CHAPTER 3

THE INTERACTION PRINCIPLE

Gird up now thy loins like a man;
for I will demand of thee, and answer
thou me.
Where wast thou when I laid the
foundations of the earth? declare, if thou
hast understanding...
Whereupon are the foundations thereof
fastened? or who laid the cornerstone
thereof ...

JOB XXXVIII, 3-6.

3.0 Introduction

Radiative transfer theory is distinguished by the fact that it is one of the branches of theoretical physics that can be made to rest on a single principle from which all the salient structures of the theory may be systematically deduced. In this sense it is a closed subsystem of electromagnetic theory. The principle that permits this mode of construction of radiative transfer theory is called the *interaction principle*. The interaction principle is a distillate of many diverse conceptual constructions concerned with radiative transfer which have arisen during the past seven decades of evolution of the theory. In this chapter we shall state the principle and present various instances of it for a selected range of physical situations customarily encountered in practical applications of radiative transfer theory. It will be demonstrated that these physical situations can all be formulated within the theory in a uniform manner using a method which we call *the method of the interaction principle*. By means of examples we shall verify, on the one hand, that the salient theoretical structures of the theory do indeed fall under the domain of the principle, and, on the other hand, we shall prepare the groundwork for the various applications of the principle in the subsequent chapters of this work.

The principle of interaction in its essential form is a statement of the linearity of the classical radiative transfer processes. Thus radiative transfer theory, a complex webwork of deductions following from the principle, is at its core a linear theory of the interaction of light with matter on a phenomenological level. The linearity of the theory arises from the confluence of two main points of view adopted by its principal developers and investigators since the turn of the century.

The first of these views is that the theory is concerned in the main either with radiant energy phenomena within the relatively hair-thin visible wavelength interval from 4×10^2 mμ to 7×10^2 mμ, or within a wider band of wavelengths from 10 to 10^5 mμ. Radiant energy phenomena within this four-orders of magnitude spread of the electromagnetic spectrum are, as we shall see below, associated with energies which barely tap the electronic energy levels of common atomic structures. The resultant interactions of radiant energy with matter are thereby limited essentially to elastic scatter activity, photoelectric effects, and simple absorption-emission phenomena. Inelastic scatter interactions of photons with matter are virtually ruled out within the $10\text{-}10^5$ mμ range of wavelengths. Within this domain the radiant energy interactions are manifestly linear, and thereby set one part of the stage for the linear structure of the interaction principle.

The second viewpoint adopted by the founders of the theory is that the interaction of light with matter is to be viewed on the phenomenological level, i.e., on the macroscopic level, with instruments which mimic normal human vision in its essential geometric characteristics. Therefore the delicate effects of wave phenomena, such as diffraction, interference, and other coherence activities are automatically excluded, by fiat, from the domain of classical radiative transfer theory. (See problems I-V of Sec. 141, Ref. [251]). In adopting this approach, we have 'shut our eyes completely' and have thought about all that we have seen. The linearities resulting from this predominantly geometrical viewpoint form the basis for the various additive and continuity properties of radiant flux discussed and developed at length in Chapter 2. These two views, one physical, the other geometrical, combine to act as effective linearization forces on the formulations of the concepts designed to describe radiative transfer processes in geophysical optics and great stretches of astrophysical optics.

The Physical Basis of the Linearity of the Interaction Principle

Before going on to state and illustrate the interaction principle, it will be instructive to examine in more detail the preceding physical assertions about the types of radiative processes limited to the purview of radiative transfer theory. In contemplating the consequences of the modern view that radiant energy is carried by quantized electromagnetic fields--i.e., by photons--we encounter a great number of possible types of interactions of photons with matter. Adopting a suggestion by Fano [90], we can usefully classify all of these variations into five main types of photon *interactions:*

I Interactions with atomic electrons.

II Interactions with atomic nucleons (protons, neutrons).

III Interactions with electric fields around charged atomic particles (electrons, charged nucleons).

IV Interactions with meson fields surrounding nucleons.

V Interactions with other photons.

The effects of these interactions are also greatly varied. But again for our present purposes, we need distinguish only three broad types of *effects:*

A. Outright absorption

B. Elastic scatter

C. Inelastic scatter

A word or two on the meaning of these terms is in order. Suppose we picture a photon as a small colored fuzzy ball, and an atom or a molecule of an optical medium as a relatively large complex spherical maze of thin, widely spaced fuzzy wires (electronic orbits or electron bonds) with tiny relatively dense central cores. Then in the case of effect A, the colored ball either zooms into the wire cage and becomes enmeshed in the maze of wires or is captured by a dense core, there to stay for a period of time far greater than that normally required to traverse the diameter of the cage at its initial speed. If it is ultimately released, we say an *emission process* has occured. In this captured state the ball, in effect, has been *absorbed* by the atom, and loses its identity as such, resulting momentarily in a higher orbit of one of the atom's electrons or in a higher stationary energy state of a molecule or in an increase in kinetic heat energy of the atom, or some combination of these. In the case of effect B, the colored ball caroms off (or skims through) the electronic shells of the atom, the net effect being a change of direction of travel of the photon with no change of its color, and we say that the photon is *scattered without change in wavelength.* In the final case, C, the ball becomes very briefly enmeshed in the electronic shell, or glances off the dense core, with greater or lesser wavelength than before, the net effect being a change of color and direction of travel, and we refer to the photon as *scattered with change in wavelength.*

Returning now to the interactions and their effects, we see that there are, in the present view, five possible types of interaction of a photon with matter and three possible types of effect. There are then in all fifteen possible interaction-effect pairs we can form: IA, IB, IC, IIA, IIB, IIC,..., VC. We shall call any of these fifteen interaction-effect pairs a *radiative process.* In Table 1 the fifteen general radiative processes are displayed by their characteristic interaction energies and by name whenever possible. For example, the class of processes we know as Rayleigh scatter is subsumed by the process IB. In this process a photon interacts with an atomic electron with the effect that it is scattered elastically. The inequalities that are indicated in the entries of the Table specify the interaction energies for

TABLE 1

GENERAL RADIATIVE PROCESSES

Photons Interacting with	Outright Absorption A	Elastic Scatter B	Inelastic Scatter C
Atomic Electrons I	Photoelectric Effect ≤ 0.1 Mev	Rayleigh Scatter ≤0.1 Mev	Compton Scatter ≥0.1 Mev
Atomic Nucleons II	Nuclear Photoelectric Effect ≥10 Mev	Nuclear Scatter ≥ 10 Mev	Nuclear Resonance Scatter ≥ 10 Mev
Electric field around Electrons, Nucleons III	Pair Production ≥1 Mev	Delbruck Scatter ≥ 3 Mev	Delbruck Resonance Scatter ≥ 3 Mev
Meson field around Nucleons IV	Meson Production ≥ 150 Mev	≥ 150 Mev	≥ 150 Mev
Other Photons V	Pair Production ≥ 1 Mev	≥ 1 Mev	≥ 1 Mev

which the associated process takes place. For example, "≤0.1 Mev" means that the associated process takes place at 0.1 million electron volts or lower. Further, "≥0.1 Mev" means that the associated process takes place at 0.1 million electron volts or higher. The unnamed processes and some of the other processes (IIIC, IVB, IVC, and the photonic interactions) have not been observed at this time of writing.

It will be instructive to correlate the Mev means of specifying the energy of a photon with its associated wavelength. By doing so, we shall be able to see clearly where the interaction energies common to radiative transfer theory stand in the arena of all this activity. To facilitate comparisons, we convert Mev units to wavelength units. The transition from Mev to wavelength is made by first recalling that the basic quantum of energy E associated with a photon of frequency ν is

$$E = h\nu$$

where the frequency is related to wavelength λ by:

$$\lambda\nu = v$$

and where "v" denotes the speed of light. If we let $v = c = 3 \times 10^8$ m/sec, and recall that $h = 6.625 \times 10^{-27}$ ergsec, then from the preceding relations:

$$\lambda = hc/E \quad \text{meters}$$

or

$$\lambda = \frac{1.24 \times 10^{-3}}{E} \text{ m}\mu \quad ,$$

where E is in Mev units. Thus if E = 1, then the associated energy is one million electron volts. The form in which we require this formula is:

$$\boxed{E = \frac{1.24 \times 10^{-3}}{\lambda} \text{ Mev}}$$

where λ is in mμ, i.e., millimicrons (10^{-9} meter), or as they are also called, *nanometers*. Assuming that our present interests lie mainly with processes in the wavelength range $10 \leq \lambda \leq 10^5$ mμ, we can now estimate the associated energies of interaction. Then by looking over the table of processes we can judge which of the areas of the main interaction arena are of primary interest. Thus we are interested in the energy range:

$$\frac{1.24 \times 10^{-3}}{10^5} \leq E \leq \frac{1.24 \times 10^{-3}}{10} \quad ,$$

i.e.,

$$1.24 \times 10^{-8} \leq E \leq 1.24 \times 10^{-4} \text{ Mev} .$$

In particular, green light (555 mμ) is on the order of 2×10^{-6} Mev.

What a tiny corner of the interaction arena we find ourselves in. A glance at the table shows that our world of radiant phenomena lies well within classes IA and IB. We shall call IA and IB the *classical radiative processes*. The classical radiative processes are, of course, replete with special radiative processes which include the various well-known absorption and scattering processes such as Raman, Rayleigh, Tyndall and resonant scatter; also fluorescence, and phosphorescence.

The simple calculation just performed shows that we need not be overly concerned in this work with such phenomena as Compton scatter--a relativistic phenomenon; pair production --a quantum electrodynamics phenomenon; or scattering of light by light--a quantum relativistic phenomenon. Even if we extend our interests down three orders of magnitude to wavelengths of the order of 10^{-2} mμ, we still remain essentially within parts IA, IB, and IC of the interaction arena. The classical radiative process region is the domain of the classical Maxwell equations. We need not at present use any other models of the light field such as the Schrödinger or Dirac models, or those of general relativity to describe the activity in that part of the interaction arena in which we have found our current interests to lie.

It follows from the preceding analysis that the Maxwell equations in quantized, special relativistic form will suffice for most conceivable applications of radiative transfer theory in geophysical settings. Actually, it has been found that the classical (non quantified, non relativistic, linear) Maxwellian theory of electromagnetic fields may for all usual purposes encountered at present, serve as the nearest point on the mainland of physics to which radiative transfer theory may be adjoined when desired (see Chap. XIV of Ref. [251]). In this way is set the predominantly linear cast of the phenomenological theory, at the base of which may be found the interaction principle.

Plan of the Chapter

The plan of the remaining part of this chapter is as follows: we present in the next section a preliminary example of the interaction principle. This will serve to focus attention on a relatively concrete but yet typical instance of the use of the principle. From the example we shall extract the essence of the principle and state and discuss the result in Sec. 3.2. Beginning with Sec. 3.3, further examples of the interaction principle will be given. The examples of application will proceed in a systematic manner from relatively simple cases to progressively more complex cases until all the main tools of radiative transfer, as needed in the present work, have been formed.

Thus in Sections 3.3 to 3.5 we apply the interaction principle to the development of the reflectance and transmittance operators for plane and curved surfaces, with detailed examples presented to help fix the main ideas of the derivations and applications. In Sections 3.6 and 3.7 the reflectance and transmittance operators for plane-parallel media are developed and applications are given. The next step in the ascending scale of applications is taken in Sections 3.8 and 3.9 in which the interaction operators for general media are defined, functional relations governing the resulting operators are derived, and applications of the operators illustrated. Then the sequence of five sections 3.10-3.14 goes on to apply the preceding theory to the problem of constructing the basic inherent optical properties and radiance functions of radiative transfer theory (volume attenuation function, volume scattering function, path function, path radiance) and in Sec. 3.15 these are all assembled into the fundamental integral equation for radiance. At this point all the main tools of radiative transfer theory will have been constructed by means of the methodical use of the interaction principle. This use of the interaction principle is systematized and summarized in Sections 3.16-3.18 in such a way as to aid the student of radiative transfer theory in attempting further applications and development of the method.

Throughout all the examples of this chapter--regardless of their level of complexity--runs a common thread of method: the *method of the interaction principle*. This method begins to form in Example 1 of Sec. 3.4; crystallizes in Example 2 of that section; and then recurs repeatedly, in the manner just outlined, through all the remaining illustrations of the chapter.

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3.1 A Preliminary Example

We shall develop an example of the interaction principle in this section with the purpose in mind of fixing, on a relatively simple intuitive level, the salient features of the principle preparatory to stating the principle in its full form.

Empirical Reflectances and Transmittances for Surfaces

A prerequisite for the development of the example is the definition of the empirical reflectance of a small plane surface S. Figure 3.1 depicts such a surface S with unit outward normal **k**, which is irradiated at each point by radiant flux* through a narrow solid angle D', the flux passing through a hypothetical collecting surface S' on its way to S. The observed (empirical) field radiance of the incident flux is N(S',D') and the observed (empirical) surface radiance--arising from reflection of N(S',D') by S in a narrow solid angle D--is N(S',D';S,D). We write:

$$\text{"}r(S',D';S,D)\text{"} \quad \text{for} \quad \frac{N(S',D';S,D)}{N(S',D')\Omega(D')} ,$$

and call r(S',D';S,D) the (empirical) *reflectance* of surface S for the incident and reflected directions D' and D, respectively. Here S' is the projection of S on a plane perpendicular to a direction ξ', the central direction of D'. The function which assigns to (S',D') and (S,D) the number r(S',D';S,D) is called the (empirical) *reflectance function* for S. For the purpose of the present example, we assume r(S',D';S,D) is known for all pairs (D',D) of incident and response (reflected)

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*For simplicity in exposition, throughout this work all radiant flux quantities will be assumed unpolarized, unless specifically stated otherwise. For an outline of the task of extending all results below to the polarized context, see Chapter XII of [251]. The interaction principle, however, holds implicitly for the polarized case. For the relative mathematical consistency of the assumption of the unpolarized light field with respect to the complete theory of the polarized field, see Sec. 13.11.