such as a ship, or patch of sky or ocean surface, etc. In short, according to the preceding definition, Y will be a point source of flux only if the inverse square law and cosine law for irradiance holds with respect to it to within one percent. (See Example 5, Sec. 2.11.) It might be of interest to take note of the logical structure of the preceding statement. In particular, we *do not* assert that "if a part Y of X is such that the inverse square law and cosine law hold with respect to it, then Y is a radiometric point source". By considering a spherical body Y, the reason for this may be seen (cf., Example 4, Sec. 2.11). Finally, we shall henceforth assume that in the determination of the surface radiance of a point source Y, the solid angle opening of the radiance meter can be adjusted so as to fit exactly the set D(Y,x).

Consider now a radiometric point source Y in a medium X. For definiteness, let the point source Y be a spherical region of radius a within a vacuum and which steadily emits radiant flux. Further, Y is such that it can be observed from all directions. Suppose it is required to estimate the radiant flux output of Y but the measurements are constrained for various reasons to take place a distance r not less than a units from the center y of Y. Figure 2.22 (a) depicts the present situation. By adjusting the meter's solid angle

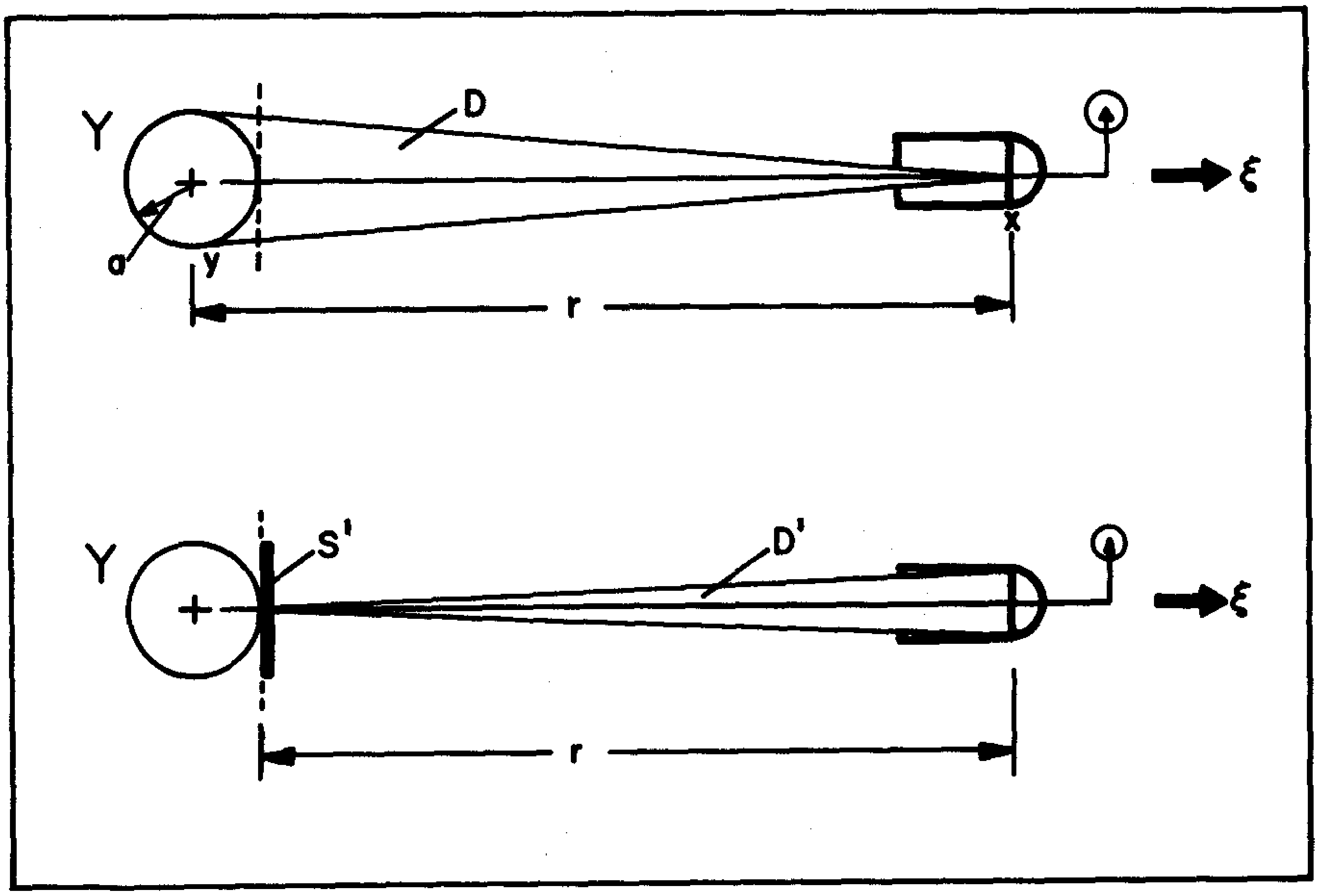

FIG. 2.22 Operational definition of a point source

opening so that the set D of directions from x to Y just fills the field of view, the field radiance N(S,D) associated with Y is read directly from the meter. Here S is the collecting surface of the meter. By the radiance invariance law (1) of Sec. 2.6, it follows that N(S,D) = N(S',D') where S' and D' are as shown in Fig. 2.22 (b), and are completely analogous to the observed surface and direction sets shown in Fig. 2.14. Hence the radiant flux output of Y across the projected surface S' of Y and within the set D' of directions is estimable as:

$$N(S',D')A(S')\Omega(D')$$

after using the measured radiance N(S,D) for N(S',D').

Now we have agreed to write:

$$\text{"}N(S',D')\text{"} \quad \text{for} \quad P(S',D')/A(S')\Omega(D')$$

where P(S',D') is the desired radiant flux output of Y in the direction D'. Since this radiance may be written as:

$$J(S',D')/A(S') \quad ,$$

we can now set:

$$P(S',D') = J(S',D')\Omega(D') \quad .$$

At this juncture the reader should first observe how the number N(S',D') 'belongs' to Y; that is, it is (by the radiance invariance law) independent of the mode of measurement. Secondly, it should be noted that of the two numbers A(S') and Ω(D'), the area A(S') 'belongs' to Y whereas Ω(D') depends on the mode of measurement (i.e., the distance r between x and y). It follows that the product N(S',D')A(S') 'belongs' to Y. But this product is simply J(S',D'), the radiant intensity (watts per steradian) of S' in the directions within D' from x to y. Hence the number J(S',D') is an intrinsic property of Y in the sense that it is independent of the mode of measurement. Finally, by recalling that the dimensions of J(S',D') contain no linear (i.e., length) terms, it becomes manifest that J(S',D') can be conceptually associated with the radiant flux output of the *point* y (the center of Y) in the direction ξ (the central direction of D'). In this way we arrive at the classical conception of radiant intensity as the radiant flux emitted by a point x per unit solid angle about a given direction ξ.

We can now use (13) as a basis for the classical formula relating the radiant intensity and radiant flux output of the point source Y. Since Y is a sphere, the projected area A(S') of Y on a plane normal to a direction ξ is independent of the direction ξ. More generally, in the point source context, we will agree to write:

$$\text{"}J(x,\xi)\text{"} \quad \text{for} \quad J(S',\xi)$$

$$\text{"}P(x,D')\text{"} \quad \text{for} \quad P(S',D')$$

whenever S' is the projection of part (or all of) the boundary of the point source Y on a plane normal to ξ, and x is some point within Y. Then, with this understanding, (13) becomes:

$$P(x,D') = \int_{D'} J(x,\xi)\, d\Omega(\xi)$$

$$= \int_{D'} J(x,\theta,\phi) \sin\theta\, d\theta\, d\phi \qquad (15)$$

where (for terrestrially-based coordinate systems) we have used (9) of Sec. 2.5, and have written "(θ,ϕ)" for ξ. If the radiant intensity output of Y is independent of ξ over D', then we can make the following statement:

If $D' = \Xi$, then,

$$P(x,D') = J(x)\Omega(D') \qquad (16)$$

i.e.,

$$P(x) = 4\pi J(x) \quad , \qquad (17)$$

where we have written:

"P(x)" for $P(x,\Xi)$

and

"J(x)" for $J(x,\xi)$.

Equation (17) is the customary form of the connection between the radiant intensity J(x) of a (directionally) uniformly emitting point source at x and its total power output P(x). By retracing the definitions of "P(x)" and "J(x)" the reader will see that the emitting object referred to is not a geometric point but rather a small finite part Y of an optical medium X, and that x is a point of X in or near Y. In this way we conceptually simplify the description of point sources to the form exhibited in (17) without contradicting the basic tenets of radiometry, in particular the S-continuity of Φ in Sec. 2.3.

Cosine Law for Radiant Intensity

The cosine law for radiant intensity (Lambert's law) can be stated as follows (cf., Fig. 2.6): *If the surface radiance* $N(S',\xi)$ *of point source surface* S' *is independent of direction* ξ *in* $\Xi(\xi')$, *where* ξ' *is the unit outward normal to* S' *at y, then the surface intensity* $J(S',\xi)$ *of* S' *varies as the cosine of the angle between* ξ' *and* ξ, *i.e.:*

$$\boxed{J(S',\xi) = J(S',\xi')\xi'\cdot\xi} \qquad (18)$$

The proof of statement (18) rests on (4). For by hypothesis we now can write:

$$N(S',D) = N(S',D')$$

where D' is a narrow conical set of directions whose central direction ξ' is normal to S', as in Fig. 2.6. Hence (4) becomes:

$$J(S',D) = J(S',D') \cos \vartheta \quad .$$

Letting D and D' become smaller and smaller with limit $\{\xi\}$ and $\{\xi'\}$, respectively, we arrive at (18).

We have deliberately retained the notation of Fig. 2.6, despite the fact that (18) can be written with less primes adorning "S" and "ξ", for the purpose of encouraging a detailed comparison of (16) of Sec. 2.4 and (18) above. Close study will again reveal the interesting duality between intensity J and irradiance H already discerned by a comparison of (8) of Sec. 2.5 and (14) above. By dwelling on this recurrent duality between J and H, one is moved to inquire whether the cosine law for radiant intensity can be generalized to a form which would constitute a dual statement to the generalized cosine law for irradiance in the form of (8) or (16) of Sec. 2.8. It turns out that an exact dual statement to (8) of Sec. 2.8 can indeed be made for radiant intensity. Now, since the basis for the generalized cosine law for irradiance can be viewed as embodied in (8) of Sec. 2.5, we should expect the basis for the generalized cosine law for radiant intensity to rest in (14) above. We now show that this expectation is correct. We begin with deriving a result, of intermediate generality, from (14), a result which provides an interesting insight into the structure of the classical Lambert law.

Let Y be a region of an optical medium X. The region Y may be of arbitrary shape. Suppose further that from vantage point x, Y is a point source and that the observed surface radiance of its boundary surface is independent of the direction of observation of Y. For simplicity, we assume that the paths of sight from x to points of Y lie in a vacuum. The current geometric situation is depicted in Fig. 2.23. Let N(S,D) and N(S',D') be the observed surface radiances seen from two arbitrary vantage points x and x' at both of which Y is a point source. The surfaces S,S' and direction sets D and D' are as shown in the figure. Thus S is the projection of Y on a plane normal to the axis of the radiance meter located at x. Similarly for S'. Then by hypothesis and by the radiance invariance law:

$$N(S,D) = N(S',D') \quad .$$

This radiance equality can be written in terms of radiant intensity:

$$\frac{J(S,D)}{A(S)} = \frac{J(S',D')}{A(S')} \tag{19}$$

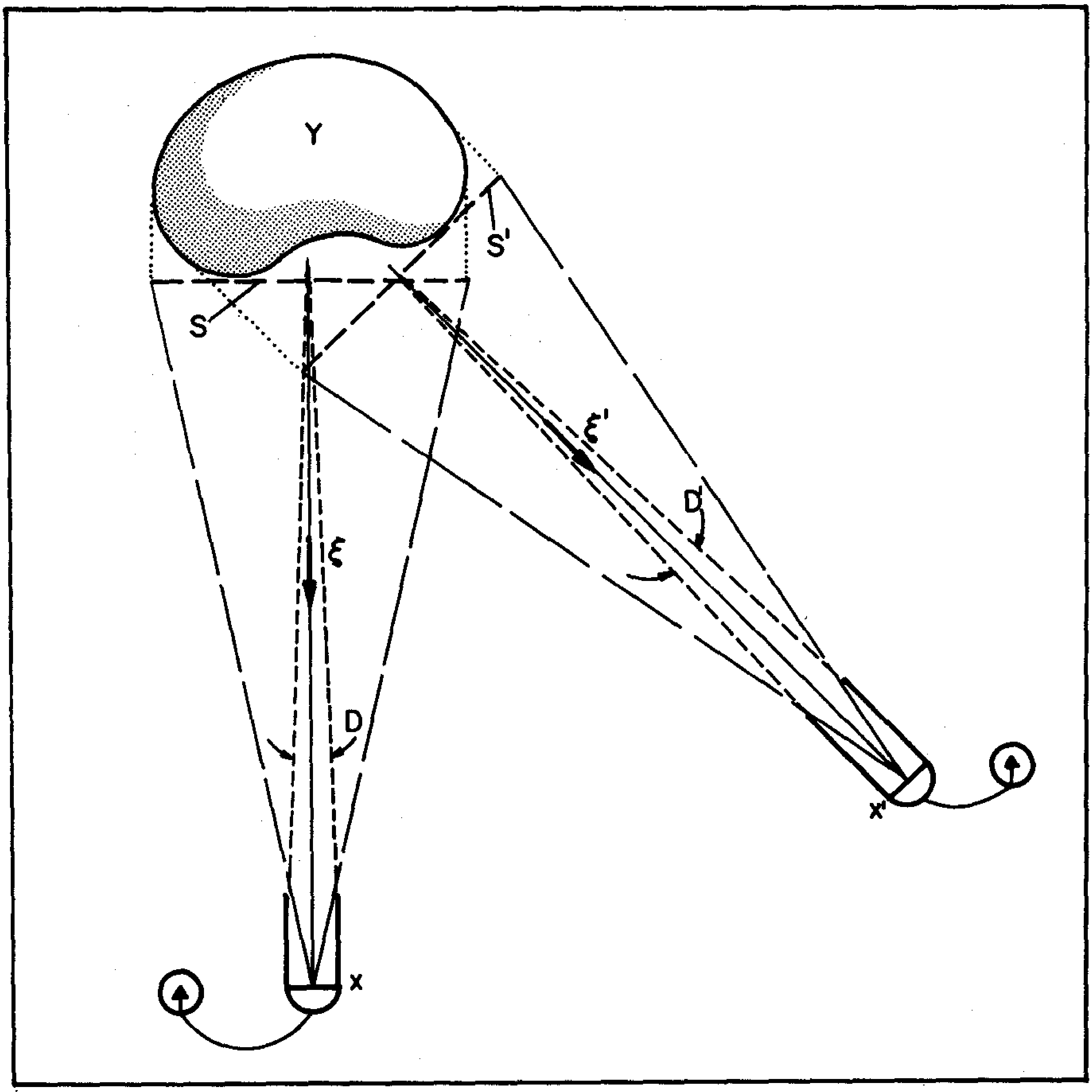

FIG. 2.23 Establishing the Cosine law for radiant intensity in the context of point sources.

Since the x and x' are arbitrary locations subject only to the requirement that Y is a point source with respect to these points, we arrive at the following slight generalization of the cosine law for radiant intensity:

> *If a part* Y *of an optical medium* X *has uniform surface radiance for all directions and all points on the boundary of* Y, *and* Y *is a point source with respect to points* x *in some subset* X_0 *of* X, *and if the paths of sight from points of* X_0 *to* Y *lie in a vacuum, then the quotient* J(S,D)/A(S) *is invariant for every point* x *in* X_0, *where* S *and* D *are defined as in Fig. 2.23.*

It is clear how the classical form of Lambert's law (18) follows from this new statement and its analytic form (19); one now lets Y be a plane surface and lets X_0 be all the appropriate points of X lying to one side of S.

It is of interest to note still one more variant of (19), one which has considerable intuitive value. Let "N" denote the hypothesized fixed radiance associated with Y. Then (19) implies that:

$$\boxed{J(S,D) = NA(S) \quad ,} \tag{20}$$

i.e., that J(S,D) varies directly as the projected area A(S) of Y on a plane perpendicular to the central direction ξ of D. This may be compared with (3). From (20) we can deduce by inspection the *direct square law*--or *area law*--for radiant intensity which states that: *for areas which are radiometric point sources with respect to some observation point, the associated intensity varies directly as the apparent (projected) area of the surface as seen from that point.* If the area is compared with geometrically similar areas, then the associated radiant intensity varies directly as the square of a common transverse dimension of these areas. This observation brings to light still another facet in the duality between irradiance and radiant intensity, the dual law for irradiance in this case being the *inverse square law*.

Generalized Cosine Law for Radiant Intensity

The preceding discussions on point sources and radiant intensity lead us to formulate several useful alternative versions of the classical Lambert law for radiant intensity. During those discussions it was observed how the concepts of irradiance and intensity played the roles of dual concepts in a sense made clear in those discussions. This duality of irradiance and intensity is capable of being expressed in a precise fashion and on a level of generality comparable to that established for the general cosine law for irradiance (8) of Sec. 2.8. We now pause briefly in our developments of geometrical radiometry to establish this interesting generalization of Lambert's law. In doing so we round out and make formal the recurrent theme of duality between surface intensity J and irradiance H encountered throughout this section.

Let Y be a region of an optical medium X such that at each point x of the closed boundary surface S of Y the surface radiance is uniform over the hemisphere $\Xi(\xi'(x))$ where $\xi'(x)$ is the unit outward normal to S at x. Let "N(x)" denote the common value of the uniform radiance distribution at x on X over the set $\Xi(\xi'(x))$. Observe that the variation of the values N(x) over S is left to be quite arbitrary. For the present discussion the only restriction on the radiance function is that it be uniform over $\Xi(\xi'(x))$ at each point x of S. Some approximate physical realizations of such a region Y are: an opaque irregularly shaped body painted with matte paints such that the paints have an arbitrary spatial pattern over S; a luminous, dense region of space such as the sun which, for practical purposes, has a directionally nearly uniform radiance distribution at each boundary point, but which still may be mottled with lighter or darker regions; the moon's surface forms still another example. However, when each of these objects is examined with extreme accuracy of radiometric detail in mind, a wealth of departures from these ideals is encountered.

Now returning to Equation (14) and using the present radiance function in the integral, we consider the particular integral:

$$\int_S N(x)\,\xi\cdot\xi'(x)\;dA(x) \quad . \qquad (21)$$

Our studies of the duality between J and H led us to believe that we may be able to do for J what we did for H when going from (8) of Sec. 2.5 to (2) of Sec. 2.8. Therefore, we are led to take (21) as a base and write:

$$\text{"}\mathbf{J}(S)\text{"} \quad \text{for} \quad \int_S N(x)\,\xi'(x)\;dA(x) \quad . \qquad (22)$$

We call **J**(S) the *vector intensity* for S. The definition of the integral is based on the notion of an ordered triple of integrals, using the form (3) of Sec. 2.8 as a model.

Now **J**(S) is a *bona fide* vector. As such it has three real numbers as x, y, and z components, and so a non zero magnitude $|\mathbf{J}(S)|$ and a direction $\mathbf{J}(S)/|\mathbf{J}(S)|$. This observation will allow us to state succinctly the radiant intensity analog to (8) of Sec. 2.8. However, before doing so, we explore one further facet of **J**(S).

Figure 2.24 depicts a typical region Y with boundary S for which **J**(S) is defined. If a direction ξ is chosen, then the boundary S of Y can be partitioned into two parts $S(\xi)$ and $S(-\xi)$ (or "S_+" and "S_-" for short) with the properties that $S(\xi)$ consists of all points x of S such that $\xi\cdot\xi'(x) > 0$, and $S(-\xi)$ consists of all points x of S such that $\xi\cdot\xi'(x) < 0$. There is generally, for all surfaces of use in practical radiometry, a closed curve C on S such that $\xi\cdot\xi'(x) = 0$ for every x on C. C is the boundary between S_+ and S_-. Observe how $S(\xi)$ and $S(-\xi)$ in the present context have their counterparts in the sets $\Xi(\xi)$, $\Xi(-\xi)$ used in the vector irradiance context.

Returning now to (14) we choose a ξ, determine the associated S_+ and S_- as just described, and then evaluate:

$$J(S_\pm,\xi) = \int_{S(\pm\xi)} N(x)\,\xi'(x)\;dA(x) \quad .$$

Suppose we write:

$$\text{"}\overline{J}(S,\xi)\text{"} \quad \text{for} \quad J(S_+,\xi) - J(S_-,\xi) \quad .$$

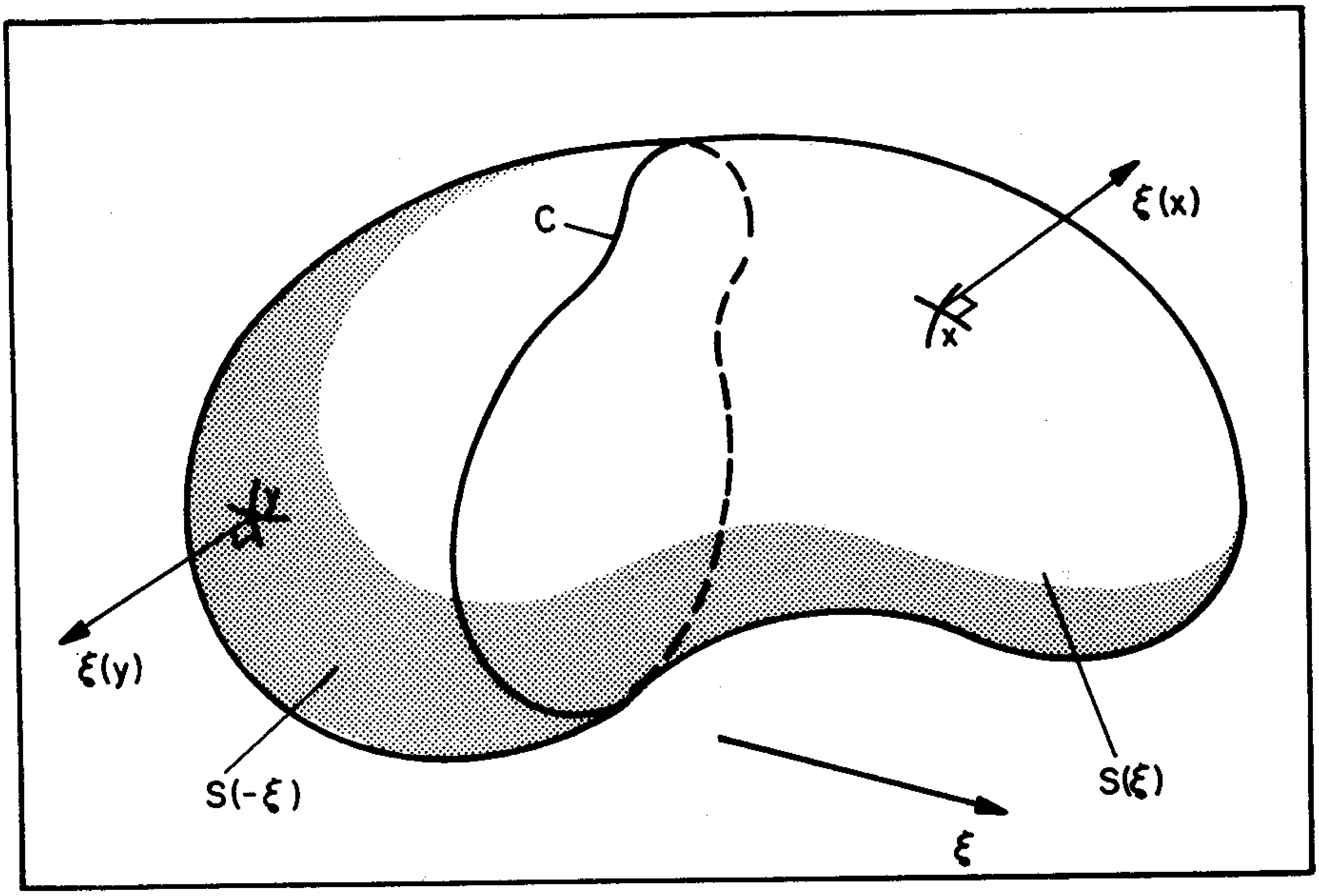

FIG. 2.24 Establishing the general Cosine law for radiant intensity.

Then from the definition of $\mathbf{J}(S)$ and $J(S_\pm,\xi)$ it follows that:

$$\xi\cdot\mathbf{J}(S) = \bar{J}(S,\xi) = J(S_+,\xi)-J(S_-,\xi) \quad . \tag{23}$$

We are now ready to state the generalized cosine law for radiant intensity.

> *Let* N(x) *be a uniform radiance distribution over the hemisphere* $\Xi(\xi'(x))$ *at each point* x *of the boundary* S *of a region* Y *in an optical medium, where* $\xi'(x)$ *is the unit outward normal to* S *at* x*. Then the vector (surface) radiant intensity* $\mathbf{J}(S)$ *as defined in (22) has the property that:*
>
> $$\boxed{\xi\cdot\mathbf{J}(S) = |\mathbf{J}(S)|\cos\vartheta} \tag{24}$$
>
> *where* "$|\mathbf{J}(S)|$" *denotes the magnitude of* $\mathbf{J}(S)$ *and* "ϑ" *denotes the angle between* ξ *and the direction of* $\mathbf{J}(S)$*. Furthermore:*
>
> $$\boxed{|\mathbf{J}(S)| = \max_\xi \bar{J}(S,\xi) \quad .} \tag{25}$$

The parallel of (24) and (25) with the irradiance case in (8) and (9) of Sec. 2.8 is exact. In particular, from (23) we can now write:

$$\boxed{\bar{J}(S,\xi) = \bar{J}(S,\mathbf{m})\mathbf{m}\cdot\xi \quad ,} \tag{26}$$

where **m** is the direction of J(S). This is the radiant intensity counterpart to (16) of Sec. 2.8. The special case (18) of Lambert's law now follows upon applying to (26) the conditions stated for (18). In particular Y now degenerates into a plane, we let $N(x) = 0$ on S_-, and $N(x)$ be constant on S_+. It should be noted in passing that (24) holds for regions including non-point sources. The duality between J and H now becomes clear upon comparison of, say (26) above with (16) of Sec. 2.8: a point x in the irradiance context is replaced by a surface S in the intensity context; the set $\Xi(\xi)$ in the irradiance context is replaced by the point ξ in the intensity context.

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2.10 Polarized Radiance

In this section we shall develop an operational definition of polarized radiance. The development shall take as a point of departure the notion of empirical radiance introduced in Sec. 2.5. The details of the development shall be kept to a minimum, as we will not in this work make extensive use of the concept of polarized radiance. For a somewhat more detailed theoretical discussion of polarized radiance suitable for geophysical applications, the reader is referred to Chapter XII of Ref. [251].

Before going into the technical details of how to measure polarized radiance, a few comments may be made on the reason for wanting to measure polarized radiance in natural optical media. The first and most important reason is that the systematic documentation of the state of polarization of submarine (and atmospheric) light fields increases our store of basic optical knowledge of the world in which we live. For those of a more practical turn of mind, it may suffice to add that knowledge of the kind and amount of polarization extant in a natural light field could yield efficient means of increasing visibility in both the atmosphere and the sea. For, the contrast of objects seen against a sky or underwater background is occasionally increased when viewed through a material which can transmit polarized light in various amounts depending on how the material is held and oriented. If we possess systematic tabulations of polarized light fields and some workable theoretical models of such fields, these empirical observations can be more deeply explored and applied. Finally, there is the question, still not fully resolved--especially for the hydrologic optics branch of geophysical optics--of whether and to what extent polarized light is used by creatures in navigating, in foraging, and in their biological growth cycles.