

on $\mathcal{O}_r(x, \xi)$ of a beam of unit radiance, the loss being due to two physical mechanisms: (i) the scattering of some of the incident radiant flux into radiant flux of a different wavelength than that of the incident beam (*inelastic scatter or transpectral scatter*); (ii) the conversion of some of the incident radiant flux into non-radiant energy (*true absorption*). Some forms of non-radiant energy pertinent here are: the potential energy of higher stationary states in atomic systems, and the kinetic energy of motion of the molecules of the optical medium. Since $\alpha(z, \xi)$ represents losses due to all the mechanisms namely elastic scatter, inelastic scatter, and true absorption, we expect on physical grounds that $a(z, \xi)$ is nonnegative for every z and ξ in its domain of definition, and we hypothesize the appropriate inequality to hold henceforth between α and s so that this nonnegativity of $a(z, \xi)$ is the case.

It is worthwhile to bring explicitness to the reader's attention the particular role played by the volume absorption function in radiative transfer theory. The function plays the role of a catchall of all radiant flux losses undergone by a beam of radiant flux other than by the mechanism of elastic scatter. The two fundamental (or primary) optical properties of a medium X are α and σ . The concept a as defined in (4) is a secondary property, that is, one that is derived from α and σ as shown. The secondary nature of the concept a follows from the fact that in practice absorption cannot be observed directly, but only indirectly by means of monitoring the initial and final states of a beam in transmission and scattering arrangements in experimental settings.

Using the definition (4) of the function a , we can write (1) or (2) in the form:

$$N_r(z, \xi) = N_0(x, \xi) \exp \left\{ - \int_0^r a(x', \xi) dr' \right\} \quad (5)$$

$$\frac{dN(z, \xi)}{dr} = -a(x, \xi) N(x, \xi)$$

where the integration is along the path $\mathcal{O}_r(x, \xi)$ with $z = x + r\xi$ (see Fig. 3.33).

4.3 Koschmieder's Equation for Radiance

A classical problem of radiative transfer theory in either the atmosphere or in the sea is to determine the apparent radiance of an object as seen along a path of sight $\mathcal{O}_r(x, \xi)$ which lies in a homogeneous and uniformly lighted region of an optical medium. Specifically, the problem is to determine the apparent radiance $N_r(z, \xi)$ given α and σ along $\mathcal{O}_r(x, \xi)$, and $N_0(x, \xi)$ at the initial endpoint x of the path, along with the fact that each point of $\mathcal{O}_r(x, \xi)$ is irradiated by the same radiance distribution (which may, however, depend arbitrarily on ξ'). This situation (or some reasonable approximation of it) arises often in the atmosphere and the sea, notably along horizontal paths of sight,

and the reader should be able to cite many personally observed instances of it. Koschmieder studied this classical setting in detail, and in 1924 published in [141] his analytic expression for $N_r(z, \xi)$ which was derived after lengthy preliminaries and under the radiometric conditions stipulated above. We turn now to a modern derivation of the expression for $N_r(z, \xi)$.

Returning to (1) of Sec. 3.15 we assume α and σ are independent of z along $\mathcal{P}_r(x, \xi)$. Then:

$$T_r(x, \xi) = e^{-\alpha r}$$

where " α " denotes the assumed fixed value of the volume attenuation function along $\mathcal{P}_r(x, \xi)$. Furthermore, since the radiance distribution $N(z, \cdot)$ is independent of z along the path then $N_*(z, \xi)$ is also independent of z along the path and we shall abbreviate this fixed value by " N_* ". Equation (1) of Sec. 3.15 then reduces to:

$$N_r(z, \xi) = N_0(x, \xi)e^{-\alpha r} + N_* \int_0^r e^{-\alpha(r-r')} dr'$$

and with the abbreviations " N_r " for $N_r(z, \xi)$ and " N_0 " for $N_0(x, \xi)$, this simplifies immediately to:

$$\boxed{N_r = N_0 e^{-\alpha r} + N_q (1 - e^{-\alpha r})} \quad (1)$$

where we have written:

$$"N_q" \text{ for } N_*/\alpha \quad (2)$$

Equation (1) is *Koschmieder's equation* which relates apparent radiance N_r to N_0 on a path \mathcal{P}_r in an optical medium along which α and σ are constant valued and along which the value N_* of the path function is constant. The radiance N_q is called the *equilibrium radiance* for \mathcal{P}_r . The significance of N_q is seen by letting $r \rightarrow \infty$ in (1), or alternately by contemplating the integrodifferential equation for N_r associated with \mathcal{P}_r as given in (3) of Sec. 3.15:

$$\frac{dN_r}{dr} = -\alpha N_r + N_* \quad (3)$$

Under our present assumptions, (3) is a relatively innocuous first order differential equation in which α and N_* are constants and N_r is the unknown function. Using (2) we can rewrite (3) as:

$$\frac{dN_r}{dr} = \alpha(N_q - N_r) \quad (4)$$

from which we can immediately read the physical significance of N_q : If $N_r < N_q$ at a point on the path, then $dN_r/dr > 0$, i.e., N_r is increasing at that point. In general, N_r always tends toward the fixed radiance N_q , and $dN_r/dr = 0$ if and only if $N_r = N_q$. Therefore N_q takes on the aspect of an equilibrium value (in an every day sense) toward which the values N_r unceasingly tend. The equilibrium radiance N_q is often observable over long horizontal uniformly lighted paths through a homogeneous natural aerosol or hydrosol.

It should be observed that the derivation of (1) places no conditions on the orientation or the location of the path \mathcal{P}_r in an optical medium. The essential point to observe in the derivation is that (1) follows from (1) of Sec. 3.15 upon assuming only that α , σ and N_* are constant long \mathcal{P}_r . This leaves \mathcal{P}_r free to be vertical, inclined, or horizontal, as the case may be. An interesting example of (1) for inclined paths of sight in the atmosphere may be obtained from the results in [71].

4.4 The Classical Canonical Equation

In this section we continue to ascend the ladder of generality and derive still further instances of canonical radiance equations. We still have not reached the most general physical setting in which the canonical equation can hold, but we have reached the point where the full canonical structure of the equation finally emerges, and we turn now to the derivation of that form.

Let $\mathcal{P}_r(x, \xi)$ be an arbitrary line of sight in a homogeneous optical medium X . To fix ideas, let the medium X be a natural hydrosol, and let us adopt the standard coordinate frame for such a setting (Sec. 2.4 and Fig. 2.3). Let $\mathcal{P}_r(x, \xi)$ be positioned as shown in Fig. 4.1.

With the geometry fixed as in Fig. 4.1, we now assume α and σ to be independent of location along the generally inclined path $\mathcal{P}_r(x, \xi)$, and that the light field does not vary over a given horizontal plane, i.e., the light field is stratified. The new feature of the canonical equation appears by assuming that there exists a nonnegative real number K (which is less than α) such that:

$$N_*(z, \xi) = N_*(z_0, \xi) e^{-K(z-z_0)} \quad (1)$$

for every path $\mathcal{P}_r(x, \xi)$ in X . This means that we are hypothesizing an exponential decrease of $N_*(z, \xi)$ with depth z in X . The justification for this assumption rests on both experimental and theoretical grounds. For an experimental justification, see Sec. 1.2; for theoretical justifications see Secs. 1.3, 7.10, 8.5, 8.6 and Sec. 10.7. For the present, we are concerned primarily with the resultant form of (6) of Sec. 3.13 to which this assumption leads us. Thus starting with (6) of Sec. 3.13, we have:

$$N_r(z, \xi) = N_o(x, \xi) T_r(x, \xi) + \int_0^r N_*(x', \xi) T_{r-r'}(x', \xi) dr'$$