which is the desired integral representation of the path radiance $N_r^*(z,\xi)$ for the path $\mathscr{P}_r(x,\xi)$ in terms of its path function N_* and beam transmittance function T_r. In the integral we have written:

$$\text{"}z\text{" for } x + r\xi$$

$$\text{"}x'\text{" for } x + r'\xi \quad ,$$

and have derived the formula under the conditions of constant index of refraction over $\mathscr{P}_r(x,\xi)$, and no internal sources over $\mathscr{P}_r(x,\xi)$. This completes the derivation.

The form of (15) is unchanged when the index of refraction is non constant, providing the appropriate form of the beam transmittance for such a case is used (see, e.g., Sec. 16 of Ref. [251]). The case of internal sources is covered by introducing the emission function N_η. (See, e.g., Sec. 19 of Ref. [251] and (1) of Sec. 5.8 below.)

It might be well to explicitly summarize what has transpired between (5) and (15). We started with (5) which is the precise formula for the observable path radiance $N_-(b)$ over path $\mathscr{P}_r(x,\xi)$ in an optical medium X. Here we have used one very convenient feature of the interaction principle, namely that it provides the exact mathematical rendition of all possible radiometric operations with radiance meters and other light measuring devices in natural optical media. Equation (5) summarizes how we can view the $N_-(b)$ as the response to incident flux $N_-(c)$, over the boundary c, of a subset C of X. This formula is then rearranged, using some of the laws of radiative transfer and the properties of the interaction operators we have deduced so far, into the classical representation (15) of the path function integral for path radiance. It should be emphasized that no intuitive observations were used as integral parts of the derivation between (5) and (15); however, they were occasionally invoked merely to make intuitively clear or to motivate the various steps in the derivation. The interaction principle has been stocked at the outset with all the intuition of the phenomenological point of view needed to establish radiative transfer theory. The principle thus provides the formal machinery for recovering all the known intuitions, and perhaps some new ones yet to be generated.

3.13 Derivation of Apparent-Radiance Equation

We turn next to the task of deducing from the interaction principle the well-known intuitive decomposition of the radiance of a distant object into two parts: the residual radiance transmitted from the object over the path of sight to the observer, and the path radiance generated over the extent of the path of sight.

The setting of Fig. 3.29 will serve for the first stage of the present discussion. We imagine the observer at point z of the path of sight $\mathscr{P}_r(x,\xi)$ in an optical medium X and that he is recording the radiance $N(z,\xi)$. For example the path of sight might begin with point x in a mountainside, or on a lake bottom, or then again it could simply be a path segment beginning and ending in midair or midwater. Now for the purposes of the present derivation, we again imagine the cylindrical subset C of X about $\mathcal{P}_r(x,\xi)$ as axis. The set C is considered once again as a one-parameter optical medium with the usual direction conventions. The actual light field in X around $\mathcal{P}_r(x,\xi)$ is considered to be incident on the surface Y of C. As usual we imagine C isolated from X without disturbing the structure of the light field on and in C. This incident light field on Y can conveniently be thought of as incident on the bases a and b and the flank c of C. We are currently interested in the corresponding response radiance $N_-(b)$ emerging from the parameter surface with index b, as a result of the incident radiances over a, b, and c. In particular, we shall eventually concentrate on the value of $N_-(b)$ at point z in the direction ξ. For the present, the interaction principle yields three operators and the statement:

$$N_-(b) = N_-(a)\mathscr{I}(C;a,b) + N_+(b)\mathscr{I}(C;b,b) + N_-(c)\mathscr{I}(C;c,b). \quad (1)$$

Our goal is to obtain a limiting form of this statement as $C\to\mathcal{P}_r(x,\xi)$. The result will be the desired equation for apparent radiance. Toward this end, let us examine in turn each of the three terms in (1).

The limit of $N_-(a)\mathscr{I}(C;a,b)$ evaluated at z and ξ, as $C\to\mathcal{P}_r(x,\xi)$, is none other than the transmitted radiance $N_r^0(z,\xi)$ over the paths $\mathcal{P}_r(x,\xi)$. For we have:

$$\mathscr{I}(C;a,b) = T(a,b) \quad (2)$$

by definition (6) of 3.8 and the definition of T(a,b) given in Sec. 3.9. The observation now follows from (2) of Sec. 3.10.

The limit of $N_+(b)\mathscr{I}(C;b,b)$ evaluated at z and ξ, as $C\to\mathcal{P}_r(x,\xi)$, is zero. This may be seen by observing first of all that:

$$\mathscr{I}(C;b,b) = R(b,a) \quad (3)$$

by definition (6) of 3.8 and the definition of R(b,a) given in Example 2 of Sec. 3.9. The observation now follows from (5b) of Sec. 3.10.

The limit of $N_-(c)\mathscr{I}(C;c,b)$ evaluated at z and ξ as $C\to\mathcal{P}_r(x,\xi)$ is the path radiance $N_r(z,\xi)$, according to (1) of Sec. 3.12. Let us write,

$$\text{"}N_r(z,\xi)\text{"} \quad \text{for} \quad \lim_{C\to\mathcal{P}_r(x,\xi)} N_-(b)(z,\xi) \quad (4)$$

where $N_-(b)$ is as given in (1) and where "$N_-(b)(z,\xi)$" denotes as usual the value of $N_-(b)$ at z, and ξ. We call $N_r(z,\xi)$ the *apparent radiance* associated with $\mathcal{P}_r(x,\xi)$. Applying the limit operation $C\to\mathcal{P}_r(x,\xi)$ to each side of (1) and using the preceding observations we have:

$$N_r(z,\xi) = N_r^o(z,\xi) + N_r^*(z,\xi) \tag{5}$$

or in compact functional form:

$$N_r = N_r^o + N_r^* \quad .$$

Equation (5) is the desired equation for the apparent radiance associated with $\mathscr{P}_r(x,\xi)$.

Equation (5) is the exact statement of the primitive intuition we have about the apparent radiance of distant objects. Let us call the radiance $N_o(x,\xi)$ at the initial point of the path $\mathscr{P}_r(x,\xi)$ the *inherent radiance* of the field at x in the direction ξ. The point x may be on a tangible surface, or it may hang in empty space, mid air, or water. Furthermore, for a given fixed z and ξ the path $\mathscr{P}_r(x,\xi)$ may vary its length r without changing $N_o(z,\xi)$, but the associated $T_r(x,\xi)$ and $N_r^*(x,\xi)$ will vary accordingly. The apparent radiance of the field at x in the direction ξ as seen at z in the direction ξ is customarily thought of as consisting of two parts: the transmitted (or reduced or residual) inherent radiance $N_r^o(z,\xi) = N_o(x,\xi)T_r(x,\xi)$, and the path radiance (or "space light" or diffuse radiance) $N_r^*(z,\xi)$ scattered into the path between x and z. The preceding derivation has established a precise rendition of this intuitive judgment by means of a formal deduction from the interaction principle.

Equation (5) may be written in more detail using the results (4) of Sec. 3.10 and (15) of Sec. 3.12:

$$N_r(z,\xi) = N_o(x,\xi)T_r(x,\xi) + \int_0^r N_*(x',\xi)T_{r-r'}(x',\xi)\, dr' \tag{6}$$

To show the logical ancestry of the concepts in (6) in a striking way, the reader is invited to replace each of the five main terms in (6) by its definition as developed in this and in the preceding three sections. When these notational disguises are removed what should be left is an expression of the incident radiance distributions on a one-parameter cylindrical optical medium C being acted on by interaction operators and the result being embedded in a formidable battery of neatly interlocking limit processes. The interaction operators used for C are supplied by the interaction principle.

<< Table of Contents << Title Page >> Next Page >>