(ii) Path Function Operator for Polarized Radiance (and hence the genesis of the volume scattering matrix --see Sec. 112 of Ref. [251]).

(iii) The Path Radiance Operator for Polarized Radiance (and hence the genesis of the beam transmittance matrix--see Sec. 112 of Ref. [251]).

(iv) Time Dependent Operators--the time dependent versions of all the kinds of operators considered so far. (See (4) of Sec. 3.15 and Sec. 127 of Ref. [251]).

(v) The Photometric Operators $Y(\mathcal{Q},M)$, $Z(\mathcal{Q},M)$. (See (13) of Sec. 2.12 and (1) of Sec. 2.13.)

(vi) The Operator C(x). (See Sec. 2.11.)

(vii) The Operators of the Mueller Phenomenological Algebra (Refs. [192], [193], [194], and Sec. 137 of Ref. [251]).

3.18 Summary of the Interaction Method

The interaction method is a method of formulating radiative transfer problems by means of the interaction principle. After some preliminary examples, the steps of the method were listed following Example 2 of Sec. 3.4. The method was then extensively applied throughout the remaining part of the chapter. In this section we summarize the method as developed throughout this chapter and include the steps of Sec. 3.17 leading to the integral representation of the interaction operators used in the method. The section concludes with some observations on the relative roles played by the interaction principle in this work and in Ref. [251].

Summary of the Interaction Method

There are three main stages of the Interaction Method. Let X be an optical medium and S be a subset of X. Then:

Stage I

(i) *Isolate the subset* S *of the optical medium. If* S *is concave decide how* S *is to be convexified (Sec. 3.8).*

(ii) *Enumerate the incident radiometric quantities* a_i *on* S. *This determines* A_j, $i = 1,\ldots,m$. *(Sec. 3.2)*

(iii) *Enumerate the requisite response radiometric quantities* b_j *on* S. *This determines* B_j, $j = 1,\ldots,n$. *(Sec. 3.2)*

(iv) *Enumerate the* mn *operators* s_{ij}, $i= 1,\ldots,m$; $j = 1,\ldots,n$, *supplied by the interaction principle (Sec. 3.2).*

(v) *Write the interaction equations:*

$$b_j = \sum_{i=1}^{m} a_i s_{ij}$$

for $j = 1,\dots,n$.

(vi) *Append to (v) any auxiliary equations connecting various chosen* a_i *and* b_j *so that it is possible to algebraically solve the system of* n *functional equations in (v) for the requisite response functions* b_j. *Invariably, these auxiliary equations may be based on one or the other of the following radiometric laws:*

(a) *The radiance invariance law over a path in a vacuum (Sec. 2.6).*

(b) *The equality of field and surface radiance distributions at a point in a general optical medium (Sec. 2.5).*

Stage II *If the structure of* A, B, *and* S *indicate the possibility of an integral representation of the interaction operators* s_{ij}, *then use the technique of the interaction measures and interaction kernels of Sec. 3.16 to obtain:*

$$s_{ij} = \int_C [\ ] K_{ij}(x,y)\ d\nu(x)$$

for $i = 1,\dots,m$: $j = 1,\dots,n$.

Stage III *Determine by means of suitable functional relations the explicit structure of the* mn *interaction kernels* K_{ij}, *if they exist, and use the results in (vi) of Stage I to obtain a solution of the interaction problem.*

Remarks on the Stages of the Interaction Method

Stage I was fully illustrated in the present chapter; however, some further aspects of the details of Stage II and Stage III beyond those covered in Sec. 3.17, remain to be observed. As regards Stage III, the interaction kernels arising in homogeneous plane-parallel media and their governing functional relations have been exhaustively studied by Chandrasekhar (Ref. [43]). Further functional relations were given in Refs. [13], [14], by Bellman and Kalaba for inhomogeneous media. The functional relations for the complete set of four interactions kernels in non homogeneous one-parameter media, (i.e., the four reflectance and transmittance functions R and

T) were introduced and derived in Refs. [233] and [234]. (See also Sec. 7.1.) The functional relations governing the interaction kernel for the general operators $\mathscr{S}(X;a,b)$ were derived in Ref. [251]. The general procedures for the solution of the functional relations governing the operators R(a,b), T(a,b), R(b,a), T(b,a) for general one-parameter optical media for $\mathscr{S}(X;a,b)$ are given in Chapter 7 of this work.

Now that the conceptual structure of radiative transfer theory has been elucidated by the interaction principle, and its mathematical foundations established (ref. [251]) it remains to solve the important mathematical problems of modern radiative transfer theory centering around the functional relations governing the interaction kernels (see problem VIII, Sec. 141 of Ref. [251]).

One final remark on Stage II must be made. This concerns the AC property of an interaction measure. If the AC property is valid for a given interaction measure, then the interaction kernel of that measure is, according to Theorem B of Sec. 3.16, the Radon-Nikodym derivative of that measure. In this regard the development of interaction kernels will be occasionally simplified if the transmittance-type operators are decomposed into their residual and diffuse parts, i.e., into parts which, respectively, describe radiant flux which has not been scattered (i.e., beam transmitted) and which has been scattered. It turns out that transmittance operators for *diffuse* flux always have the AC property. (Reflectance-type operators generally have the AC property outright since they describe only diffuse flux.) The basis for these remarks rests in Ia, Ib of Sec. 23, Ref. [251], which, in the present work, may be taken as basic postulated regularity properties of interaction kernels. A model for this procedure of decomposing operators will be found in Sec. 7.1. The decomposition of the light field, which is a natural prerequisite to the decomposition of interaction operators, can easily be done in general since the concept of scattered and non scattered radiant flux is now rigorously definable by means of the path function and path radiance operators of Examples 1 and 2 of Sec. 3.17. This decomposition will be studied as a matter of course in Chapter 5. The net result of Stage II of the interaction method will be that the diffuse component of a transmittance operator (rather than the undecomposed operator) will be passed on to Stage III for the determination of its kernel. The prototype of this procedure may be found in Ref. [43], and in Refs. [234], [235].

The Interaction Method and Quantum Theory

We append here some final observations on the general methodology of the interaction method, an observation which will point up some points of similarity between the interaction method and two basic methods of solving dynamical problems in classical and modern physics. The observation is designed to be of especial interest to physicists, rather than radiative transferists per se. Nevertheless, since radiative transfer is ultimately derivable from quantum mechanics, the latter workers may peruse the following with some profit.

The first point of similarity was noted in the discussion following Example 2 of Sec. 3.4 where a comparison was made between the Newtonian laws of motion and the interaction principle, and note was made of the applicability of the method to linear hydrodynamics and general wave guide phenomena. We need not repeat it here. The second point of comparison appears to be even deeper than the first when we note the similarity between the interaction method and the formulation of the quantum mechanics of many-state atomic systems. To facilitate the comparison, the reader may consult, e.g., [92]. Here are the parallel correspondents: to an atomic or molecular system we pair an optical medium (step (i)). To the various base states of the atomic system we pair the sets of incident and response radiometric quantities (steps (ii), (iii)). To the Hamiltonian matrix of the atomic system we pair the set $(s_{ij})$of interaction operators (step (iv)). To the transition probability equation (the linear superposition of amplitude functions) we pair the interaction equation (step (v)). To the finding of either the Hamiltonian matrix (using conservation laws and auxiliary physical arguments) or S-matrix, we pair the finding of the interaction operators (step (vi) and Stages II and III). The mystery of this remarkable similarity between the quantum mechanical and radiative transfer formalisms is only apparent and is resolved by noting that *each discipline is founded (for its own particular experimental reasons) on a set of linear superposition principles. Hence both methodologies come under the single unifying framework of vector space theory.* The salient difference between the two formalisms is that the possibility of interference of amplitudes exists in quantum theory, whereas interference of radiant fluxes is ruled out by fiat from radiative transfer theory (cf. Sec. 2.2). In the preceding point by point parallelism of the mathematics of quantum theory and radiative transfer theory lie the keys to the solutions of the basic problems II, and IV in Sec. 142 of Ref. [251]. It may be noted in passing that the applications of the linear interaction principle to quantum mechanics, linear hydrodynamics, acoustics, and electromagnetic theory, e.g., introduce complex-valued interfering amplitudes, and on this level the theoretical and numerical methodologies of all these fields are strikingly alike.

The Interaction Principle as a Means and as an End

Throughout this chapter there have been several occasions to refer to the developments of radiative transfer theory in Ref. [251] and in particular to the interaction principle in that work. A few words may be in order to help place in perspective the relative roles of the interaction principle in these two works.

The interaction principle in Ref. [251] was the end of a long series of generalizations and abstractions starting mainly with the work of Schuster, on through classical principles of invariance of Ambarzumian and Chandrasekhar, and up through the principle of invariant imbedding of Bellman and Kalaba as applied to transport phenomena, and finally on to the invariant imbedding relation, and the interaction principle itself. It was shown, in particular, how all these principles could be deduced from the classical equation of transfer, and how the equation of transfer could itself be viewed as a local form of the principles of invariance. *Hence, in a word, the interaction principle was viewed in Ref. [251] as an end of a set of long conceptual and deductive trails, the main trail starting from Schuster's initial insight in 1905.* Thus in [251] the roots of interaction principle were established in the classical origins of the subject along with electromagnetic and axiomatic bases of the principle. *With this in mind we have taken the alternate view in the present chapter that the interaction principle is a basic means of formulating radiative transfer theory, a single working principle from which the salient algebraic structures of the theory may be deduced.* The thirty-eight enumerated examples throughout this chapter, starting in Sec. 3.4 and ending in Sec. 3.17, have shown that the interaction principle can indeed be used as a starting point for the construction of the principles of invariance on all types of three-dimensional media, the various classical interreflection problems of surfaces, the beam transmittance function for paths, the classical attenuation and scattering functions of the media used in the equation of transfer, and the equation of transfer itself.

Conclusion

In sum, then, the work of the monograph [251] constituted a necessary prerequisite for the establishment of the interaction principle. The present work no longer views the interaction principle as an end of research but rather as a means of application and a source of new research in radiative transfer theory and general linear transport theories (even beyond radiative transfer, as in hydrodynamics, acoustics, e. m. wave propagation, etc.). The first application of the interaction principle was to the development of the discrete-space theory of radiative transfer in Ref. [251]. These applications are continued in this chapter, and the following chapters of the present work.

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3.19 Bibliographic Notes for Chapter 3

The interaction principle as given in Sec. 3.2 was first formulated in Ref. [251], the end result of an extended series of generalizations. A historical sketch of the evolution of the main lines of radiative transfer *theory* (not its manifold applications) which are pertinent to the interaction principle is given cumulatively in the bibliographic notes for the chapters in Ref. [251]. The formulation of the interaction method, as summarized in Sec. 3.18, is new.