$$B(x) = ae^{-\alpha x} + b(1-e^{-\alpha x}) \quad ." \qquad (1)$$

The salient features of this equation, those that make it "canonical" in the technical sense, can be described in terms of the concepts developed in Chapter 3. First of all we observe that (1) has the Gestalt of (5) of Sec. 3.13, where the term $ae^{-\alpha x}$ corresponds to N_r^o in equation (5) of Sec. 3.13, the term $b(1-e^{-\alpha x})$ corresponds to N_r^*, and the term $B(x)$ to N_r. Thus $B(x)$ is interpretable as the apparent radiance of an object (Sec. 3.13) as seen over a path of length x, where the path radiance of the path is $b(1-e^{-\alpha x})$ and the inherent radiance of the object is a. The particular manner in which a, b, and $e^{-\alpha x}$ occur in the algebraic form of (1) characterize (1) as *canonical*. Equation (1) is substantially the algebraic form of B(x) deduced by Bouguer from empirical observations. According to Middleton, however, Bouguer ostensibly missed the full physical significance of the terms a and b. Hindsight and a fully developed theory now let us view a and b in quite simple terms. Thus a in (1) is the inherent radiance of the object which is transmitted over the path with beam transmittance $e^{-\alpha x}$. Hence α must be the attenuation coefficient of the path (our α of Sec. 3.11). The term b is a simple instance of the general concept of equilibrium radiance which will be introduced and studied in detail in this chapter. Physically, b is the radiance of a very long uniformly lighted homogeneous path. Mathematically, b is simply the limit of B(x) as $x\to\infty$. The radiance b is independent of location along the uniformly lighted homogeneous path, and in real life is closely approximated by the horizon radiance under suitable atmospheric conditions. The horizon radiance remains ostensibly constant, for example, on a transcontinental jet flight at 10,000 m altitude over large segments of the flight path. The observed horizon radiance seen by the jet pilot is the real counterpart to the equilibrium radiance b in (1). Of course similar interpretations of a,b and corresponding interpretations of (1) apply to horizontal lines of light in the sea, under suitable conditions.

In the present chapter we shall develop a hierarchy of canonical equations of transfer for radiance starting with the simplest of applied situations and concluding with what appears to be the most comprehensive canonical equation of transfer for physically meaningful contexts. Equation (1) will fall somewhere in the lower middle of this hierarchy, that is, somewhere in the neighborhood of the Koschmieder equation of Sec. 4.3. Throughout this chapter, unless specifically noted otherwise, all optical media will be considered emission-free, in the steady state, and of constant index of refraction. This condition does not constitute any significant loss of generality in terrestrial settings while permitting a simple exposition of the main idea of the canonical equation.

4.1 Radiance in Transparent Media

We take up first the simplest case in which the canonical equation of transfer can occur: transparent optical media. A *transparent* optical medium X is one in which $\alpha(x,\xi) = 0$ and $\sigma(x;\xi';\xi) = 0$ for every x in X and ξ',ξ in Ξ. An example of a transparent optical medium is a block of glass which does not appreciably absorb or scatter radiant energy. Under these conditions, the integral equation of transfer (2) of Sec. 3.15 associated with a path $\mathcal{P}_r(x,\xi)$ in a vacuum takes the form:

$$\boxed{N(z,\xi) = N(x,\xi)} \quad . \tag{1}$$

Where $z = x + \xi r$. This instance of the equation of transfer is clearly interpretable also as an instance of the radiance invariance law(2) of Sec. 2.6.

In the case of a transparent optical medium in which the index of refraction varies with location along $\mathcal{P}_r(x,\xi)$, the n^2-law for radiance (4) of Sec. 2.6

$$N(z,\xi)/n^2(z) = N(x,\xi)/n^2(x) \tag{2}$$

governs the magnitude of $N(z,\xi)$ along $\mathcal{P}_r(x,\xi)$.

The preceding two laws also can be made to follow from the appropriate integrodifferential form of the equation of transfer. This would be equation II of Sec. 21 in Ref. [251], which in turn is deducible from the interaction principle. Thus we would deduce from this equation that

$$\boxed{\frac{d\ N(x,\xi)/n^2(x)}{dr} = 0} \quad , \tag{3}$$

from which follows (2). Equation (3) of Sec. 3.15 yields in particular:

$$\frac{dN(x,\xi)}{dr} = 0 \tag{4}$$

for the case of a transparent medium with constant index of refraction. From this follows (1). Clearly (4) is a special case of (3), so that (3) may be considered the basic equation for radiative transfer in transparent media.

<< Table of Contents << Title Page >> Next Page >>

4.2 Radiance in Absorbing Media

The next simplest case of an optical medium containing a radiative transfer process is that of a purely absorbing medium. A *purely absorbing* optical medium X is one in which $\sigma(x;\xi',\xi) = 0$ for every x in X and ξ',ξ in Ξ. An everyday example of a purely absorbing medium is a uniformly exposed photographic negative. By holding such a negative to the eye and viewing one's surroundings through it, the principal radiative transfer feature of a purely absorbing medium is readily perceived: Such media characteristically *decrease* the radiance of a scene by a factor which depends only on the inherent optical and geometric makeup of the medium and which does not depend on the surrounding light field. If the absorption properties of an optical medium X are uniform throughout