constituting $N_0(x,\xi)$ which have not been absorbed or scattered from $\mathcal{P}_r(x,\xi)$ as they travelled from x to $x+r\xi = z$. This is the *phenomenological interpretation* of $N_r^0(x,\xi)$, and we shall adopt it in the present work.

The generalization of the foregoing results to the case of non-constant index of refraction is effected by repeating all steps with $(N_-(a)/n^2)$ instead of $N_-(a)$, and building a tube of natural paths around $\mathcal{P}_r(x,\xi)$ using cross section a as a base from which the paths begin with normal incidence. The motivation for using the quotient $N_-(a)/n^2$ rests in the n^2-law for radiance in (4) of Sec. 2.6.

This discussion is concluded with the observation that the beam transmittance $T_r(x,\xi)$ associated with the path $\mathcal{P}_r(x,\xi)$ in X is independent of the radiance distribution in X. This may be seen by returning to (3) and recalling that the standard transmittance operator R(a,b) is an integral operator whose kernel function S_b is derived from an interaction operator obtained via the interaction principle. A re-examination of the conclusion of the interaction principle will show that an interaction operator is independent of the members of its domain sets A_i and range sets B_j, in other words interaction operators do not depend on the radiance distributions (i.e., the light fields) in X.

Inherent and Apparent Optical Properties

The foregoing observation is of considerable importance in establishing the basic optical properties of a natural optical medium X. *An optical property* P *of an optical medium* X *(*P *in the form of a number, function, or operator) which is independent of the light fields (in the form of radiance distributions) in* X *will be called an inherent optical property of* X*; otherwise,* P *is an apparent optical property.* Hence, the beam transmittance function T_r which assigns to the path $\mathcal{P}_r(x,\xi)$ in X the beam transmittance $T_r(x,\xi)$, being independent of the radiance distribution in X, is an inherent optical property of X. We shall return to the systematic study of inherent and apparent optical properties in Chapter 9.

3.11 Derivation of the Volume Attenuation Function

The volume attenuation function is a measure of how much radiance a light beam loses per unit length of travel under the joint action of scattering and absorption processes. In this section we shall develop the concept of the volume attenuation function with the beam transmittance function as a starting point.

Let $\mathcal{P}_r(x,\xi)$ be a natural path in an optical medium X with associated beam transmittance $T_r(x,\xi)$. If a parcel of radiant flux of unit initial radiance traverses $\mathcal{P}_r(x,\xi)$, then on the one hand $T_r(x,\xi)$ is the amount of radiance transmitted over $\mathcal{P}_r(x,\xi)$, and on the other hand:

$$1 - T_r(x,\xi)$$

is the amount of radiance lost over $\mathscr{P}_r(x,\xi)$. Hence if we write:

$$"\alpha_r(x,\xi)" \quad \text{for} \quad \frac{1 - T_r(x,\xi)}{r}$$

then we can say that $\alpha_r(x,\xi)$ is the average amount of radiance lost per unit length for a beam of initial unit radiance traversing $\mathscr{P}_r(x,\xi)$.

We are almost at our goal. It remains to write:

$$\boxed{"\alpha(x,\xi)" \quad \text{for} \quad \lim_{r\to 0} \alpha_r(x,\xi)} \quad . \qquad (1)$$

The function α which assigns to each x in X and ξ in Ξ the non negative value $\alpha(x,\xi)$ given in (1) is called the *volume attenuation function.* The dimensions of $\alpha(x,\xi)$ are L_r^{-1} (inverse radial length--see note (h) to Table III of Sec. 2.12). That $\alpha(x,\xi)$ is a non negative number follows from the contraction property of $T_r(x,\xi)$, ((8) of Sec. 3.10).

Once again we have assumed the existence of a limit without appropriate mathematical preamble. As in the case of (2), (3) and (5) of Sec. 3.10, we are concerned here only with the formal conceptual content of the interaction principle. There should not be any concern at present about the existence of the limit (1) above and the limits in (2), (3), and (5) of Sec. 3.10. These limits can always be made to exist in an acceptable and workable setting by postulating physically reasonable regularity properties of the underlying radiative process. However, what is of greatest importance here is the fact that there now exists a formal deductive chain of arguments connecting the volume attenuation function with the interaction principle. In this way we have shown that the volume attenuation function is an inherent optical property of an optical medium and a property whose conceptual roots are logically linked to the same principle which yields the reflectance and transmittance operators for surfaces and general subsets in the medium.

A useful connection between α and T_r is the exponential representation of T_r using α. This connection is derived by using the multiplicative property of T_r ((6) of Sec. 3.10) to write:

$$\frac{T_{r+s} - T_r}{s} = \frac{T_s - 1}{s} \cdot T_r = (-\alpha_s)T_r \quad .$$

The definition of α_s was used to obtain the second equality. Letting s→0, we obtain:

$$\boxed{\frac{dT_r}{dr} = -\alpha T_r} \qquad (2)$$

For given x and ξ this is an elementary differential equation for T_r, with known function α, whose solution is:

$$T_r = \exp\left\{-\int_0^r \alpha \, dr'\right\} \quad ,$$

or in more explicit notation:

$$\boxed{T_r(x,\xi) = \exp\left\{-\int_0^r \alpha\big(x(r'),\xi(r')\big)\, dr'\right\}} \quad . \tag{3}$$

We have used the identity property for T_r ((7) of Sec. 3.10) to find the integration constant for the particular solution (3) of equation (2). Here $x(r')$ and $\xi(r')$ are the location and direction of a variable point within $\mathscr{P}_r(x,\xi)$, at distance r' from x along the path. If the index of refraction were constant, then $x(r') = x + r'\xi$; $\xi(r') = \xi$ for every r', $0 \leq r' \leq r$; and (3) would become:

$$T_r(x,\xi) = \exp\left\{-\int_0^r \alpha(x + r'\xi,\xi)\, dr'\right\} \quad , \tag{4}$$

and (2) would take the form:

$$\frac{dT_r(x,\xi)}{dr} = -\alpha(x + r\xi,\xi)T_r(x,\xi) \quad . \tag{5}$$

For a discussion of (5) in the case of variable index of refraction, see Sec. 17 of Ref. [251]. In that section there is also an alternative derivation of the function α using empirical radiances and empirical attenuating volumes. An experimental procedure for determining α is given in Sec. 13.4 of this work.

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3.12 Derivation of Path Radiance and Path Function

We continue the sequence of derivations, begun in Sec. 3.10, leading to the derivation of the integral equation of transfer for radiance along a path $\mathscr{P}_r(x,\xi)$ in an optical medium X. In this section we give a derivation of two important components of this equation: the path radiance, and the path function associated with $\mathscr{P}_r(x,\xi)$, and conclude with a derivation of an important connection between them.

The Path Radiance

Let $\mathscr{P}_r(x,\xi)$ be a path in an optical medium X such as that depicted in Fig. 3.29. Once again, for simplicity, we assume constant index of refraction over X and no internal