As a specific example, let $\theta = 30°$, $B = 120$ blondels, and $A = 4\ m^2$. Then:

$$F = (3.14) \times (120) \times 4 \times (1/2)^2$$

$$= 378 \text{ lumens.}$$

Example 5. A red-orange appearing filter is known to have a band pass of 10 mμ, but it is not known precisely what wavelengths of radiant flux it transmits. An experiment is suggested and tried in which it is inferred that an irradiance of 2 watts/m^2 over the transmission interval $\Delta\lambda = 10$ mμ produces an illuminance of 1360 lumens. Can the transmission wavelengths of the filter also be inferred from this information? To answer this, consider the following observations.

From the definitional identity:

$$E = Y(H,\Lambda)$$

$$= K_m \int_0^\infty H(\lambda)\bar{y}(\lambda)\, d\lambda$$

and the fact that $H(\lambda) = 0$ outside the interval $\Delta\lambda$ about the unknown λ, we have very nearly:

$$E = K_m H(\lambda)\bar{y}(\lambda)\Delta\lambda \quad .$$

Hence:

$$\bar{y}(\lambda) = E/\left(K_m H(\lambda)\Delta\lambda\right)$$

$$= 1360/(680 \times 2 \times 10)$$

$$= 1/10 = .10 \quad .$$

From Table 1, by linear interpolation, we infer that $\lambda = 472$ mμ or 652 mμ. From the given general appearance of the filter's color, we infer that $\lambda = 652$ mμ.

2.13 Generalized Photometries

We conclude this chapter with a few observations on the necessary forms of certain generalized photometries which arise in an attempt to extend the salient ideas of classical photometry. The directions of extension to which we subject the ideas of photometry in this discussion are toward a more general class of 'luminosity' functions. The class we envision here is to contain not only the classical luminosity functions of human eyes, as briefly discussed in 2.11, but also irradiation-response functions describing photographic, phototransmissive, photovoltaic, photoemissive, and photocurrent phenomena. In short, we attempt to sketch in broad terms certain possible generalizations of the 'lumen' concept with reference to irradiations which can be measurably effective on both organic and inorganic levels. Our discussion will consider in turn linear and nonlinear generalized photometries.

Linear Photometries

Let us begin with the simpler of the two generalizations: the *linear photometry*. The classical photometry discussed in Sec. 2.12 is an instance of a linear photometry. Using that discussion as a suitable motivation and background, we can initially and broadly define *theoretical linear photometry* to be the study of the properties of the effects $Z(\mathscr{R},M)$, on some physical object, of radiometric causes $\mathscr{R}$ over a wavelength set M, and under the premise that the numbers $Z(\mathscr{R},M)$ have certain postulated general properties. Specifically, for a given physical object (eye, skin, selenium cell, etc.), let $Z(\cdot,\cdot)$ be a function which assigns to each radiometric concept $\mathscr{R}$ and part M of the spectrum Λ a real number $Z(\mathscr{R},M)$ with the following properties*:

(i) $\mathscr{R}$*-Linearity*: For every two radiometric concepts $\mathscr{R}_1$ and $\mathscr{R}_2$ and nonnegative real numbers c_1 and c_2 for which $c_1\mathscr{R}_1 + c_2\mathscr{R}_2$ is defined, and for every part M of the spectrum Λ,

$$Z(c_1\mathscr{R}_1+c_2\mathscr{R}_2,M) = c_1 Z(\mathscr{R}_1,M)+c_2 Z(\mathscr{R}_2,M)$$

(ii) M-*Additivity*: For every radiometric function and every two disjoint parts M_1 and M_2 of Λ,

$$Z(\mathscr{R},M_1 \cup M_2) = Z(\mathscr{R},M_1)+Z(\mathscr{R},M_2)$$

(iii) M-*Continuity*: For every radiometric function

$$\text{if } \quad l(M) = 0, \quad \text{then} \quad Z(\mathscr{R},M) = 0 \ .$$

An example of $Z(\mathscr{R},M)$ would be the amount of reddening (suitably measured) of human skin under irradiation (so that $\mathscr{R}$ can be irradiance H) over a certain portion of the far infrared (so that M consists of all wavelengths from, e.g., $\lambda = 800$ mμ to $\lambda = 850$ mμ). Another example of $Z(\mathscr{R},M)$ would be the rate of oxygen production by a leaf of some type of vegetation under irradiation (so that $\mathscr{R}$ can be scalar irradiance h) and over some part M of the spectrum. Marine biological contexts appear also to present potential areas for generalized photometries.

At any rate, the landmarks of an incipient linear photometry are properties (i), (ii), (iii) above. The concept of a linear photometry is certainly not empty since we have

*The footnote to the discussion of (3) and (4) of Sec. 2.3 applies also to the present discussion and should be consulted before proceeding.

classical photometry; and many additional photometric phenomena appear to be linear over given ranges within Λ. One of the useful facts about a linear photometry is the provable existence of a generalized luminosity function $\overline{z}(\cdot)$ within that photometry with the property that:

$$Z(\mathscr{R},M) = \int_M \mathscr{R}(\lambda)\overline{z}(\lambda)\ d\lambda \quad . \tag{1}$$

We call this the *canonical representation* of $Z(\cdot,\cdot)$ for a linear photometry. The mathematical basis for this fact rests in general measure theory (the Radon-Nikodym theorem), and was alluded to earlier in (31) of Sec. 2.12 in connection with $\overline{y}(\cdot)$.

In sum then, it is possible to carry over to any general linear photometry the useful notion of a general 'luminosity function' which describes a general 'relative luminosity' of $\mathscr{R}$ over M (cf. (6) and (7) of Sec. 2.12). As a result it also appears possible to generate the concept of generalized 'lumens', so that one can initially place on a firm scientific footing such generalized linear photometries.

Nonlinear Photometries

Turning now to consider the prospects of forming a foundation for nonlinear photometries we are faced with the usual arresting fact about nonlinear phenomena: there are so many types of them. Were the world built so that there was only one type of nonlinearity--say of the power-exponential type or the sinusoidal type, etc.--then the problem of representing nonlinear phenomena would long ago have been thoroughly subdued, analytically speaking. However, since man's finite amount of attention must be spread over an apparently infinite class of nonlinear phenomena, this layer of attention must be nearly 'monomolecular' in depth wherever it exists.

To make a small start into the wilderness of nonlinear photometries, let us consider the first and logically the simplest types of departure from linearity. The preceding three statements (i)-(iii), constituting the defining properties of a linear photometry, may not all hold for given photometric phenomena. The three main types of departure from linearity would be:

Type I nonlinearity:	(i) does not hold; (ii) and (iii) hold
Type II nonlinearity:	(ii) does not hold; (i) and (iii) hold
Type III nonlinearity:	(i) and (ii) do not hold: (iii) holds

This choice of classification is based on the plausible feeling that: "if $l(M) = 0$, then $Z(\mathscr{R},M) = 0$" will always hold in any reasonable designed measure $Z(\cdot,\cdot)$ of a radiometric effect. Therefore, if a nonlinearity is encountered, it is likely to be traceable to a violation of either (i) or (ii), or both. Each of the three types of nonlinearity will now be briefly discussed with the purpose in mind of suggesting possible routes toward linearization.

One very promising mode of approach to Type I nonlinearities is to find a function f which would linearize $Z(\cdot,M)$ for every M. Specifically, we suggest finding a real valued function f, defined on the real numbers, such that:

$$\text{(iv)} \quad f\big(Z(c_1\mathscr{R}_1 \cdot {}_{-2}\mathscr{R}_2,M)\big) = c_1 f\big(Z(\mathscr{R}_1,M)\big) + c_2 f\big(Z(\mathscr{R}_2,M)\big) \quad .$$

Many logarithmic and power nonlinearities are linearized away in this manner by the time-tested technique of plotting on logarithmic or exponential, or power coordinates. Whenever a linearizing function f can be found so that (iv) holds, then we say that the Type I nonlinearity is *removable*. The functional composition $f\circ Z$ of the linearizer f and the Z suffering a Type I removable nonlinearity, is now linear. Thus (i)-(iii) hold for $f\circ Z$ and so the canonical form (1) is available for use with $f\circ Z$. Summarizing: *whenever a Type I nonlinearity of a photometric measure* $Z(\cdot,M)$ *is removable by a linearizer* f *such that (iv) holds, then the composition* $f\circ Z(\cdot,M)$ *has a canonical representation (1).*

Let us consider now the Type II nonlinearity. We ask: if (ii) does not hold, *in what way* is it most likely not to hold? Imagine an erythemal phenomenon: a bit of living animal tissue is irradiated simultaneously by two distinct sets of radiation of non-overlapping wavelength sets M_1 and M_2. The effect $Z(\mathscr{R},M_1 \cup M_2)$ is noted. Then a biologically equivalent piece of tissue is irradiated in turn by samples of wavelength sets M_1 and M_2, and $Z(\mathscr{R},M_1)$ and $Z(\mathscr{R},M_2)$ are noted. Since M_1 and M_2 are allowed to be active separately, more effect-activity say, may take place in the tissue under each irradiation by M_i than when they act simultaneously. Thus, it may be that while the effects are not additive, they are M-*subadditive*:

$$\text{(v)} \qquad Z(\mathscr{R},M_1 \cup M_2) \le Z(\mathscr{R},M_1) + Z(\mathscr{R},M_2) \quad .$$

Whenever a Type II linearity is encountered so that (ii) does not hold, it may be the case that M-subadditivity subsists. If subadditivity is indicated in a Type II nonlinearity, then it may be shown (cf. [103]) that for every $\mathscr{R}$ there exists an extended measure $Z^*(\mathscr{R},\cdot)$ which is additive in the sense of (ii). The net result we have reached may be stated as follows: *Every photometric measure* $Z(\mathscr{R},\cdot)$ *which exhibits nonlinearity of Type II and which is subadditive (i.e., (v) holds) may be extended to a linear photometric measure* $Z_*(\mathscr{R},\cdot)$ *for which a canonical representation (1) is possible.*

The immediate attempt at linearization of a Type III nonlinearity is to seek a linearizer f such that (iv) holds. Some Type III nonlinearities will surely succumb to these very general modes of attack. Beyond these few approaches lies an unknown field of potential modes of study of nonlinear photometries.

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2.14 Bibliographic Notes for Chapter 2

This chapter is based in the main on unpublished lecture notes (Refs. [210], [211]) in radiometry and photometry given in 1953 and 1954 at the Visibility Laboratory of the University of California, San Diego. The characterization of the foundations of radiometry in terms of a systematic use of additivity and continuity properties of the radiant flux function Φ, as given in Sec. 2.3, is derived from a similar treatment given in Ref. [251], and which in turn is based on the general measure-theoretic approach to radiometry and radiative transfer theory introduced in [216]. An important paper on photometry is that of Gershun, [98] who introduced and made precise the concept of the light field (our vector illuminance $\mathbf{E}$). Gershun also introduced the operational definition of radiance in the form $N = H/\Omega$ (re: (2) of Sec. 2.5). An important source of photometric wisdom may be found in the writings of Moon. In particular, the radiometric lectures cited above drew inspiration from some of the ideas of Refs. [184] and [185], especially in connection with developing general photometries. An old standard work on photometry and still valuable is Walsh's treatise [311]. The work by Le Grand, Ref. [153], is a relatively modern work on the optical-physiological properties of human vision which may be used to supplement the discussions of Sec. 2.12.