

media. A *transparent* optical medium X is one in which $\alpha(x, \xi) = 0$ and $\sigma(x; \xi'; \xi) = 0$ for every x in X and ξ', ξ in Ξ . An example of a transparent optical medium is a block of glass which does not appreciably absorb or scatter radiant energy. Under these conditions, the integral equation of transfer (2) of Sec. 3.15 associated with a path $\mathcal{Q}_T(x, \xi)$ in a vacuum takes the form:

$$\boxed{N(z, \xi) = N(x, \xi)} \quad (1)$$

Where $z = x + \xi r$. This instance of the equation of transfer is clearly interpretable also as an instance of the radiance invariance law(2) of Sec. 2.6.

In the case of a transparent optical medium in which the index of refraction varies with location along $\mathcal{Q}_T(x, \xi)$, the n^2 -law for radiance (4) of Sec. 2.6

$$N(z, \xi)/n^2(z) = N(x, \xi)/n^2(x) \quad (2)$$

governs the magnitude of $N(z, \xi)$ along $\mathcal{Q}_T(x, \xi)$.

The preceding two laws also can be made to follow from the appropriate integrodifferential form of the equation of transfer. This would be equation II of Sec. 21 in Ref. [251], which in turn is deducible from the interaction principle. Thus we would deduce from this equation that

$$\boxed{\frac{d N(x, \xi)/n^2(x)}{dr} = 0} \quad (3)$$

from which follows (2). Equation (3) of Sec. 3.15 yields in particular:

$$\frac{dN(x, \xi)}{dr} = 0 \quad (4)$$

for the case of a transparent medium with constant index of refraction. From this follows (1). Clearly (4) is a special case of (3), so that (3) may be considered the basic equation for radiative transfer in transparent media.

4.2 Radiance in Absorbing Media

The next simplest case of an optical medium containing a radiative transfer process is that of a purely absorbing medium. A *purely absorbing* optical medium X is one in which $\sigma(x; \xi', \xi) = 0$ for every x in X and ξ', ξ in Ξ . An everyday example of a purely absorbing medium is a uniformly exposed photographic negative. By holding such a negative to the eye and viewing one's surroundings through it, the principal radiative transfer feature of a purely absorbing medium is readily perceived: Such media characteristically *decrease* the radiance of a scene by a factor which depends only on the inherent optical and geometric makeup of the medium and which does not depend on the surrounding light field. If the absorption properties of an optical medium X are uniform throughout

X , then the factor of the observed decrease is a simple exponential factor $\exp \{-\alpha r\}$ depending only on the attenuation coefficient α and the length r of one's path of sight through the medium. In particular no light from the surrounds of the path will be added to that of the path. Indeed, if the universe were made up only of absorbing material, radiative transfer theory beyond the use of the exponential function would not exist, so simple and straightforward is the form of (1) of Sec. 3.15 when reduced to pure absorption case:

$$N_r(z, \xi) = N_o(x, \xi) T_r(x, \xi) \quad (1)$$

Equivalently (3) of Sec. 3.15 reduces to:

$$\frac{dN(z, \xi)}{dr} = -\alpha(z, \xi)N(z, \xi) \quad (2)$$

However, in all real media, absorption mechanisms are accompanied by scattering mechanisms in the radiative processes within such media. Hence, the losses summarized by the volume attenuation function α include scattering losses in addition to the absorption losses. The losses due to scattering at a typical point of a path $\mathcal{Q}_r(x, \xi)$ in general optical medium X are readily characterized using the volume scattering function of Sec. 3.14. Indeed the integral:

$$\int_{\Xi} \sigma(z; \xi; \xi') d\Omega(\xi')$$

represents the total radiance loss by a beam of given wavelength and unit radiance, under scattering without change in wavelength (elastic scatter) and per unit length at z , along the direction ξ of the path $\mathcal{Q}_r(x, \xi)$ at that point. This interpretation follows readily from the developments in Sec. 3.14.

Let us write:

$$"s(z, \xi)" \text{ for } \int_{\Xi} \sigma(z; \xi; \xi') d\Omega(\xi') \quad (3)$$

We call s the *volume total scattering function on X* . Further, let us write:

$$"a(z, \xi)" \text{ for } \alpha(z, \xi) - s(z, \xi) \quad (4)$$

so that:

$$\alpha(z, \xi) = a(z, \xi) + s(z, \xi)$$

We call the function a which assigns to each point z on $\mathcal{Q}_r(x, \xi)$ the value $a(z, \xi)$, the *volume absorption function on X* . The interpretation of $a(z, \xi)$ is straightforward: it represents the loss of radiance per unit length at point z

on $\mathcal{O}_r(x, \xi)$ of a beam of unit radiance, the loss being due to two physical mechanisms: (i) the scattering of some of the incident radiant flux into radiant flux of a different wavelength than that of the incident beam (*inelastic scatter or transpectral scatter*); (ii) the conversion of some of the incident radiant flux into non-radiant energy (*true absorption*). Some forms of non-radiant energy pertinent here are: the potential energy of higher stationary states in atomic systems, and the kinetic energy of motion of the molecules of the optical medium. Since $\alpha(z, \xi)$ represents losses due to all the mechanisms namely elastic scatter, inelastic scatter, and true absorption, we expect on physical grounds that $a(z, \xi)$ is nonnegative for every z and ξ in its domain of definition, and we hypothesize the appropriate inequality to hold henceforth between α and s so that this nonnegativity of $a(z, \xi)$ is the case.

It is worthwhile to bring explicitness to the reader's attention the particular role played by the volume absorption function in radiative transfer theory. The function plays the role of a catchall of all radiant flux losses undergone by a beam of radiant flux other than by the mechanism of elastic scatter. The two fundamental (or primary) optical properties of a medium X are α and σ . The concept a as defined in (4) is a secondary property, that is, one that is derived from α and σ as shown. The secondary nature of the concept a follows from the fact that in practice absorption cannot be observed directly, but only indirectly by means of monitoring the initial and final states of a beam in transmission and scattering arrangements in experimental settings.

Using the definition (4) of the function a , we can write (1) or (2) in the form:

$$N_r(z, \xi) = N_0(x, \xi) \exp \left\{ - \int_0^r a(x', \xi) dr' \right\} \quad (5)$$

$$\frac{dN(z, \xi)}{dr} = -a(x, \xi) N(x, \xi)$$

where the integration is along the path $\mathcal{O}_r(x, \xi)$ with $z = x + r\xi$ (see Fig. 3.33).

4.3 Koschmieder's Equation for Radiance

A classical problem of radiative transfer theory in either the atmosphere or in the sea is to determine the apparent radiance of an object as seen along a path of sight $\mathcal{O}_r(x, \xi)$ which lies in a homogeneous and uniformly lighted region of an optical medium. Specifically, the problem is to determine the apparent radiance $N_r(z, \xi)$ given α and σ along $\mathcal{O}_r(x, \xi)$, and $N_0(x, \xi)$ at the initial endpoint x of the path, along with the fact that each point of $\mathcal{O}_r(x, \xi)$ is irradiated by the same radiance distribution (which may, however, depend arbitrarily on ξ'). This situation (or some reasonable approximation of it) arises often in the atmosphere and the sea, notably along horizontal paths of sight,