summarized in Sections 3.16-3.18 in such a way as to aid the student of radiative transfer theory in attempting further applications and development of the method.

Throughout all the examples of this chapter--regardless of their level of complexity--runs a common thread of method: the *method of the interaction principle*. This method begins to form in Example 1 of Sec. 3.4; crystallizes in Example 2 of that section; and then recurs repeatedly, in the manner just outlined, through all the remaining illustrations of the chapter.

3.1 A Preliminary Example

We shall develop an example of the interaction principle in this section with the purpose in mind of fixing, on a relatively simple intuitive level, the salient features of the principle preparatory to stating the principle in its full form.

Empirical Reflectances and Transmittances for Surfaces

A prerequisite for the development of the example is the definition of the empirical reflectance of a small plane surface S. Figure 3.1 depicts such a surface S with unit outward normal **k**, which is irradiated at each point by radiant flux* through a narrow solid angle D', the flux passing through a hypothetical collecting surface S' on its way to S. The observed (empirical) field radiance of the incident flux is N(S',D') and the observed (empirical) surface radiance--arising from reflection of N(S',D') by S in a narrow solid angle D--is N(S',D';S,D). We write:

$$\text{"r(S',D';S,D)"} \quad \text{for} \quad \frac{N(S',D';S,D)}{N(S',D')\,\Omega(D')} \quad ,$$

and call r(S',D';S,D) the (empirical) *reflectance* of surface S for the incident and reflected directions D' and D, respectively. Here S' is the projection of S on a plane perpendicular to a direction ξ', the central direction of D'. The function which assigns to (S',D') and (S,D) the number r(S',D';S,D) is called the (empirical) *reflectance function* for S. For the purpose of the present example, we assume r(S',D';S,D) is known for all pairs (D',D) of incident and response (reflected)

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*For simplicity in exposition, throughout this work all radiant flux quantities will be assumed unpolarized, unless specifically stated otherwise. For an outline of the task of extending all results below to the polarized context, see Chapter XII of [251]. The interaction principle, however, holds implicitly for the polarized case. For the relative mathematical consistency of the assumption of the unpolarized light field with respect to the complete theory of the polarized field, see Sec. 13.11.

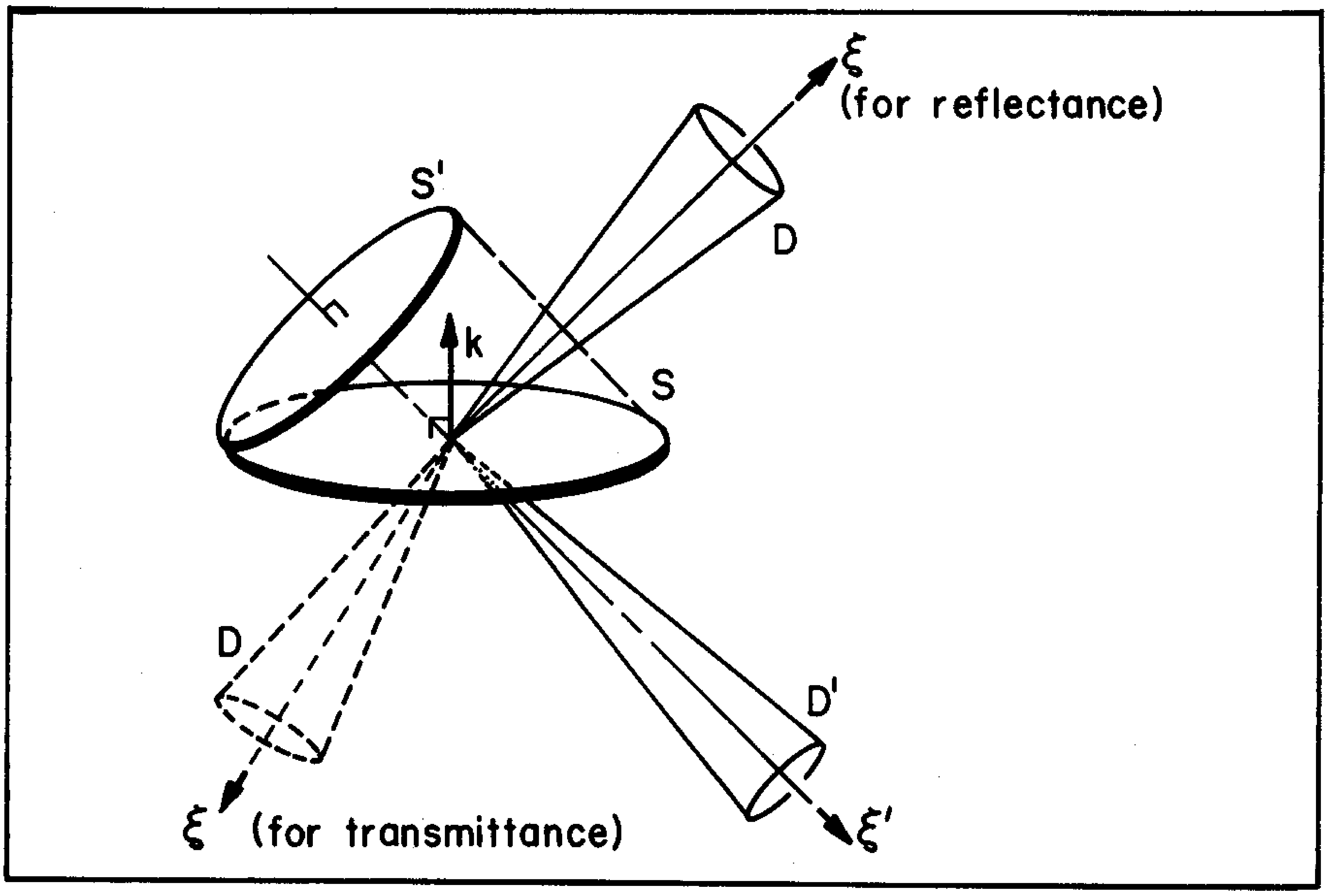

FIG. 3.1 Setting for empirical reflectances and transmittances of surfaces.

direction sets such that $\xi' \cdot \mathbf{k} < 0$ and $\xi \cdot \mathbf{k} > 0$, respectively. Here ξ' and ξ are arbitrary central directions of the sets D' and D, respectively. These two conditions merely require D' and D to lie on opposite sides of S, as in Fig. 3.1.

Now the essential property of the response radiance N(S',D';S,D) is that it is additive with respect to D'. More precisely, experimental evidence indicates that we may assert the following property of N(S',D';D,S). In each case let the sets D,D' of directions be circular, conical sets with central directions ξ,ξ', respectively. Then:

(i) *(D'-Additivity)* If S is a surface in an optical medium X and S is irradiated in turn by radiances $N(S_1',D_1')$ and $N(S_2',D_2')$, with $N(S_1',D_1';S,D)$ and $N(S_2',D_2';S,D)$ as the respective observed response radiances, then $N(S_1',D_1';S,D)+N(S_2',D_2';S,D)$ is the observed radiance of the S under simultaneous irradiation.

Furthermore:

(ii) *(D'-Continuity)* Let the geometric setting be defined as in (i). If $\Omega(D') = 0$ and $\xi' \neq \xi$, then $N(S',D'; S,D) = 0$.

By letting D lie on the same side of S as D', these two empirically-based properties of reflected radiant flux are readily turned into the corresponding laws for transmitted flux (see dotted direction cone in Fig. 3.1). Observe that, by virtue of (i), r(S',D';S,D) is independent of the magnitude of N(S',D'). It should be particularly noted that (i) and (ii) are new laws which are independent of the D-additivity and D-continuity properties of Φ in Sec. 2.3. The present laws are intended to characterize a particular type of interaction of radiant flux with matter, whereas the earlier laws were intended to characterize certain intrinsic radiometric (principally geometric) properties of radiant flux regardless of its interaction with matter. These two properties permit a limiting process to culminate in rigorously defined reflectance and transmittance functions for surfaces. The details of such definitions will be considered in (6)-(9) of Sec. 3.3. For the present we use (i) and (ii) as they stand to help solve the following radiometric interaction problem.

The Problem

Two plane surfaces, S_1 and S_2, in a vacuum are mutual point sources. In addition, they are mutually visible and are irradiated by sources of radiance N_1^0 and N_2^0 over solid angles, D_{01} and D_{02}, respectively, as shown in Fig. 3.2. These incident radiances initiate an interreflection process between S_1 and S_2 with resultant surface radiances $N(S_1,D_{12})$ and $N(S_2,D_{21})$. Here D_{12} is the set of directions from a point in S_1 to every point in S_2. Since S_1 and S_2 are mutual point sources (i.e., each is a point source as seen from the points of the other), D_{12} does not vary appreciably as location is varied over S_1, and so may be assumed constant over S_1.

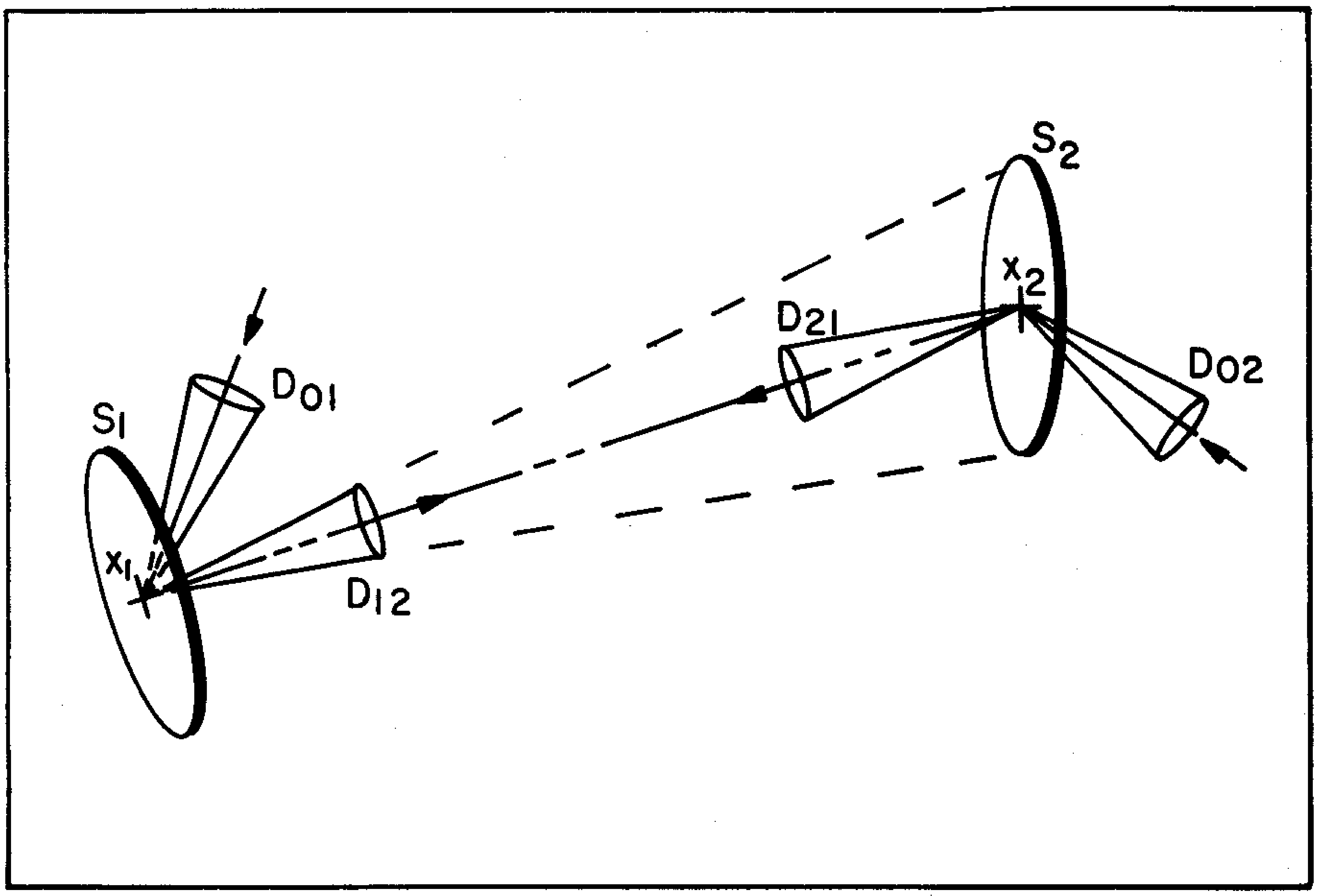

FIG. 3.2 Setting up an interaction calculation for surfaces S_1 and S_2.

Similarly D_{21} is the set of directions from a fixed point of S_2 to every point of S_1; and D_{21} has the same general property as D_{12}. With these preliminaries out of the way, we can now state the present problem: *Given:* S_1 *and* S_2*, as above, with known empirical reflectance functions,* r_1 *and* r_2*, and initial irradiations,* N_1^0, N_2^0. *Required: the steady state empirical radiances* $N(S_1,D_{12})$ *and* $N(S_2,D_{21})$.

The Present Instance of the Interaction Principle

To facilitate the present discussion let us write:

"N_{12}" for $N(S_1,D_{12})$

"N_{21}" for $N(S_2,D_{21})$

"r_{212}" for $r_1(S_{12}',-D_{12};S_1,D_{12})$

"r_{121}" for $r_2(S_{21}',-D_{21};S_2,D_{21})$

"r_{012}" for $r_1(S_{01}',D_{01};S_1,D_{12})$

"r_{021}" for $r_2(S_{02}',D_{02};S_2,D_{21})$

where S_{12}', e.g., is the projection of S_1 on the plane normal to the direction from x_1 to x_2. Similarly, for S_{21}', S_{01}', and S_{02}'. In the case of S_{01}', e.g., imagine an external source point x_o. The set $-D_{12}$ consists of all negatives of directions in D_{12}. Thus if ξ is in D_{12}, then $-D_{12}$ contains $-\xi$. Now by virtue of the definition of empirical reflectance, the D'-additivity property (i) above, and the fact that the intervening space between S_1 and S_2 is a vacuum (so that the radiance invariance law can be used) we have:

$$N_{12} = N_1^o r_{012}\Omega_{01} + N_{21} r_{212}\Omega_{12} \tag{2}$$

$$N_{21} = N_2^o r_{021}\Omega_{02} + N_{12} r_{121}\Omega_{21} \tag{3}$$

where we have written:

"Ω_{oi}" for $\Omega(D_{oi})$, $i = 1,2$

and

"Ω_{ij}" for $\Omega(D_{ij})$, $i,j = 1,2$

For later purposes it is convenient to make one final set of definitions. We write:

"Σ_{ij}^o" for $r_{oij}\Omega_{oi}$ $i,j = 1,2$

"Σ_{iji}" for $r_{iji}\Omega_{ji}$ $i,j = 1,2$.

Then (2) and (3) become:

$$N_{12} = N_1^0\Sigma_{12}^0 + N_{21}\Sigma_{212} \qquad (4)$$

$$N_{21} = N_2^0\Sigma_{21}^0 + N_{12}\Sigma_{121} \qquad (5)$$

Equations (4) and (5) together constitute the algebraic core of the statement of the present form of the interaction principle. In the present case we have two relatively small plane surfaces which are interacting radiometrically. Each surface S_i ($i=1$ or 2) is irradiated by two incident parcels of radiant flux in the form of the empirical radiances, N_i^0 and N_{ji}, and S_i itself has a resultant surface radiance N_{ij}. *To the sets of such incident radiances,* N_i^0 *and* N_{ji} *and response radiances* N_{ij} *associated with* S_i *(*$i=1$ or 2*), there correspond four interaction operators (numbers in this case), namely* Σ_{ij}^0 *and* Σ_{iji}*, such that (4) and (5) hold.* The main role of the interaction principle in the present case would be to assert the existence of these operators and to yield the interaction equations (4), (5).

Solution of the Problem

The interaction principle formulation (4), (5) of the present problem leads to the solution of the problem by means of the theory of simultaneous algebraic equations. Thus, multiplying each side of (4) by Σ_{121}:

$$N_{12}\Sigma_{121} = N_1^0\Sigma_{12}^0\Sigma_{121} + N_{21}\Sigma_{212}\Sigma_{121}$$

and using this representation of $N_{12}\Sigma_{121}$ in (5):

$$N_{21} = N_2^0\Sigma_{21}^0 + \left(N_1^0\Sigma_{12}^0\Sigma_{121} + N_{21}\Sigma_{212}\Sigma_{121}\right)$$

whence:

$$N_{21} = \frac{N_2^0\Sigma_{21}^0 + N_1^0\Sigma_{12}^0\Sigma_{121}}{1 - \Sigma_{212}\Sigma_{121}} \quad .$$

The radiance N_{12} can be found by permuting the symbols "1" and "2" in this equation. The complete solution of the system (4), (5) is then:

$$N_{12} = \frac{N_1^0\Sigma_{12}^0 + N_2^0\Sigma_{21}^0\Sigma_{212}}{1 - \Sigma_{121}\Sigma_{212}} \qquad (6)$$

$$N_{21} = \frac{N_2^0\Sigma_{21}^0 + N_1^0\Sigma_{12}^0\Sigma_{121}}{1 - \Sigma_{121}\Sigma_{212}} \qquad (7)$$

Discussion of Solution

A sufficient condition that N_{12} and N_{21} are determinable via (6) and (7) is that the product $\Sigma_{121}\Sigma_{212}$ is less than 1. We shall now show that a sufficient condition that this latter property holds is that at least one of Σ_{121} and Σ_{212} is strictly less than 1. An examination of these definitions of Σ_{121} and Σ_{212} shows that this condition may be based on a particular form of the principle of conservation of energy. To see this in the case of Σ_{212}, we need only systematically unfold the definitions leading to it. Thus, Σ_{212} is:

$$r_{212}\Omega_{12}$$

The quantity r_{212} is:

$$r_1(S_{12}',-D_{12};S_1,D_{12}) \quad .$$

This in turn is the value of the empirical reflectance function for S_1. By (1), and the fact that this value of r_1 is independent of the magnitude of the irradiating flux, we can select any incident radiance, say N_{12}^- over $-D_{12}$, and let N_{12}^+ be the response (reflected) radiance over D_{12}. Then:

$$r_1(S_{12}',-D_{12};S_1,D_{12})\Omega_{12} = \frac{N_{12}^+}{N_{12}^-} \quad .$$

If $A(S_{12}')$ is the projected area of S_1 on a plane normal to the direction from x_1 to x_2 (see Fig. 3.2), and $P(S_1,-D_{12})$ and $P(S_1, D_{12})$ are the radiant fluxes associated with $N_{12}^{\pm}$, then the incident radiant flux is given by:

$$P(S_1,-D_{12}) = N_{12}^-A(S_{12}')\Omega_{12}$$

and the surface (response) radiant flux is given by:

$$P(S_1,D_{12}) = N_{12}^+A(S_{12}')\Omega_{12} \quad .$$

Here we have used the fact that $\Omega(-D_{12}) = \Omega(D_{12}) = \Omega_{12}$, and also the operational definition of surface radiance (Sec. 2.6). Hence:

$$r_1(S_{12}',-D_{12};S_1,D_{12})\Omega_{12} = \frac{P(S_1,D_{12})}{P(S_1,-D_{12})}$$

At this point we choose the *energy conservation principle* in the form which states that: *if* P^- *is the total radiant flux incident on a given surface* S *and* P^+ *is the total radiant flux leaving the surface* S *and* P^+ *and* P^- *are independent of time, then* $P^+ \leq P^-$. We shall assume this statement is true. From this we deduce in particular that:

$$r_1(S_{12}',-D_{12};S_1,D_{12})\Omega_{12} \leq 1 \quad ,$$

so that:

$$\Sigma_{212} \leq 1 \quad .$$

A similar inequality now follows for Σ_{121}. These inequalities are the most we can say, without further qualifications, about any reflectance (or transmittance) operator occurring in the theory of radiative transfer. *Thus in a particular geometrical situation we must explicitly postulate or demonstrate that at least one of Σ_{121} and Σ_{212} in (6) and (7) is strictly less than 1;* and as our analysis has now made clear, this is a sufficient condition that (6) and (7) uniquely determine N_{12} and N_{21}.

Related Problems and their Solutions

The solutions (6) and (7) of the problem considered above can be used to solve related problems centering on the radiometric interaction of S_1 and S_2. Suppose, for example, we require the surface radiance of S_1 in some set $D_{1\beta}$ of directions other than D_{12}. Here "β" is associated with point $x\beta$ which may be any point in the surrounding medium either in or not in S_1 or S_2. Toward this end we write:

"$N_{1\beta}$" for $N(S_1, D_{1\beta})$

"$r_{21\beta}$" for $r_1(S'_{12}, -D_{12}; S_1, D_{1\beta})$

"$r_{01\beta}$" for $r_1(S'_{01}; D_{01}; S_1, D_{1\beta})$.

Then by the D'-additive property (i) above and the radiance invariance law we have:

$$N_{1\beta} = N_1^O r_{01\beta}\Omega_{01} + N_{21} r_{21\beta}\Omega_{12} \quad .$$

In an exactly similar manner we arrive at the surface radiance of S_2:

$$N_{2\beta} = N_2^O r_{02\beta}\Omega_{01} + N_{12} r_{12\beta}\Omega_{21} \quad .$$

Once again we can contract these solutions into a fixed form which clearly reveals the underlying unity of the interaction concept. Thus by writing:

"$\Sigma^O_{i\beta}$" for $r_{oi\beta}\Omega_{oi}$, $i = 1,2$

and

"$\Sigma_{ij\beta}$" for $r_{ij\beta}\Omega_{ji}$, $i,j = 1,2$

the preceding equations become:

$$N_{1\beta} = N_1^o \Sigma_{1\beta}^o + N_{21}\Sigma_{21\beta} \tag{8}$$

$$N_{2\beta} = N_2^o \Sigma_{2\beta}^o + N_{12}\Sigma_{12\beta} \tag{9}$$

where N_{12} and N_{21} are as given in (6) and (7). For the purposes of later comparison with the general statement of the interaction principle we observe that: *to the incident radiances* N_i^o *and* N_{ji} *on* S_i *(i = 1 or 2) and response radiance* $N_{i\beta}$ *there correspond four interaction operators (numbers in this case), namely* $\Sigma_{i\beta}^o$ *and* $\Sigma_{ij\beta}$ *such that (8) or (9) hold.* The index β in (8) and (9) (and in the equations below) may be replaced by distinct indices, if desired. In other words, surfaces S_1 and S_2 may give off radiances in distinct directions, which may be computed by (8), (9) by replacing index β in (9), say, by a new index γ.

An Alternate Form of the Principle

We now abruptly change our conceptual orientation in Fig. 3.2 from that of two radiometrically interacting surfaces S_1 and S_2 to that of a single subset S of the optical medium irradiated from without by radiant flux. This change in orientation can be encouraged by imagining S_1 and S_2 in Fig. 3.2 to be encircled by a closed dashed curve and to think of the curve as holding a *single subset* S of space (that is, S is a disconnected subset which happens to consist of two separate surfaces, S_1 and S_2). This subset S is irradiated at two places by incident radiances N_1^o and N_2^o, and the response of S is imagined in the form of two streams of flux characterized by $N_{1\beta}$ and $N_{2\beta}$. This conceptual compression of S_1 and S_2 into a single radiometrically responsive entity can be expressed symbolically as follows. We first write the system (8) and (9) in matrix form (replacing "β" in (9) by "γ", for generality):

$$(N_{1\beta}, N_{2\gamma}) = (N_1^o, N_2^o)\begin{pmatrix} \Sigma_{1\beta} & 0 \\ 0 & \Sigma_{2\gamma} \end{pmatrix} + (N_{21}, N_{12})\begin{pmatrix} \Sigma_{21\beta} & 0 \\ 0 & \Sigma_{12\gamma} \end{pmatrix}.$$

Further, from (6) and (7) we can write:

$$(N_{21}, N_{12}) = (N_1^o, N_2^o)\begin{pmatrix} \Psi_{11}^o & \Psi_{12}^o \\ \Psi_{21}^o & \Psi_{22}^o \end{pmatrix}$$

where, in turn, we have written:

$$"\Psi^o_{12}" \quad \text{for} \quad \frac{\Sigma^o_{12}}{(1-\Sigma_{121}\Sigma_{212})}$$

$$"\Psi^o_{21}" \quad \text{for} \quad \frac{\Sigma^o_{21}}{(1-\Sigma_{121}\Sigma_{212})}$$

$$"\Psi^o_{11}" \quad \text{for} \quad \frac{\Sigma^o_{12}\Sigma_{121}}{(1-\Sigma_{121}\Sigma_{212})}$$

$$"\Psi^o_{22}" \quad \text{for} \quad \frac{\Sigma^o_{21}\Sigma_{212}}{(1-\Sigma_{121}\Sigma_{212})} \quad .$$

Then going one step further and writing:

$$"\Psi^o" \quad \text{for} \quad \begin{pmatrix} \Psi^o_{11} & \Psi^o_{12} \\ \Psi^o_{21} & \Psi^o_{22} \end{pmatrix}$$

and:

$$"\Psi^o_{\beta\gamma}" \quad \text{for} \quad \begin{pmatrix} \Sigma^o_{1\beta} & 0 \\ 0 & \Sigma^o_{2\gamma} \end{pmatrix}$$

and:

$$"\Psi_{\beta\gamma}" \quad \text{for} \quad \begin{pmatrix} \Sigma_{21\beta} & 0 \\ 0 & \Sigma_{12\gamma} \end{pmatrix} \quad ,$$

we arrive at the following alternate representation of the system (8) and (9):

$$(N_{1\beta},N_{2\gamma}) = (N^o_1,N^o_2)(\Psi^o_{\beta\gamma} + \Psi^o\Psi_{\beta\gamma}) \quad .$$

Let us write:

$$"\Phi^o_{\beta\gamma}" \quad \text{for} \quad \Psi^o_{\beta\gamma} + \Psi^o\Psi_{\beta\gamma}$$

and thereby arrive at the desired form of the system (8), (9):

$$\boxed{(N_{1\beta},N_{2\gamma}) = (N^o_1,N^o_2)\Phi^o_{\beta\gamma}} \quad . \qquad (10)$$

The significance of (10) may be discerned as follows: for the given subset S we have shown that *to an arbitrary pair of incident radiances* (N^o_1,N^o_2) *and response radiances* $(N_{1\beta},N_{2\gamma})$ *there corresponds a unique interaction operator (a 2×2 matrix* *of real numbers in this case), namely* $\Phi^{0}_{\beta\gamma}$*, such that (10) holds.*

Equation (10) constitutes an alternate form of the interaction principle to that displayed in (4), (5) or in (8), (9). This alternate form is designed to give an indication of the potential internal complexity of an object S to which the interaction principle may assign an interaction operator. It is not too great an extension of ideas from the setting of (10) to the setting of an arbitrary finite number of interacting surfaces. However, the systematic study of such systems of interacting surfaces or solids is the domain of discrete space radiative transfer theory and lies far beyond our present concerns. For those interested in pursuing this matter further, we observe that the complete theory of such systems is developed in Ref. [251].

The Natural Mode of Solution

We conclude this preliminary example of the interaction principle by displaying an alternate mode of solution of the problem of the radiometric interaction of the two surfaces S_1 and S_2 considered above. Our purpose is to show that this alternate mode of solution and the interaction principle mode of solution are equivalent. As our developments proceed into the next chapter, we shall also see that each mode of solution possesses a valuable conceptual kernel which is capable of extension to quite wide domains of application in radiative transfer theory in general, and hydrologic optics in particular. This alternate mode of solution we call the *natural mode* of solution, for it appears to be conceptually the simplest and most natural approach to interreflection problems.

The natural mode of solution may be described quite briefly as follows. We imagine a hyper-fast camera filming the radiometric interaction of two surfaces, S_1 and S_2. The filmed episode begins the instant the incident radiances $N_1{}^{o}$ and $N_2{}^{o}$ simultaneously impinge on S_1 and S_2, respectively. In a playback of the filmed episode in slow motion, we see part of $N_1{}^{o}$ reflected from S_1 and start to travel toward S_2. This reflected flux eventually reaches S_2 and part of it is redirected back toward S_1. In the meantime $N_2{}^{o}$ has been reflected at S_2 and part of the reflected flux moves on to S_1, there to be reflected and to have some flux begin to return to S_2. As the film continues, the sources $N_1{}^{o}$ and $N_2{}^{o}$ continue to steadily pour flux on S_1 and S_2. After a while S_1 is being irradiated by photons, some of which come directly from $N_1{}^{o}$, some of which are making their first arrival from S_2, and some their second arrival from S_2, etc. By and by the fluxing and interfluxing reaches a measurable steady state (while, in principle, however, there will always be some interreflection number which has not yet been attained). The following argument develops the symbolic representation of this steady state interreflection process.

Retaining the notation of the preceding discussions, let us go on to write:

$$\text{"}N^1_{12}\text{"} \quad \text{for} \quad N^0_1\Sigma^0_{12}$$

and

$$\text{"}N^1_{21}\text{"} \quad \text{for} \quad N^0_2\Sigma^0_{21} \quad .$$

Further, for every $j = 2,3,\ldots,$ we write:

$$\text{"}N^j_{12}\text{"} \quad \text{for} \quad N^{j-1}_{21}\Sigma_{212}$$

$$\text{"}N^j_{21}\text{"} \quad \text{for} \quad N^{j-1}_{12}\Sigma_{121} \quad .$$

By recalling the moving-picture allusion it is easy to see that N_{12}^j is interpretable as the surface radiance of S_1 in the directions of S_2 consisting of radiant flux having undergone precisely j reflections. Again, by means of the analogy, we are led to write:

$$\text{"}N_{12}\text{"} \quad \text{for} \quad \sum_{j=1}^{\infty} N^j_{12} \tag{11}$$

$$\text{"}N_{21}\text{"} \quad \text{for} \quad \sum_{j=1}^{\infty} N^j_{21} \tag{12}$$

The numbers N_{12} and N_{21} obtained in this way are called the *natural solution* of the present problem of the radiometrically interacting surfaces S_1 and S_2. That N_{12} and N_{21} are indeed solutions of the steady-state interreflection problem associated with S_1 and S_2 will now be shown. By starting with the definitional identity arising from (11):

$$N_{12} = \sum_{j=1}^{\infty} N^j_{12} \quad ,$$

we deduce the following chain of equalities:

$$
\begin{aligned}
N_{12} &= \sum_{j=1}^{\infty} N_{12}^{j} \\
&= N_{12}^{1} + \sum_{j=2}^{\infty} N_{12}^{j} \\
&= N_{12}^{1} + \sum_{j=2}^{\infty} \left(N_{21}^{j-1} \Sigma_{212}\right) \\
&= N_{12}^{1} + \left(\sum_{j=1}^{\infty} N_{21}^{j}\right) \Sigma_{212} \\
&= N_1^{o}\Sigma_{12}^{o} + N_{21}\Sigma_{212} \qquad (13)
\end{aligned}
$$

The last equality follows from (12) and the preceding definition of N_{12}^{1}. By comparing (13) and (4) we see that the natural mode of solution implies the interaction mode of solution. Evidently the steps in (13) are reversible, so that the interaction mode of solution implies the natural mode of solution. Thus the two modes of solution are equivalent in this case. Since the interaction mode of solution clearly represents the solution of the interreflection problem of S_1 and S_2, the natural mode of solution therefore is also, by virtue of the preceding equivalence, a solution of the interreflection problem. This equivalence actually holds in very general settings and has been established in detail for these settings, in Ref. [251]. We shall have occasion to study and use once again this equivalence of the two techniques later in the present work. Finally, we observe that the sums in (11) and (12), being reducible to a simple geometric series with ratio $\Sigma_{121}\Sigma_{212}$ and initial term of the form $(N_i^o\Sigma_{ij}^o + N_j^o\Sigma_{ji}^o\Sigma_{jij})$ (i=1, j=2 for (11); i=2, j=1 for (12)), are readily evaluated; these sums are given by (6) and (7).

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3.2 The Interaction Principle

With the preliminary example complete, we turn now to the statement of the central principle of radiative transfer theory:

The Interaction Principle: For every X, S, A, B, m *and* n, *if* X *is an optical medium and* S *is a subset of* X, *and* A *(= (*$A_1, \ldots, A_m$*)) is a class of sets* A_i *consisting of incident radiometric functions on* S, *and* B *(= (*$B_1, \ldots, B_n$*)) is a class of sets* B_j *consisting of response radiometric functions on* S, *and* m *and* n *are positive integers, then there exists a unique set* $\{s_{ij}: i=1,\ldots,m,\ j=1,\ldots,n\}$ *of linear (interaction)*